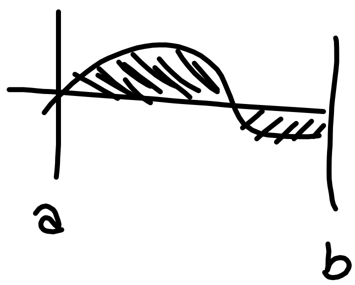
$f(x) = \sqrt{x}$ Today §4.2

DEFINITE INTEGRAL $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x = \int_a^b f(x)\,dx$

What if $f(x_k) < 0$?

This is area, if $f(x) \ge 0 \ \forall x \in [a,b]$.

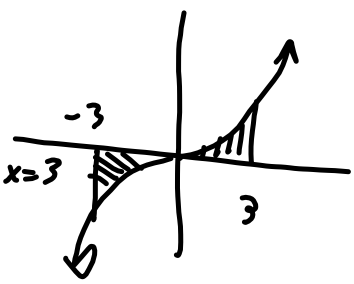
Otherwise, it's "signed area"



$\int_a^b f(x)\,dx$ is signed area

$$\int_{-3}^{3} x^3\,dx = 0$$

What IS the shaded area?



① Double the area from $x=0$ to $x=3$ exploiting x^3 is an odd function

② Area from -3 to 0 plus area from 0 to 3

$$\int_{-3}^{0} |x^3|\,dx + \int_{0}^{3} |x^3|\,dx$$

$$= \int_{-3}^{0} -x^3\,dx + \int x^3\,dx$$

$$|x^3| = \begin{cases} x^3 & \text{if } x^3 \geq 0 \\ -x^3 & \text{if } x^3 < 0 \end{cases}$$

$x^3 \geq 0 \Rightarrow$

$x \geq 0$

$$|x^2-3x+2| = \begin{cases} x^2-3x+2 & \text{if } x^2-3x+2 \geq 0 \\ -(x^2-3x+2) & \text{if } x^2-3x+2 < 0 \end{cases}$$

$2 \leq x \leq 1$

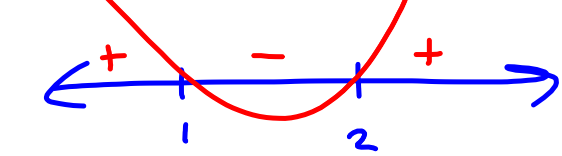
$$= \begin{cases} x^2-3x+2 & \text{if } x \leq 1 \text{ OR } x \geq 2 \\ -x^2+3x-2 & \text{if } 1 < x < 2 \end{cases}$$

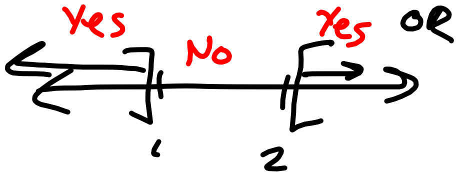
$x > 1$ and $x < 2$

$x^2-3x+2 \geq 0$

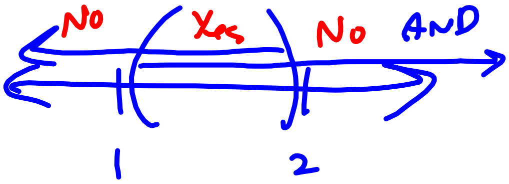
$(x-2)(x-1) \geq 0$

$x-2 \geq 0$ OR $x-1 \geq 0$ No!

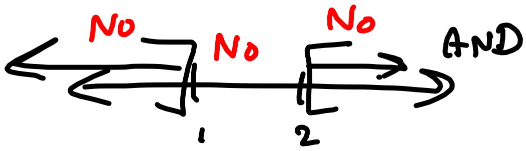




$(-\infty, 1] \cup [2, \infty)$



$(1, 2)$



$\varnothing$

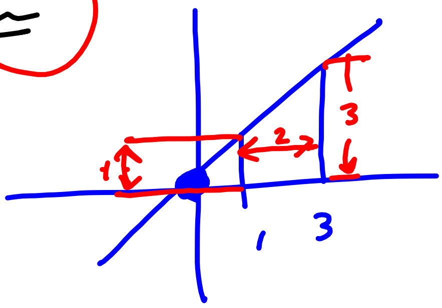
$$\int_{-3}^{5} |x^2-3x+2|\, dx$$

$$= \int_{-3}^{1} (x^2-3x+2)\, dx - \int_{1}^{2} (x^2+3x+2)\, dx$$

$$+ \int_{2}^{5} (x^2-3x+2)\, dx$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} = \frac{n^2+n}{2} = \frac{n^2+\cdots}{2}$$

$$\int_1^3 x\,dx = 4$$



By the definition

$[a,b]=[1,3]$

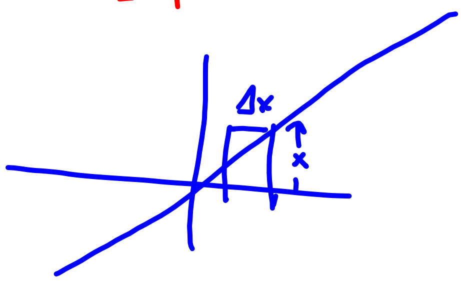
$$\frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n} = \Delta x$$

$$\boxed{x_k = a + k\Delta x}$$

$$= 1 + k\cdot\frac{2}{n} = 1+\frac{2k}{n} = \frac{2k}{n}+1$$

$$A = \frac{1}{2}(b_1+b_2)h$$
$$= \frac{1}{2}(1+3)(2)$$
$$= \frac{1}{2}(4)(2)$$
$$= 4$$

