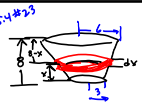
§5.4#23



Full of water
$62.5\ \text{lb}/\text{ft}^3$

Pumping these circular "slices" of water to top of tank

Distance $= 8-x$

$F = (62.5\ \text{lb}/\text{ft}^3)(\text{Volume})$

$= (62.5\ \text{lb}/\text{ft}^3)(\pi r^2 \Delta x)$

$= 62.5\pi r^2 \Delta x$ lbs

$= 62.5\pi \left(\frac{3}{8}x+3\right)^2 \Delta x$

$r = 3$ when $x=0$
$r = 6$ when $x=8$
$(0,3), (8,6)$
$m = \frac{6-3}{8} = \frac{3}{8}$
$r = \frac{3}{8}(x-0)+3$
$= \frac{3}{8}x+3$

Work $=$ $FD = 62.5\pi\left(\frac{3}{8}x+3\right)^2 \Delta x \cdot (8-x)$

what a mess!

$$\text{Work} = \int_0^8 62.5\pi \left(\frac{3}{8}x+3\right)^2 (8-x)\,dx$$

Maple says the integral evaluates as follows:

$$62.5 \cdot \text{Pi} \cdot \int_0^8 \left(\frac{3}{8}\cdot x+3\right)^2 \cdot (8-x)\,dx = 33000.0\,\pi \approx 1.036725576 \times 10^5 \text{ ft-lbs}$$

Book says 1.04×10^5, which is in agreement. But there's something(s) wrong with my work, below.

$$= 62.5\pi\left[\int_0^8 8\left((3/8)x+3\right)^2 dx - \int_0^8 x\left(\frac{9}{64}x^2+\frac{9}{4}x+9\right)dx\right]$$

$u = \frac{3}{8}x+3 \rightarrow dx = \frac{8}{3}du$

$$= 62.5\pi\left[\left[8\cdot\frac{8}{3}\cdot\frac{1}{3}\left(\frac{3}{8}x+3\right)^3\right]_0^8 - \int_0^8\left(\frac{9}{64}x^3+\frac{9}{4}x^2+9x\right)dx\right]$$

$$= \frac{(62.5\pi)(64)}{9}\left[\left(\frac{3}{8}(8)+3\right)^3 - 3^3\right] - \left[62.5\pi\left[\frac{9}{16}x^4+\frac{9}{12}x^3+\frac{9}{2}x^2\right]_0^8\right]$$

$= \frac{4000\pi}{9}\left[(6)^3 - 3^3\right] - 62.5\pi\left[\frac{9}{16}(8)^4+\frac{3}{4}(8)^3+\frac{9}{2}(8)^2\right]$

1.86×10^5
$= 186000$

$= \frac{4000\pi}{9}(216-27) - 62.5\pi\left[\frac{9}{16}(4096)+\frac{3}{4}(512)+\frac{9}{2}(64)\right]$

$= \frac{4000\pi}{9}(189) - 62.5\pi\left[2304+384+288\right]$

$= 84000\pi - 62.5\pi\left[2976\right]$

$= 84000\pi - 186000\pi$

-85000

Apr 23-12:45 PM