$$\int_a^b x^2\,dx = \frac{b^3-a^3}{3}$$

$$(x+y)^2 = x^2+2xy+y^2$$

$$\sum_{k=1}^{n} f(x_k)\,\Delta x = \sum_{k=1}^{n} x_k^2\,\Delta x = \sum_{k=1}^{n}\left[\left(\left(\frac{b-a}{n}\right)k+a\right)^2\left(\frac{b-a}{n}\right)\right]$$

$$\Delta x = \frac{b-a}{n}$$

$$\frac{b-a}{n}\sum_{k=1}^{n}\left(\left(\left(\frac{b-a}{n}\right)k\right)^2+2\left(\left(\frac{b-a}{n}\right)k\right)a+a^2\right)$$

$$x_k = a+k\Delta x$$

$$= a+k\left(\frac{b-a}{n}\right)$$

$$= \left(\frac{b-a}{n}\right)k+a$$

$$\frac{b-a}{n}\left[\left(\frac{b-a}{n}\right)^2\sum_{k=1}^{n}k^2+2a\left(\frac{b-a}{n}\right)\sum_{k=1}^{n}k+\sum_{k=1}^{n}a^2\right]$$

$$= \frac{b-a}{n}\left[\left(\frac{b-a}{n}\right)^2\left(\frac{n^3+n}{3}\right)+\left(\frac{2a(b-a)}{n}\right)\left(\frac{n^2+n}{2}\right)+na^2\right]$$

$$= \left(\frac{b-a}{n}\right)^3\left(\frac{n^3+n}{3}\right)+2a\left(\frac{b-a}{n}\right)^2\left(\frac{n^2+n}{2}\right)+\frac{b-a}{n}\cdot na^2$$

3

$$= \frac{(b-a)^3}{3n^3}\left(\frac{n^3+\cdots}{1}\right) + a\left(\frac{(b-a)^2}{n^2}\right)\left(\frac{n^2+\cdots}{1}\right) + a^2(b-a)$$

$$\xrightarrow{n\to\infty} \frac{(b-a)^3 n^3}{3n^3} + a\,\frac{(b-a)^2 n^2}{n^2} + a^2(b-a)$$

$$= \frac{(b-a)^3}{3} + a(b-a)^2 + a^2(b-a)$$

$$= \frac{(b-a)^3 + 3a(b-a)^2 + 3a^2(b-a)}{3}$$

$$= \frac{b^3 - 3b^2a + 3ba^2 - a^3 + 3a(b^2 - 2ab + a^2) + 3a^2(b-a)}{3}$$

$$= \frac{b^3 - 3b^2a + 3ba^2 - a^3 + 3b^2a - 6ba^2 + 3a^3 + 3ba^2 - 3a^3}{3}$$

$$= \frac{b^3 - a^3}{3}$$

4.2 #37 Fixed

$$\int_{-3}^{0} (1 + \sqrt{9-x^2})\,dx$$

missed it!

Area

$= \frac{1}{4}(\pi(3^2)) + \text{Rectangle}$

$= \frac{9\pi}{4} + 1 \cdot 3$

$= \frac{9\pi}{4} + 3$

Missing on Soln's

$y = 1 + \sqrt{9-x^2}$

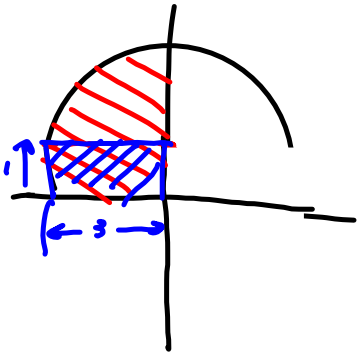
$y - 1 = \sqrt{9-x^2}$

$(y-1)^2 = 9 - x^2$

$x^2 + (y-1)^2 = 9$

$(x-h)^2 + (y-k)^2 = 3^2$

$(h,k) = (0,1)$, $r = 3$



4.3 FTC II

$$\int_2^5 x^2\,dx = \frac{1}{3}x^3\Big]_2^5 = \frac{1}{3}\left[5^3 - 2^3\right]$$

$$F'(x) = f(x) \Longrightarrow$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

FTC I

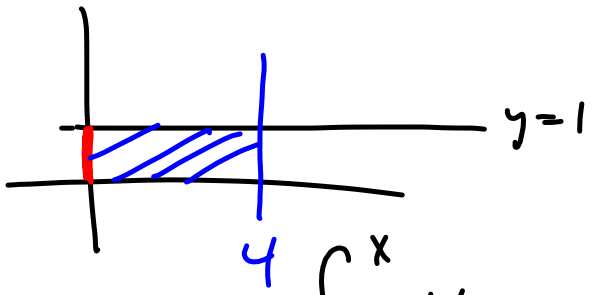
$$\frac{d}{dx}\left[\sin(x^3)\right] = \cos(x^3)\cdot 3x^2$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\frac{d}{dx}\left[\int_0^{x^3} t\,dt\right]$$

$$= (x^3)\cdot 3x^2$$

$$= 3x^5$$

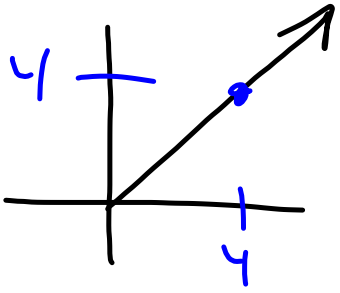


$$\int_0^x 1\,dt = t\Big]_0^x = x$$

$$\left[\frac{t^2}{2}\right]_0^{x^3}$$

$$= \frac{(x^3)^2}{2}$$

$$= \frac{x^6}{2}$$



$$\frac{d}{dx}\left[\frac{x^6}{2}\right] = \frac{6x^5}{2} = 3x^5$$