§4.3

19. 0/1 points

Find the derivative of the function.

$$g(x) = \int_{3x}^{4x} \frac{u^2 - 1}{u^2 + 1}\, du \qquad \left[\textit{Hint}: \int_{3x}^{4x} f(u)\, du = \int_{3x}^{0} f(u)\, du + \int_{0}^{4x} f(u)\, du\right]$$

$g'(x) = $ ☐ ✗ $\boxed{-3 \cdot \dfrac{9x^2 - 1}{9x^2 + 1} + 4 \cdot \dfrac{16x^2 - 1}{16x^2 + 1}}$

$$\int_a^b = \int_a^0 + \int_0^b = -\int_0^a + \int_0^b$$

$$g(x) = \int_{3x}^{4x} = \int_{3x}^{0} + \int_0^{4x} = -\int_0^{3x} \frac{u^2-1}{u^2+1}\,du + \int_0^{4x} \frac{u^2-1}{u^2+1}\,du$$

$$= -\left(\frac{(3x)^2-1}{(3x)^2+1}\right)(3) + \left(\frac{(4x)^2-1}{(4x)^2+1}\right)(4)$$

$$= -\left(\frac{9x^2-1}{9x^2+1}\right)(3) + \left(\frac{16x^2-1}{16x^2+1}\right)(4) = -\frac{27x^2-3}{9x^2+1} + \frac{64x^2-4}{16x^2+1}$$

$$= \frac{3-27x^2}{9x^2+1} + \frac{64x^2-4}{16x^2+1}$$

20. 0/1 points

Find the derivative of the function.

$$g(x) = \int_{\tan x}^{4x^2} \frac{1}{\sqrt{5 + t^3}}\, dt$$

$g'(x) =$ ✗ $\boxed{-\frac{\sec^2(x)}{\sqrt{5+\tan^3(x)}} + \frac{8x}{\sqrt{5+64x^6}}}$

MOAR §4.3

FTC I

$$= -\int_0^{\tan(x)} \frac{1}{\sqrt{5+t^3}}\,dt + \int_0^{4x^2} \frac{1}{\sqrt{t^3+5}}\,dt$$

$$= -\left(\frac{1}{\sqrt{\tan^3(x)+5}}\right)\left(\sec^2(x)\right) + \left(\frac{1}{\sqrt{(4x^2)^3+5}}\right)(8x)$$

$$= -\sec^2(x)\cdot\frac{1}{\sqrt{\tan^3(x)+5}} + 8x\cdot\frac{1}{\sqrt{64x^6+5}}$$

$$= \frac{-\sec^2(x)}{\sqrt{\tan^3(x)+5}} + \frac{8x}{\sqrt{64x^6+5}}$$

§4.4

$$\int \sqrt[8]{x^9}\,dx = \int x^{9/8}\,dx = \frac{x^{\frac{17}{8}}}{\frac{17}{8}} + C = \boxed{\frac{8}{17}x^{\frac{17}{8}} + C}$$

Power Rule!

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (\text{If } n \neq -1)$$

$n = -1$ is for §6.3 or §6.4

Last Section for Calc I.

6. 0/1 points

Find the general indefinite integral. (Use C for the constant of integration.)

$$\int (u+1)(2u+7)\,du = \int (2u^2 + 9u + 7)\,du$$

$$= \boxed{\frac{2u^3}{3} + \frac{9u^2}{2} + 7u + C}$$

7. 0/1 points

Find the general indefinite integral. (Use C for the constant of integration.)

$$\int \sec(t)(8\sec(t) + 9\tan(t))\,dt$$

$C + 8\tan(t) + 9\sec(t)$

$$= \int \left(8\sec^2(t) + 9\sec(t)\tan(t)\right)dt = 8\tan(t) + \sec(t) + C$$

From knowledge about trig derivatives:

