

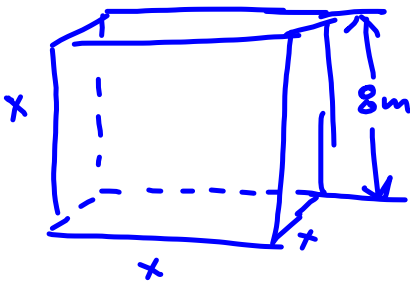
Finishing a "Final Test Review" video  
that covers Tests 1 thru 3.

Test 4 & 5 material pretty exhaustively  
covered in last few weeks.

Related Rates is big deal & tough

Differentials . . . . .

Paint a cube with .1 cm coat.  
How much paint?



cm - vs - m

$$dV$$

$$V = x^3$$

$$dV = 3x^2 dx$$

$$\Delta V \approx dV = 3(800 \text{ cm}^2)(.1 \text{ cm})$$

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Estimate volume change when  
the radius increases from 10cm to 10.1cm

$$V = \frac{4}{3}\pi r^3$$

$$dV = 4\pi r^2 dr$$

$$\Delta V \approx dV = 4\pi (10)^2 (.1)$$


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Estimate the change between  $f(9)$  and  $f(9.1)$   
for  $f(x) = \sqrt{x}$  with a differential.

$$f = \sqrt{x} = x^{\frac{1}{2}}$$

$$df = \frac{1}{2}x^{-\frac{1}{2}} dx$$

$$f'(x_1)(x-x_1)$$

$$\Delta f \approx df = \frac{1}{2}(9)^{-\frac{1}{2}}(.1) = \frac{1}{6} \cdot .1$$

Estimate  $\sqrt{9.1}$  with tangent line.

$$f(x) = \sqrt{x} = x^{\frac{1}{2}} \quad \rightarrow \text{Linearization}$$

$$x_1 = 9$$

$$f(x_1) = y_1 = \sqrt{9} = 3$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$$

$$f'(x_1) = \frac{1}{2}(9)^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

$$y = m(x - x_1) + y_1 = f'(x_1)(x - x_1) + f(x_1)$$

$$y = \frac{1}{6}(x - 9) + 3$$

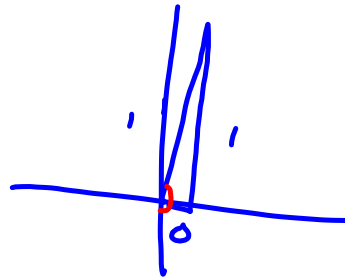
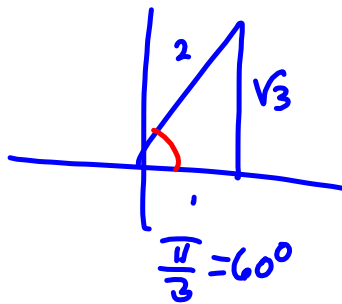
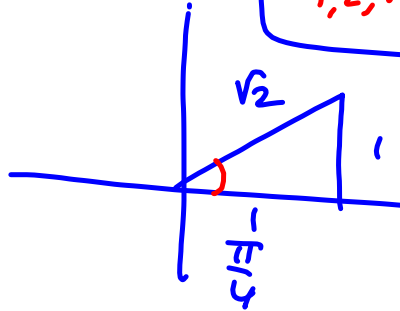
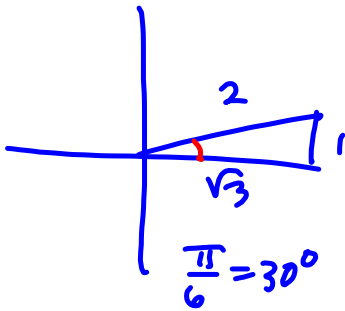
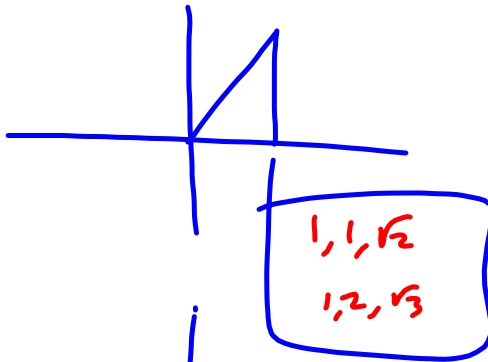
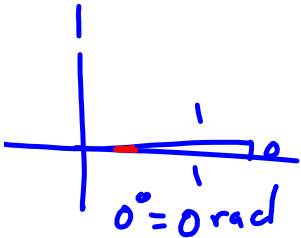
$$y(9.1) = \frac{1}{6}(9.1 - 9) + 3$$

$$= \frac{1}{6}(.1) + 3$$

$$\sin(47^\circ) \quad x_1 = 45^\circ = \frac{\pi}{4}$$

$$\Delta x = 2^\circ = (2) \left( \frac{\pi}{180} \right) = \frac{\pi}{90}$$

$$\sec\left(\frac{\pi}{3}\right)$$



$$\begin{array}{cccccc}
 & & & & & & 1 \\
 & & & & & 1 & 1 \\
 & & & 1 & 2 & 1 & \\
 & & 1 & 3 & 3 & 1 & \\
 & 1 & 4 & 6 & 4 & 1 & \\
 1 & 5 & 10 & 10 & 5 & 1 & 
 \end{array}$$

$$(3x-7y)^5 =$$

$$\begin{aligned}
 & 1(3x)^5(-7y)^0 + 5(3x)^4(-7y)^1 + 10(3x)^3(-7y)^2 \\
 & + 10(3x)^2(-7y)^3 + 5(3x)(-7y)^4 + 1(3x)^0(-7y)^5
 \end{aligned}$$

$$\begin{aligned}
 = & 3^5 x^5 y^0 - 5(3^4)(7)x^4 y + 10(3^3)(7)^2 x^3 y^2 \\
 & - 10(3^2)(7^3)x^2 y^3 + 5(3)(7^4)x y^4 - 7^5 y^5
 \end{aligned}$$