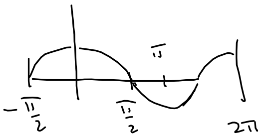
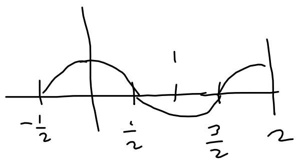
S5.1 #19 in book.

Area between $y = \cos(\pi x) = y = 4x^2 - 1$

$\cos(x)$



$\cos(\pi x)$

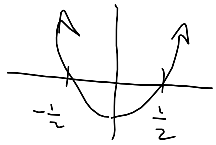


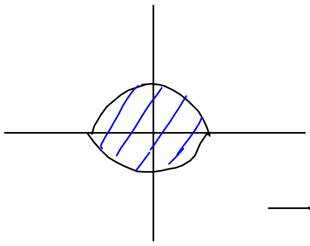
$y = 4x^2 - 1 = 0$

$1 = 4x^2$

$x^2 = \frac{1}{4}$

$x = \pm\frac{1}{2}$

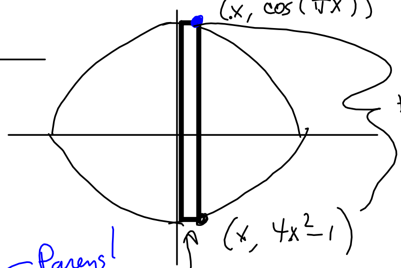




$\int_a^b (\text{Higher} - \text{Lower})\,dx$

$(x, \cos(\pi x))$

Height is $(\cos(\pi x) - 4x^2 - 1)$



$(x, 4x^2 - 1)$

Parens!

width is $\Delta x = dx$

$$\text{Area} = \int_{-\frac{1}{2}}^{\frac{1}{2}} (\cos(\pi x) - (4x^2 - 1))\,dx$$

= Note even functions $= 2\int_0^{\frac{1}{2}} (\cos(\pi x) - (4x^2-1))\,dx$

$$= 2\left[\frac{1}{\pi}\int_0^{\frac{1}{2}} \cos(\pi x)\cdot\pi\,dx - \int_0^{\frac{1}{2}} (4x^2-1)\,dx\right] =$$

$u = \pi x$

$du = \pi\,dx$

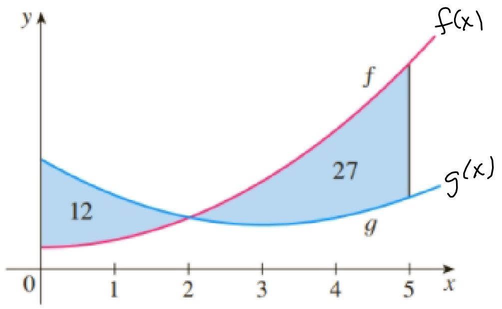
$$= 2\left[\frac{1}{\pi}\sin(\pi x)\right]_0^{\frac{1}{2}} - 2\left[\frac{4}{3}x^3 - x\right]_0^{\frac{1}{2}} = \frac{2}{\pi}[1 - 0] - 2\left[\left(\frac{4}{3}\left(\frac{1}{8}\right) - \frac{1}{2}\right) - 0\right]$$

$$= \frac{2}{\pi} - 2\left[\frac{1}{6} - \frac{1}{2}\right] = \frac{2}{\pi} - 2\left[\frac{1}{6} - \frac{3}{6}\right] = \frac{2}{\pi} - 2\left[-\frac{2}{6}\right] = \boxed{\frac{2}{\pi} + \frac{2}{3}}$$

29. The graphs of two functions are shown with the areas of the regions between the curves indicated.

(a) What is the total area between the curves for $0 \leqslant x \leqslant 5$?

(b) What is the value of $\int_0^5 [f(x) - g(x)]\,dx$?



Area between is

$$\int_0^2 (g(x)-f(x))\,dx + \int_2^5 (f(x)-g(x))\,dx$$

(a) $= 12+27 = \boxed{39}$

(b) $= -12+27 = \boxed{15}$

35–36 Evaluate the integral and interpret it as the area of a region. Sketch the region.

35. $\int_0^{\pi/2} |\sin x - \cos 2x|\, dx$ **36.** $\int_0^4 |\sqrt{x+2} - x|\, dx$

Need a picture & some analysis!