$\frac{d}{dx}[\sin(x)] = \cos(x)$

$\frac{d}{dx}[\cos(x)] = -\sin(x)$

$\frac{d}{dx}[\tan(x)] = \sec^2(x)$

$\frac{d}{dx}[\csc(x)] = -\csc(x)\cot(x)$

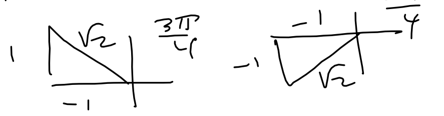
$\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$

$\frac{d}{dx}[\cot(x)] = -\csc^2(x)$

$$\int \frac{\sin(2x)}{\sin(x)}\,dx$$

MOAR §4.4

$$= \int \frac{2\cancel{\sin(x)}\cos(x)}{\cancel{\sin(x)}}\,dx = \int 2\cos(x)\,dx = 2\sin(x) + C$$



What's wrong with this:

$$\int_{\frac{3\pi}{4}}^{\frac{5\pi}{4}} \frac{\sin(2x)}{\sin(x)}\,dx = 2\sin(x)\Big]_{\frac{3\pi}{4}}^{\frac{5\pi}{4}} = 2\sin\left(\tfrac{5\pi}{4}\right) - 2\sin\left(\tfrac{3\pi}{4}\right)$$

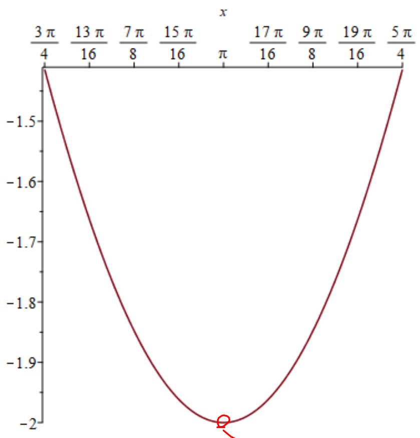
$$= 2\left(-\tfrac{1}{\sqrt{2}}\right) - 2\left(\tfrac{1}{\sqrt{2}}\right)$$

$$= -4\cdot\tfrac{1}{\sqrt{2}} = -\tfrac{4}{\sqrt{2}} = \tfrac{-4\sqrt{2}}{2} = \boxed{-2\sqrt{2}}$$

?

Problem is $\sin(x) = 0$ @ $x = \pi \in \left[\frac{3\pi}{4}, \frac{5\pi}{4}\right]$, where $\frac{\sin(2x)}{\sin(x)}$ $\nexists$. FTC II says we NEED it CONTINUOUS!

This IS the proper value of the definite integral, because a hole at one point doesn't change the area under the curve! ADVANCED CALCULUS!



Hole RIGHT Here! $\frac{\sin(2x)}{\sin(x)}$ $\nexists$ @ $x = \pi$!

$$\lim_{x\to\pi} \frac{\sin(2x)}{\sin(x)} = -2$$

10. 0/1 points

Evaluate the integral.

$$\int_1^4 \left(\frac{1}{x^2} - \frac{8}{x^3}\right) dx = \int_1^4 \left[x^{-2} - 8x^{-3}\right] dx = \left[\frac{x^{-1}}{-1} - 8 \cdot \frac{x^{-2}}{-2}\right]_1^4$$

$$= \left[-\frac{1}{x} + \frac{4}{x^2}\right]_1^4 = \left(-\frac{1}{4} + \frac{4}{4^2}\right) - \left(-\frac{1}{1} + \frac{4}{1^2}\right) = \left(-\frac{1}{4} + \frac{1}{4}\right) - (-1+4)$$

$$= \boxed{-3}$$

Try This One: $\int_{-1}^{2} \left(\frac{1}{x^2} - \frac{8}{x^3}\right) dx =$

What's wrong with this?

$\frac{1}{x^2}$ & $\frac{8}{x^3}$ aren't cont's @ $x = 0 \in [-1, 2]$,

so FTC II DNA.

