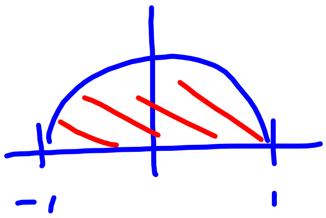
Screwups on §4.1 Video

Right Endpoints



Area under top $\frac{1}{2}$ of circle, radius $r=1$, centered @ $(0,0)$.

$x^2+y^2=1$

$\vdots$

Approx. Area by rectangles. $y=\pm\sqrt{1-x^2}$

$y=+\sqrt{1-x^2}$ is top $\frac{1}{2}$.

$$\boxed{\lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x_k = \int_a^b f(x)\,dx}$$

Works, for equal-width rectangles.

Really, we're saying make the biggest width approach zero

$\Delta x = \frac{b-a}{n}$

$[a,b]$

$n =$ # of partitions

$\boxed{x_k = a + k\Delta x}$

$[a,b] = [-1,1]$ $f(x) = \sqrt{1-x^2}$

$n = 10$

$\Delta x = \frac{b-a}{n} = \frac{1-(-1)}{10} = \frac{2}{10} = \frac{1}{5}$

$x_1 = a + 1\cdot\Delta x = -1 + \frac{1}{5} = -\frac{4}{5}$

$x_2 = a + 2\cdot\Delta x = -1 + 2\cdot\frac{1}{5} = -1 + \frac{2}{5} = -\frac{3}{5}$

$x_3 = \cdots\cdots = -\frac{2}{5}$

$x_4 = \cdots\cdots = -\frac{1}{5}$

$x_5 = \cdots = 0$

$\vdots$

$x_k = -1 + k\cdot\frac{1}{5} = -1 + \frac{k}{5}$ Express x_k as a function of k

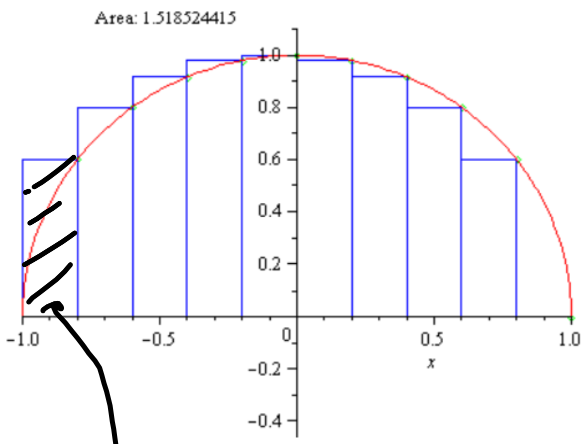
$\vdots$

$x_{10} = -1 + 10\cdot\frac{1}{5} = 1$

Area of k^{th} rectangle is $f(x_k)\Delta x$

$$= \sqrt{1-x_k^2}\;\Delta x$$

$$= \sqrt{1-\left(-1+\frac{k}{5}\right)^2}\cdot\frac{1}{5}$$



$$\sqrt{1-\left(-1+\frac{1}{5}\right)^2}\cdot\frac{1}{5}$$

$$= \sqrt{1-\left(-\frac{4}{5}\right)^2}\cdot\frac{1}{5} = \text{Area of } 1^{\text{st}} \text{ rectangle.}$$

Now add 'em up!

$$\sqrt{1-x_1^2}\,\Delta x + \sqrt{1-x_2^2}\,\Delta x + \cdots + \sqrt{1-x_{10}^2}\,\Delta x$$

$$= \Delta x\left[\sqrt{1-x_1^2} + \cdots + \sqrt{1-x_{10}^2}\right]$$

$$= \sum_{k=1}^{10} f(x_k)\Delta x = \sum_{k=1}^{10}\sqrt{1-x_k^2}\cdot\frac{1}{5} = \frac{1}{5}\sum_{k=1}^{10}\sqrt{1-\left(-1+\frac{k}{5}\right)^2}$$

Let c, d be constants.

Then $\sum_{k=1}^{n} c = cn \quad \Big(\underbrace{c + c + c + \dots + c}_{n \text{ of 'em}} \Big)$

$\sum_{k=1}^{5} 2 = 10$

$= \sum c \cdot 1$

$= c \left(\sum 1 \right) = c \Big(\underbrace{1 + 1 + 1 + \dots + 1}_{n \text{ of 'em}} \Big)$

$\sum_{k=1}^{n} c x_k = c x_1 + c x_2 + c x_3 + \dots + c x_n$

$= c \left(x_1 + \dots + x_n \right)$

$= c \sum_{k=1}^{n} x_k$

$\sum_{k=1}^{n} (a x_k + b y_k)$

$= a \sum x_k + b \sum y_k$

$1 + 2 + 3 + 4 + 5 + 6 = \sum a_k + \sum b_k$

$= 1 + 4 + 2 + 5 + 3 + 6 = \sum (a_k + b_k)$

Key sums

$$\sum_{k=1}^{n} c = cn$$

$$\sum_{k=1}^{n} k = 1+2+3+4+\dots+n = \frac{n(n+1)}{2}$$

$$\frac{n}{2}=50 \left\{ \begin{array}{l} 1+100 = 101 = n+1 \\ 2+99 = 101 = n+1 \\ 3+98 = 101 \\ + \\ \vdots \\ 49+52 \\ 50+51 \end{array} \right. \qquad \frac{n}{2}\cdot(n+1)$$

Notice

$$\sum_{k=1}^{1} k = 1 = \frac{1(1+1)}{2} = 1 \checkmark$$

Suppose it works for some n. $\frac{n(n+1)}{2} = \sum_{k=1}^{n} k$

We show it works for $n+1$ want $\frac{(n+1)(n+2)}{2} = \sum_{k=1}^{n+1} k$

Consider $\sum_{k=1}^{n+1} k = \underbrace{1+2+3+\dots+n}_{\sum_{k=1}^{n} k = \frac{n(n+1)}{2}} + (n+1)$

$$= \frac{n(n+1)}{2} + n+1$$

$$= \frac{n^2+n}{2} + \frac{n+1}{1}\cdot\frac{2}{2}$$

$$= \frac{n^2+n+2n+2}{2}$$

$$= \frac{n^2+3n+2}{2} = \frac{(n+1)(n+2)}{2}$$

$$= \frac{(n+1)((n+1)+1)}{2}$$

$\therefore$ It's true for all $n \geq 1$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} = \frac{n^2 + \sim\sim}{2}$$

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{2n^3 + \text{smaller}}{6} = \frac{n^3 + \sim\sim}{3}$$

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^4 + \sim\sim}{4}$$