$\pi r_2^2 h - \pi r_1^2 h$

$$V = 2\pi r h\,\Delta r$$

Δx

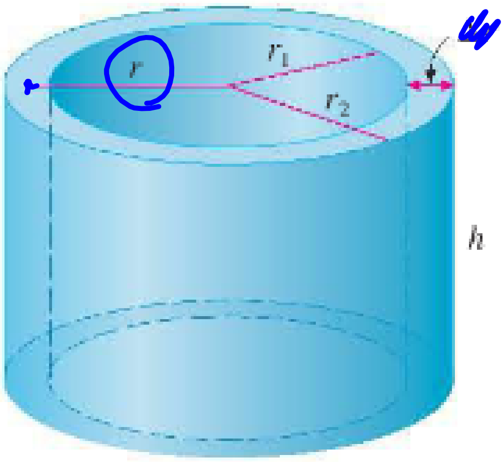


FIGURE 2

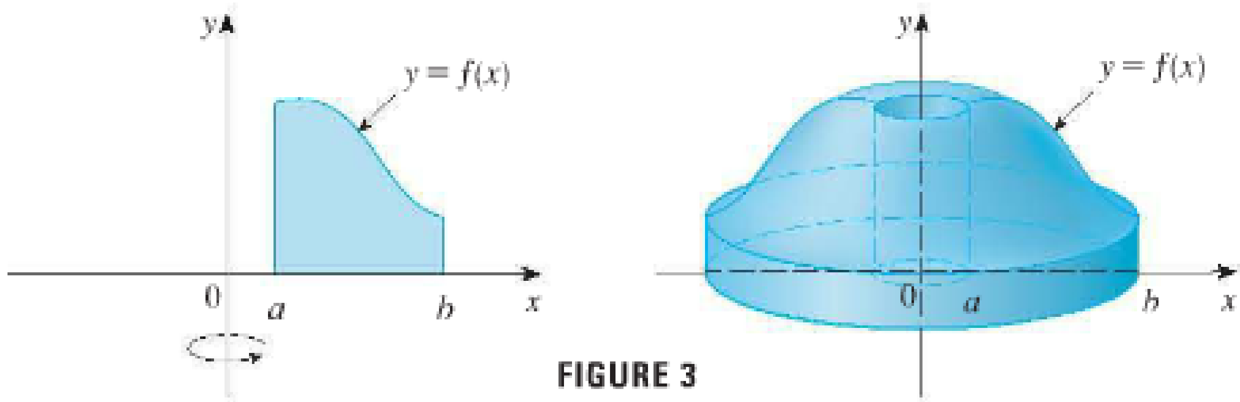


FIGURE 3

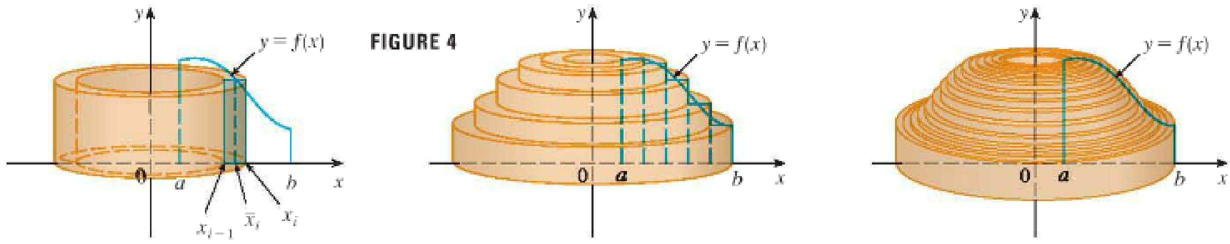
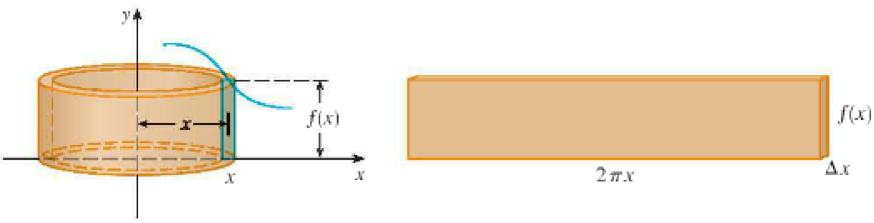


FIGURE 4

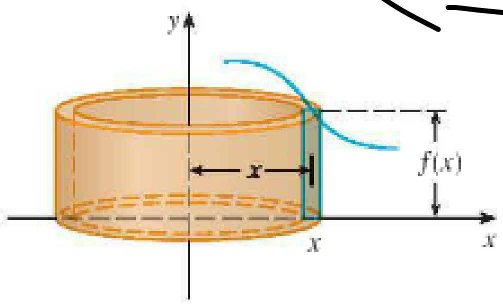
$\boxed{2}$ The volume of the solid in Figure 3, obtained by rotating about the y-axis the region under the curve $y = f(x)$ from a to b, is

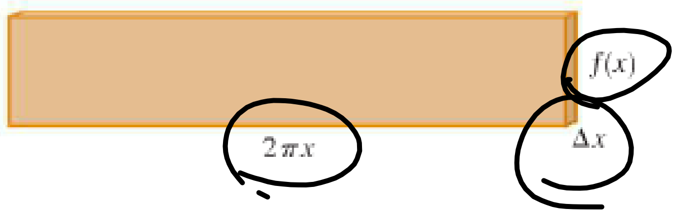
$$V = \int_a^b 2\pi x f(x)\,dx \qquad \text{where } 0 \leqslant a < b$$

$$\int_a^b \underbrace{(2\pi x)}_{\text{circumference}} \underbrace{[f(x)]}_{\text{height}} \underbrace{dx}_{\text{thickness}}$$



