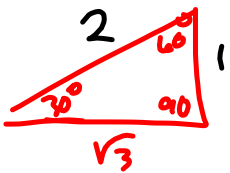


$$\int_0^2 u^4 du = \left. \frac{u^5}{5} \right|_0^2 = \frac{32}{5} \quad ?$$



$$\int_{x=0}^{x=\frac{\pi}{6}} \cos^4 x \sin x dx = \int_{x=0}^{x=\frac{\pi}{6}} u^4 \sin x \left( \frac{du}{- \sin x} \right)$$

$$\begin{aligned}
 u &= \cos x & x=0 &\Rightarrow u = \cos(0) = 1 \\
 \frac{du}{dx} &= -\sin x & x=\frac{\pi}{6} &\Rightarrow u = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \\
 du &= -\sin x dx \Rightarrow \frac{du}{-\sin x} = dx
 \end{aligned}$$

$$\begin{aligned}
 &= - \int_{u=1}^{u=\frac{\sqrt{3}}{2}} u^4 du \\
 &= - \left[ \frac{u^5}{5} \right]_{u=1}^{u=\frac{\sqrt{3}}{2}} \\
 &= - \left[ \frac{\cos^5 x}{5} \right]_0^{\frac{\pi}{6}}
 \end{aligned}$$

$$\int_0^{\frac{\pi}{6}} \cos^4 x \sin x \, dx = - \int_0^{\frac{\pi}{6}} u^4 \, du = - \left[ \frac{\left(\frac{\pi}{6}\right)^5}{5} - \frac{0^5}{5} \right]$$

is Bad

$$- \left[ \frac{\left(\frac{\sqrt{3}}{2}\right)^5}{5} - \frac{1^5}{5} \right]$$

is correct.

$$\int \cos(3x) dx = \int \cos u \cdot \frac{du}{3} = \frac{1}{3} \int \cos u du =$$
$$u = 3x \Rightarrow du = 3dx \Rightarrow dx = \frac{du}{3}$$
$$\int \cos u du = \sin u + C$$
$$\frac{1}{3} \sin(3x) = \frac{1}{3} \sin u + C$$

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Trick: Do  $\int f(x) dx$  before

$$\int_a^b f(x) dx$$

$$\int \sin^5 x \, dx \quad (1-x)^2 = (x-1)^2$$

$$u = \sin x$$

$$du = \cos x \, dx \quad \text{They ain't no } \cos x \text{ in there.}$$

Want some sort of  $u^n du$  thingie.

$$\begin{aligned} \text{Hint: } \sin^5 x &= \sin^4 x \sin x \\ &= (\sin^2 x)^2 \sin x \end{aligned}$$

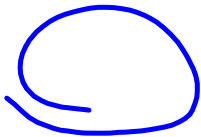
$$\begin{aligned}
\int \sin^5 x \, dx &= \int \sin^4 x \sin x \, dx \\
&= \int (1 - \cos^2 x)^2 \sin x \, dx \\
&= -\int (\cos^4 x - 2\cos^2 x + 1) \sin x \, dx \\
&= -\int \cos^4 x (-\sin x) \, dx + 2 \int \cos^2 x (-\sin x) \, dx + \int \sin x \, dx \\
&= -\frac{\cos^5 x}{5} + \frac{2\cos^3 x}{3} - \cos x + C \\
u &= \cos x \\
du &= -\sin x \, dx
\end{aligned}$$

$$\int \cos x \, dx = \sin x + C \quad \text{CAKE}$$

$$\int \cos(5x^3 - 7x) \, dx \quad \text{Tough! Ouch!}$$

$$\int \cos(5x^3 - 7x) (15x^2 - 7) \, dx \quad \text{CAKE}$$

$$= \sin(5x^3 - 7x) + C$$



$$c \quad u = 5x^3 - 7x$$

$$du = (15x^2 - 7) \, dx$$

Review Differentials