Find two numbers whose difference is 100 and whose product is a minimum.

$x - y = 100$

$-y = 100 - x$

$y = x - 100$

$x = 1^{st}$ #, $y = 2^{nd}$ #.

$x - y = 100$

minimize $xy = P$

$y = x - 100$

$\Rightarrow P = x(x-100) = x^2 - 100x$

$\Rightarrow \frac{dP}{dx} = 2x - 100 \overset{set}{=} 0$

$\Rightarrow 2x = 100$

$\boxed{x = 50}$

$y = x - 100 = -50$

$\boxed{y = -50}$

$y = x^2 - 100x$

$y' = 2x - 100$

− | +

min @ 50

3. Find two positive numbers whose product is 100 and whose sum is a minimum.

Let $x = 1^{st}$ #

$y = 2^{nd}$ #

$x, y > 0$

$y = \frac{100}{x} \Leftarrow xy = 100$

minimize

$S = x + y$

$= x + 100/x \qquad \frac{x^2+100}{x}$

$= x + 100x^{-1}$

$\frac{dS}{dx} = 1 - 100x^{-2}$

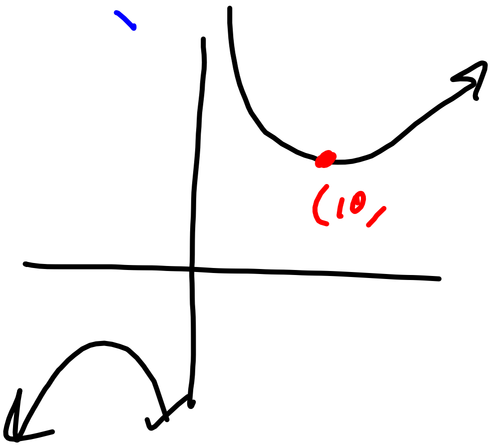
$= \frac{x^2-100}{x^2} \underset{=}{\text{set}} 0$

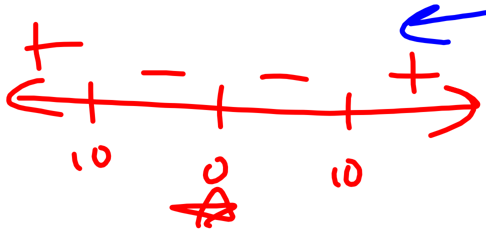
$x = \pm 10$

$\Rightarrow x = 10$

$y = \frac{100}{x} = \frac{100}{10} = 10$

"helping"

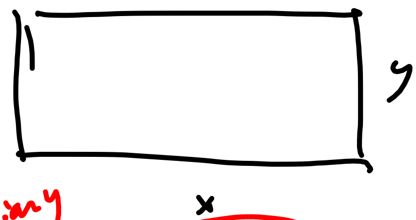




7. Find the dimensions of a rectangle with perimeter 100 m whose area is as large as possible.

Let x = length (m)

y = width (in m)



Auxiliary eq'n.

$2x + 2y = 100$

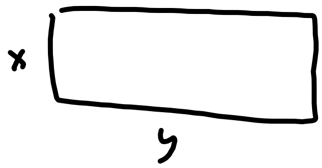
$x + y = 50$

$y = 50 - x$

Maximize

Area $= A = xy =$

$= x(50 - x)$, etc.



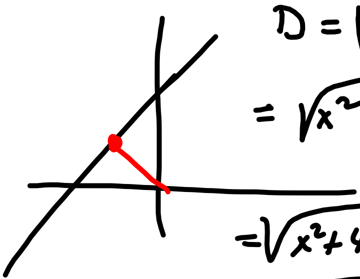
Perimeter = 100

$2x + 2y = 100$

$A = xy$ to be maximized

19. Find the point on the line $y = 2x + 3$ that is closest to the origin.

Distance from $y=2x+3$ to $(0,0)$ is to be minimized



$$D = \sqrt{(x-0)^2 + (y-0)^2}$$

$$= \sqrt{x^2 + (2x+3)^2}$$

$$= \sqrt{x^2 + 4x^2 + 12x + 9}$$

$$= \sqrt{5x^2 + 12x + 9}$$

why? ← To min. D, just min D^2!