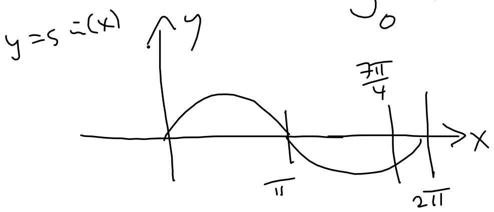
One LIKE #14: (§4.4)

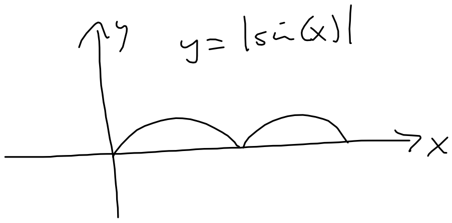
$$\int_{-1}^{3} 5|x|\,dx = \int_{-1}^{0} 5(-x)\,dx + \int_{0}^{3} 5x\,dx, \text{ b/c}$$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Manage $|f(x)|$ inside the integral!

What about $\displaystyle\int_{0}^{\frac{7\pi}{4}} |\sin(x)|\,dx = \int_{0}^{\pi} \sin(x)\,dx + \int_{\pi}^{\frac{7\pi}{4}} -\sin(x)\,dx$



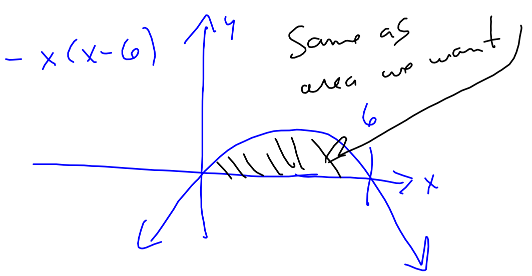


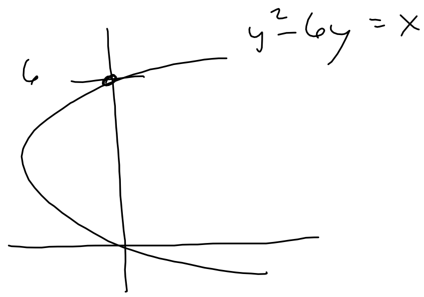
15. 0/1 points SCalc8 4.4.045. [3353673]

The area of the region that lies to the right of the y-axis and to the left of the parabola $x = 6y - y^2$ (the shaded region in the figure) is given by the integral $\int_0^6 (6y - y^2)\,dy$. (Turn your head clockwise and think of the region as lying below the curve $x = 6y - y^2$ from $y = 0$ to $y = 6$.) Find the area of the region.

36

$$6y - y^2 = -(y^2 - 6y) = -y(y-6)$$





$$\int_0^6 (6y - y^2)\,dy =$$

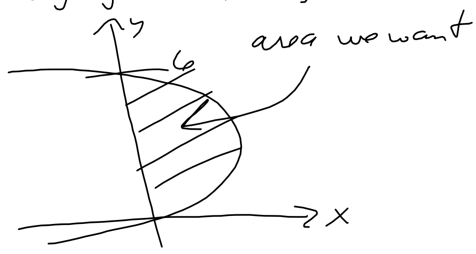
$$= \left[6 \cdot \frac{y^2}{2} - \frac{y^3}{3}\right]_0^6$$

$$= \left[3y^2 - \frac{1}{3}y^3\right]_0^6$$

$$= 3(6)^2 - \frac{1}{3}(6)^3$$

$$= 108 - \frac{1}{3}(216) = 108 - 72 = \boxed{36}$$

$$x = 6y - y^2 = -(y^2 - 6y)$$



19. 0/1 points SC

If $f(x)$ is the slope of a trail at a distance of x miles from the start of the trail, what does $\int_4^9 f(x)dx$ represent?

- The elevation at $x = 4$ miles from the start of the trail.
- The change in the elevation between $x = 4$ miles and $x = 9$ miles from the end of the trail.
- The elevation at $x = 9$ miles from the end of the trail.
- The elevation at $x = 9$ miles from the start of the trail.
- The change in the elevation between $x = 4$ miles and $x = 9$ miles from the start of the trail.

NET CHANGE

F= FORCE on an object moving in a line as a function of position x, then

$$\int_4^{10} F(x)dx = \text{work done between } x=4 \text{ \& } x=10.$$