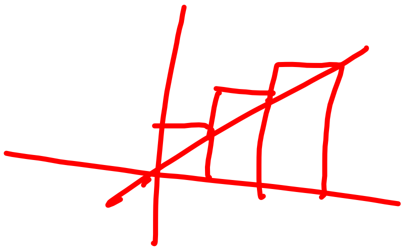
$$\sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n} x_k \Delta x$$



$f(x)=x$

$$= \sum_{k=1}^{n} \boxed{\left(\frac{2k}{n}+1\right)}\left(\frac{2}{n}\right)$$

$$\frac{2}{n}\left[\sum_{k=1}^{n} \frac{2k}{n} + \sum_{k=1}^{n} 1\right]$$



$$\frac{2}{n}\sum_{k=1}^{n}\left(\frac{2k}{n}+1\right)$$

$$=\frac{2}{n}\sum\frac{2k}{n}+\frac{2}{n}\sum 1$$

$$=\frac{4}{n^2}\sum k+\frac{2}{n}\sum 1$$

$$=\frac{4}{n^2}\cdot\frac{n^2+n}{2}+\frac{2}{n}\cdot n$$

$$\frac{3x^2}{5x^2} \xrightarrow{x\to\infty} \frac{3}{5}$$

$$=\frac{2}{n}\left[\frac{2}{n}\sum_{k=1}^{n}k+\sum_{k=1}^{n}1\right]$$

$$=\frac{2}{n}\left[\frac{2}{n}\left(\frac{n^2+\cancel{n}}{2}\right)+n\right]$$

$$=\frac{4}{n^2}\left(\frac{n^2+\cancel{n}}{2}\right)+\frac{2}{n}\cdot n$$

$$=\frac{4n^2+\cancel{4n}}{2n^2}+2$$

$$n\to\infty$$

$$\frac{4n^2}{2n^2}+2=2+2=4$$

$$=\frac{4n^2}{2n^2}+\frac{\cancel{4n}}{2n^2}+2$$

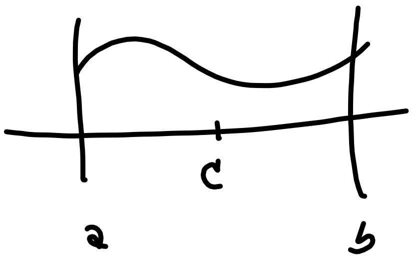
$$=\frac{4}{2}+\frac{2}{n}+2$$

$$=2+\frac{2}{n}+2=4+\frac{2}{n}\xrightarrow{n\to\infty}4$$

$$\int_a^b f = -\int_b^a$$

Reverse limits of integration: Change sign.
"Δx's are negative"

$$\int_a^a f = 0$$



$a < c < b$

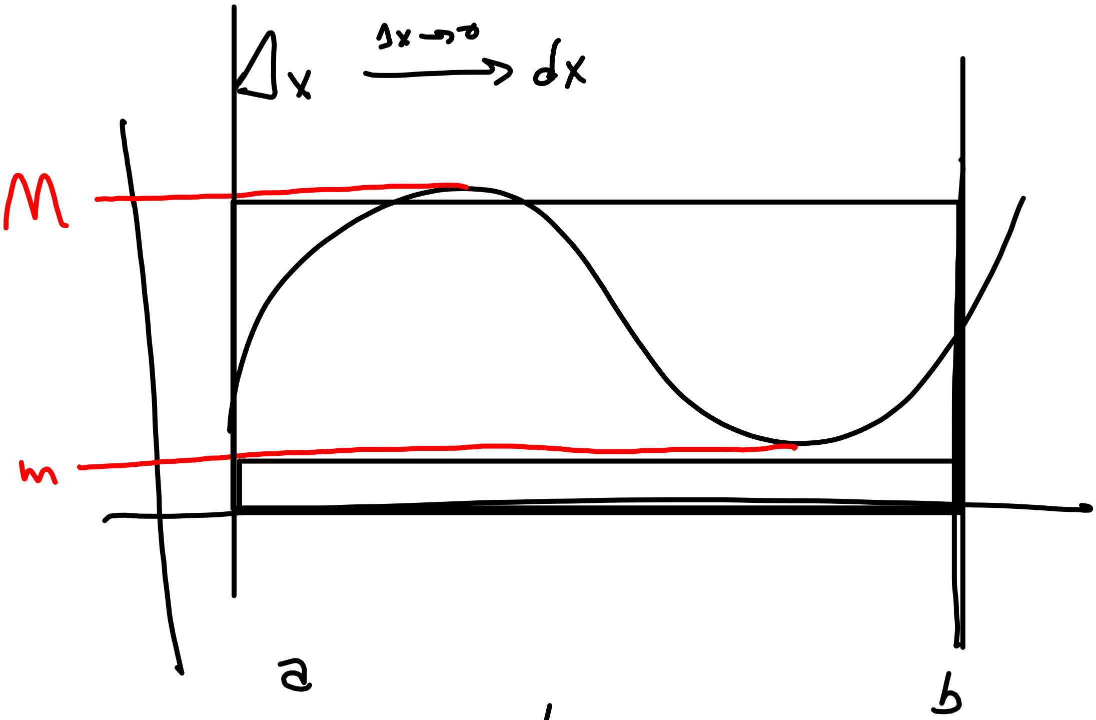
$$\int_a^b = \int_a^c + \int_c^b$$

$$\int_a^b (cf \pm dg) = c\int_a^b f \pm d\int_a^b g$$

$$\int_1^3 (3x^2+5x)\,dx = 3\int_1^3 x^2\,dx + 5\int_1^3 x\,dx$$

$\int$ is a linear operator.

(if you throw in $\int_a^b 0\,dx = 0$)



$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$$