(36)

$\sqrt{x+2} = x$

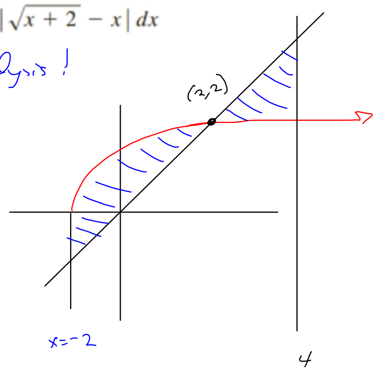
$x+2 = x^2$

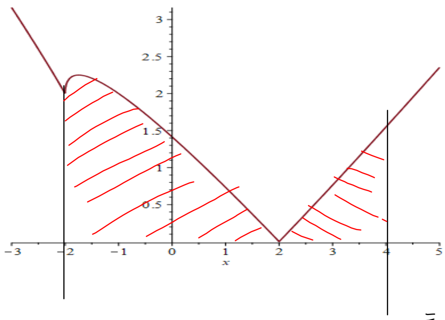
$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x \in \{-1, 2\}$

$\sqrt{-1+2} = \sqrt{1} = 1 \neq -1$





By hand:

$$\int_{-2}^{4} |\sqrt{x+2} - x|\, dx$$

$$= \int_{-2}^{2} (\sqrt{x+2} - x)\, dx + \int_{2}^{4} (x - \sqrt{x+2})\, dx$$

$u = x+2$

$du = dx$

$y = x$ is odd, $[-2,2]$ is balanced interval.

$$= \int_{x=-2}^{x=2} u^{\frac{1}{2}} du - \int_{-2}^{2} x\, dx$$

$$+ \int_2^4 x - \int_{x=2}^{x=4} u^{\frac{1}{2}} du = \frac{2}{3}u^{\frac{3}{2}}\Big]_{-2=x}^{2=x} - 0 + \frac{x^2}{2}\Big]_2^4 - \frac{2}{3}u^{\frac{3}{2}}\Big]_{x=2}^{x=4}$$

$$= \frac{2}{3}\sqrt{x+2}\Big]_{-2}^{2} + \left[\frac{4^2}{2} - \frac{2^2}{2}\right] - \frac{2}{3}\sqrt{x+2}\Big]_2^4$$

$$= \frac{2}{3}\left[\sqrt{2+2} - \sqrt{-2+2}\right] + [8-2] - \frac{2}{3}\left[\sqrt{4+2} - \sqrt{2+2}\right]$$

$$= \frac{2}{3}\left[\sqrt{4}\right] + 6 - \frac{2}{3}\left[\sqrt{6} - \sqrt{4}\right] = \frac{4}{3} + 6 - \frac{2}{3}\left[\sqrt{6} - 2\right]$$

$$= \frac{4+18}{3} - \frac{2}{3}\sqrt{6} + \frac{4}{3} = \boxed{\frac{26}{3} - \frac{2\sqrt{6}}{3}}$$

$$\int_{-2}^{4} |\sqrt{x+2} - x|\, dx = \int_{-2}^{2} (\sqrt{x+2} - x)\, dx + \int_2^4 (x - \sqrt{x+2})\, dx$$

$$= \left[\frac{(x+2)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^2}{2}\right]_{-2}^{2} + \left[\frac{x^2}{2} - \frac{2}{3}(x+2)^{\frac{3}{2}}\right]_2^4$$

$$= \left(\frac{2}{3}(4)^{\frac{3}{2}} - \cancel{\frac{4}{2}}\right) - \left(\frac{2}{3}(0)^{\frac{3}{2}} - \cancel{\frac{4}{2}}\right) + \left[\underline{\left(\frac{4^2}{2} - \frac{2}{3}(6)^{\frac{3}{2}}\right) - \left(\frac{2^2}{2} - \frac{2}{3}(4)^{\frac{3}{2}}\right)}\right]$$

$$= \frac{2}{3}(2)^3 + 8 - \frac{2}{3}\cdot 6\sqrt{6} - \left(2 - \frac{2}{3}\cdot 8\right)$$

$$= \frac{16}{3} + \frac{24}{3} - \frac{12}{3}\sqrt{6} - \left(\frac{6-16}{3}\right) = \frac{40}{3} - 4\sqrt{6} - \left(-\frac{10}{3}\right) = \boxed{\frac{50}{3} - 4\sqrt{6}}$$

The faster you work, the slower you go, sometimes! WRITE MUCH, THINK LITTLE.

$$(\sqrt{6})^3 = \sqrt{6^3} = \sqrt{6^2 \cdot 6} = 6\sqrt{6}$$

Poen for questions, until 4:10.

Will open for questions again at 5:00.