$$\frac{d}{dx}\int_0^{3x^2}(t-1)\,dt = (3x^2-1)(6x)$$

Chain Rule for FTC I

$$\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x))\cdot\frac{dg}{dx}$$

They ain't no 'x' in there.

$$\frac{d}{dx}\int_a^{f(t)} g(s)\,ds = 0$$

$$\frac{d}{dx}\left[t^2-5t\right] = 0$$

(unless someone says a relationship/dependency of t-variable on the x-variable

$$\frac{d}{dx}[5] = 0$$

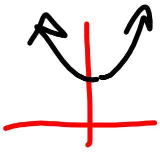
IF t is assumed to be an implicit function of x.

Then

$$\frac{d}{dx}\int_a^{f(t)} g(s)\,ds = g(f(t))\cdot\frac{df}{dt}\cdot\frac{dt}{dx}$$

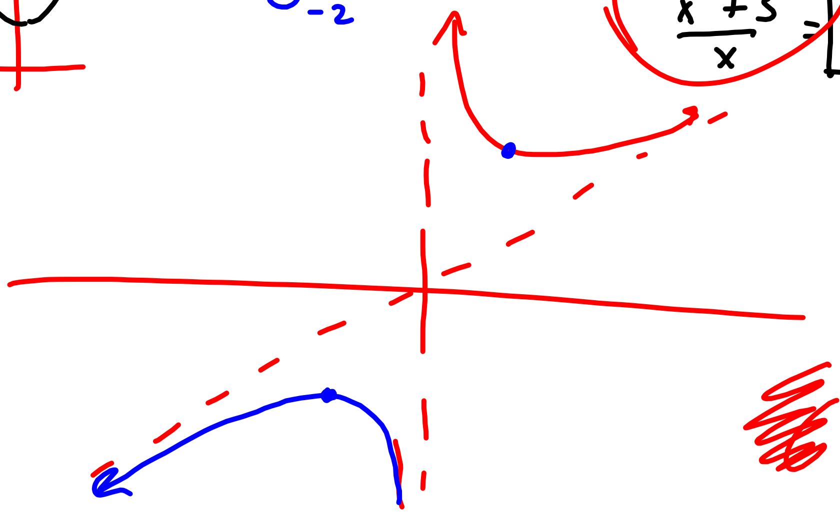
$$\frac{d}{dx}\left[3x^2y^3\right] = 6xy^3 + 9x^2y^2\frac{dy}{dx}$$

$x^2+5=0$

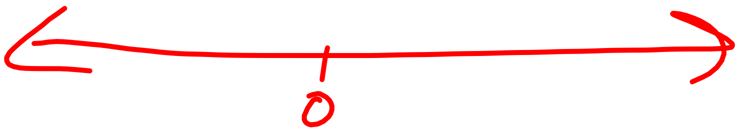


$$\int_{-2}^{4}\frac{x^2+5}{x}\,dx$$

$$\frac{x^2+5}{x} = \boxed{x + \frac{5}{x}}$$



$x=0$



$$\int_{-1}^{3} (x^2 + 3x - 1)\,dx$$

① By Limit Definition

② By FTC II

$$\Delta x = \frac{b-a}{n} = \frac{3+1}{n} = \frac{4}{n} \qquad x_k = a + k\Delta x = -1 + \frac{4k}{n} = \frac{4k}{n} - 1$$

$$\sum_{k=1}^{n} f(x_k)\,\Delta x_k = \sum_{k=1}^{n}\left[\left(\left(\frac{4k}{n}-1\right)^2 + 3\left(\frac{4k}{n}-1\right) - 1\right)\cdot\frac{4}{n}\right]$$

$$= \frac{4}{n}\sum_{k=1}^{n}\left(\frac{16k^2}{n^2} - \frac{8k}{n} + 1 + \frac{12k}{n} - 3 - 1\right)$$

$$= \frac{4}{n}\sum_{k=1}^{n}\left(\frac{16k^2}{n^2} + \frac{4k}{n} - 3\right) = \frac{4}{n}\left(\frac{16}{n^2}\cdot\frac{n^3 + \text{sm}}{3} + \frac{4}{n}\cdot\frac{n^2+\text{sm}}{2} - 3n\right)$$

$$= \frac{64}{3n^3}\cdot\left(n^3 + \text{smaller}\right) + \frac{8}{n^2}\cdot\left(n^2 + \text{smaller}\right) - \frac{12}{n}\cdot n$$

$$\xrightarrow{n\to\infty} \frac{64}{3} + 8 - 12 = \frac{64}{3} - \frac{12}{3} = \frac{52}{3}$$

(2) $$\int_{-1}^{3}(x^2+3x-1)\,dx = \left[\frac{1}{3}x^3+\frac{3}{2}x^2-x\right]_{-1}^{3}$$

$$= \frac{1}{3}(3)^3+\frac{3}{2}(3)^2-3 \quad - \left[\frac{1}{3}(-1)^3+\frac{3}{2}(-1)^2-(-1)\right]$$

$$F(b) \quad - \quad F(a)$$

$$= 9+\frac{27}{2}-3-\left[-\frac{1}{3}+\frac{3}{2}+1\right]$$

$$= 6+\frac{27}{2}+\frac{1}{3}-\frac{5}{2}$$

$$= 6+11+\frac{1}{3} \quad = 17+\frac{1}{3} = \frac{51+1}{3} = \frac{52}{3}$$

$$\frac{d}{dx}\int_{-51\pi}^{\cos(x^2)} (t^3-1)^{35}\sin(\cos(3t^2+5))\,dt$$

$$\left((\cos(x^2))^3-1\right)^{35}\sin\left(\cos\left(3(\cos(x^2))^2+5\right)\right)\cdot(-\sin(x^2))(2x)$$

$$\frac{d}{dx}\int_{-\frac{\pi}{2.1}}^{x}\tan(t)\,dt = \tan x$$ as long as we stay where tangent is defined/continuous.

$$0 \le x < \frac{\pi}{2}$$

$$\frac{1}{3} < \frac{1}{2}$$