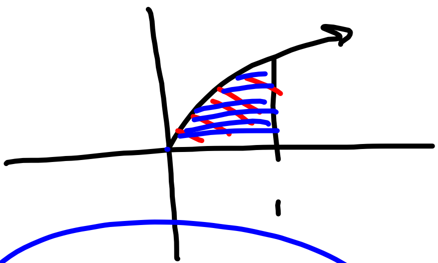
$$\int_a^b \underbrace{(2\pi x)}_{\text{circumference}} \; \underbrace{[f(x)]}_{\text{height}} \; \underbrace{dx}_{\text{thickness}}$$

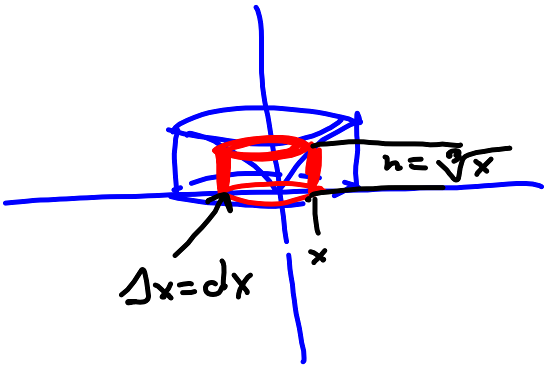




#3 5.3

$y=\sqrt[3]{x}$, $y=0$, $x=1$, about y-axis





Shells $\int_0^1 2\pi x \cdot x^{\frac{1}{3}} \cdot dx$

This is way better than shell method for revolving f(x) about y-axis

y = f(x) about x-axis:

1. Disc
2. shell

y = f(x) about y-axis

1. shell
2. Disc

Crossover: when $y=f(x)$ or $x=g(y)$ are 1-to-1 and it's "easy" to go from $y=f(x)$ to $x=g(y)=f^{-1}(x)$

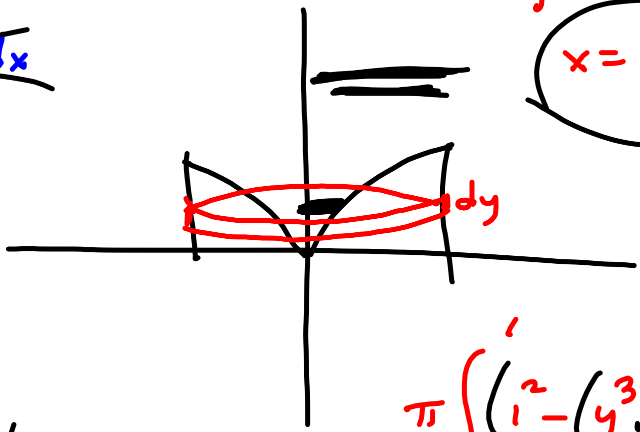
$$2\cdot \mathrm{Pi}\cdot \int_0^1 x\cdot x^{\frac{1}{3}}\,dx = \frac{6}{7}\pi$$

washer

$\cancel{\pi\int(1^2 - x^{2/3})\,dx}$

$y = \sqrt[3]{x}$

$x = y^3$



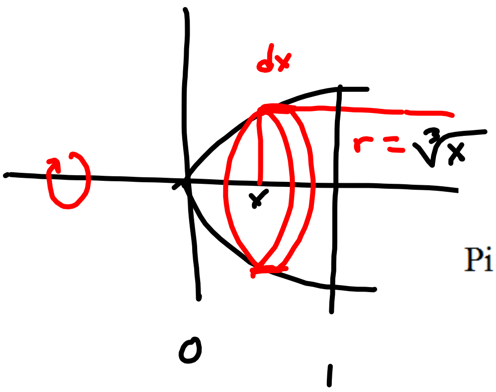
$$\pi\int_0^1 \left(1^2 - (y^3)^2\right)dy$$

$$\pi\int_0^1 dy$$

$$\text{Pi}\cdot\int_0^1 \left(1 - (y^3)^2\right)dy = \frac{6}{7}\pi$$

$$\pi\int_0^1 (y^3)^2\,dy$$

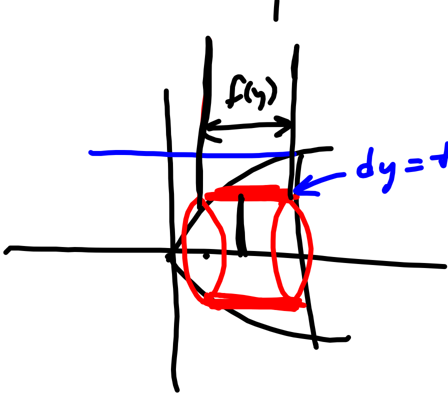
$y = \sqrt[3]{x}$, $x = 1$, $y = 0$, about the x-axis



$\pi \int_0^1 (x^{1/3})^2 \, dx$ Disc Method

$\text{Pi} \cdot \int_0^1 \left(x^{\frac{1}{3}}\right)^2 dx = \frac{3}{5}\pi$

$y = \sqrt[3]{x} \Rightarrow y^3 = x$



dy = thickness of wall
Integrating w.r.t. y.

Shell Method

$2\pi \int_0^1 y f(y) \, dy = 2\pi \int_0^1 y(1 - y^3) \, dy$

$h = 1 - y^3 = f(y)$

$\int_0^1 2 \cdot \text{Pi} \cdot y \cdot (1 - y^3) \, dy = \frac{3}{5}\pi$

$y = x$