$$D^2 = f(x) = 5x^2 + 12x + 9$$

$$f'(x) = 10x + 12 \overset{\text{SET}}{=} 0$$

$$10x = -12$$

$$x = -\frac{12}{10} = -\frac{6}{5}$$

$$y\Big|_{x=-6/5} = 2x+3\Big|_{x=-6/5}$$

$$= 2\left(-\frac{6}{5}\right) + 3$$

$$= -\frac{12}{5} + \frac{15}{5} = \frac{3}{5} \rightsquigarrow \boxed{\left(-\frac{6}{5}, \frac{3}{5}\right)}$$

20. Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(3, 0)$.

$$D^2 = \left(\sqrt{(x-3)^2 + (y-0)^2}\right)^2$$

$$= x^2 - 6x + 9 + (\sqrt{x} - 0)^2$$

$$= x^2 - 6x + 9 + \sqrt{x}^2$$

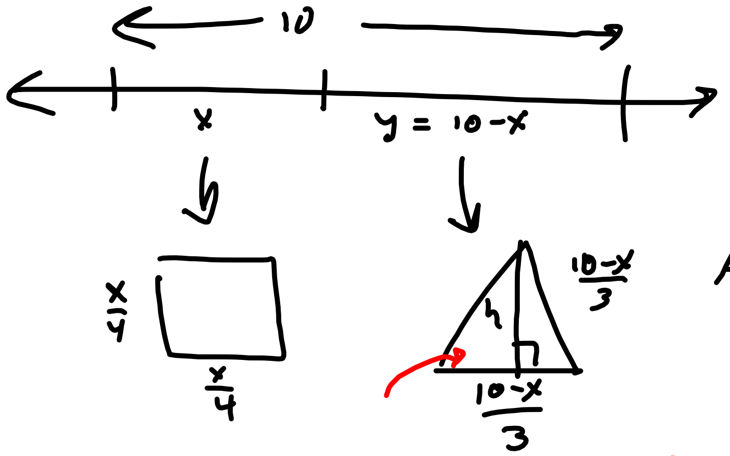
$$= x^2 - 6x + 9 + x$$

$$= x^2 - 5x + 9 \equiv f(x) \Rightarrow$$

$$f'(x) = 2x - 5 \overset{\text{SET}}{=} 0 \Rightarrow x = \frac{5}{2} \Rightarrow y = \sqrt{\frac{5}{2}} = \frac{\sqrt{5}}{\sqrt{2}}$$

$$= \frac{\sqrt{5}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{10}}{2} \rightsquigarrow \left(\frac{5}{2}, \frac{\sqrt{10}}{2}\right)$$

35. A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is (a) a maximum? (b) A minimum?



$$A = \frac{1}{2} b h$$
$$= \frac{1}{2}\left(\frac{10-x}{3}\right) h$$

(1) $$\frac{h}{\frac{10-x}{3}} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \Rightarrow h = \frac{\sqrt{3}}{2} \cdot \frac{3}{10-x}$$

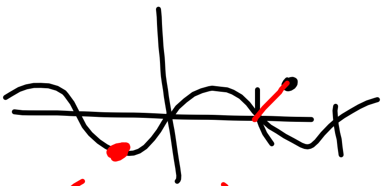
(2) $$h^2 + \left(\frac{10-x}{6}\right)^2 = \left(\frac{10-x}{3}\right)^2$$

$$h^2 = \left(\frac{10-x}{3}\right)^2 - \frac{(10-x)^2}{(3\cdot 2)^2}$$
$$= \left(\frac{10-x}{3}\right)^2 - \frac{1}{2^2}\left(\frac{10-x}{3}\right)^2$$
$$= \frac{3}{4}\left(\frac{10-x}{3}\right)^2$$

36. Answer Exercise 35 if one piece is bent into a square and the other into a circle.

37. A cylindrical can without a top is made to contain $V\ \text{cm}^3$ of liquid. Find the dimensions that will minimize the cost of the metal to make the can.

22. Find, correct to two decimal places, the coordinates of the point on the curve $y = \sin x$ that is closest to the point $(4, 2)$.



$(x, \sin x)$

Min $D = \sqrt{(x-4)^2 + (y-2)^2}$

$= \sqrt{(x-4)^2 + (\sin x - 2)^2}$

We min.

$D^2 = f(x) = \boxed{(x-4)^2 + (\sin x - 2)^2}$

$= x^2 - 8x + 16 + \sin^2 x - 4\sin x + 4$

$= x^2 - 4x + \sin^2 x - 4\sin x + 20$

$f'(x) \overset{\text{SET}}{=} 0$, etc