§3.8 video Posted

Test 3 video Posted

§3.9 is all that remains

ANTIDERIVATIVES

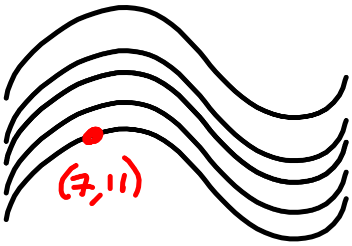
$f'(x) = x^2 - 5x + 7$

Find $f(x)$.

$f(x) = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 7x + \frac{312\pi e^{37}}{97\sqrt{5}}$

$\Rightarrow f'(x) = x^2 - 5x + 7$

$f'(x) = \sin x \Rightarrow f(x) = -\cos x + C$ for any constant $C \in \mathbb{R}$



All have the same slope
Same $f'(x)$

So $f'(x)$ does not uniquely determine $f(x)$

If I tell you what $f'(x)$ is AND tell you $f(7)=11$, then f is uniquely determined.

Suppose $f'(x) = 3x+2$ & $f(2)=5$. Find f.

$\Rightarrow f(x) = \frac{3}{2}x^2+2x-5$

$\Rightarrow f(2) = \frac{3}{2}(2)^2+2(2) = 10$ Nope

$\frac{3}{2}(2)^2+2(2)-5 = 5$ Yep

Need an initial condition on $f(x)$ to determine the "constant of integration"

Standard Method:

$f(x) = \frac{3}{2}x^2+2x+C$ &

$f(2) = \frac{3}{2}(2)^2+2(2)+C = 6+4+C = 10+C = 5 \Rightarrow$

$C=-5$

$\Rightarrow f(x) = \frac{3}{2}x^2+2x-5$

Antiderivatives in C4 will give us area

$f' = \text{rate}$

$f = \text{position}$

$F = \int f = \text{area}$!

1^{st}

FTC I

$\frac{d}{dx} \int f = f$

"sum"

$\sum \xrightarrow[\text{Calculus}]{\text{Newton}} \int$

$$\underbrace{\int x^n dx}_{\text{Antiderivative of } x^n} = \frac{x^{n+1}}{n+1} + C$$ Power Rule

is true $\forall\, n \neq -1$

$$\boxed{\int x^{-1} dx = \int \frac{dx}{x} = \ln|x| + C}$$

only exception.

$\frac{1}{x}$ DOES have $\ln|x| + C$ as its antiderivative.

No justification, except

If $\int x^{-1} dx = \frac{x^0}{0} + C$, then ?

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

Every continuous function has an antiderivative.

Recall: to have derivative, f must be smooth.

$f''(x) = x^2 - 2x + 7, \quad f'(1) = 2, \quad f(2) = 3$

Find $f(x)$.

$f'(x) = \frac{1}{3}x^3 - x^2 + 7x + C$

$f'(1) = \frac{1}{3} - 1 + 7 + C =$

$= \frac{1}{3} + 6 + C = \frac{1+18}{3} + C = \frac{19}{3} + C = 2 \Rightarrow$

$C = 2 - \frac{19}{3} = \frac{6-19}{3} = -\frac{13}{3}$

$f(x) = \frac{1}{12}x^4 - \frac{1}{3}x^3 + \frac{7}{2}x^2 - \frac{13}{3}x + D$

$f(2) = \frac{1}{12}(2)^4 - \frac{1}{3}(2)^3 + \left(\frac{7}{2}\right)(2)^2 - \frac{13}{3}(2) + D$

$= \frac{1}{12}\cdot 16 - \frac{1}{3}\cdot 8 + \frac{7}{2}\cdot 4 - \frac{13}{3}\cdot 2 + D$

$= \frac{4}{3} - \frac{8}{3} + 14 - \frac{26}{3} + D$

$= \frac{4-8+42-26}{3} + D$

$= \frac{12}{3} + D = 3 = \frac{9}{3}$

$\cancel{D = \frac{9}{3}} \quad D + 4 = 3$

$D = -1$

$f(x) = \frac{1}{12}x^4 - \frac{1}{3}x^3 + \frac{7}{2}x^2 - \frac{13}{3}x - 1$