§3.7 Problem-Solving

Google "Polya" problem-solving method.

See Step-by-steps in text.

Planning a video on 3.7 #39 - one of the tougher ones.

Two videos labeled 3.4 Rational Funcs - Oblique Asymptotes.

1^{st} one: Algebra methods

2^{nd} one: Calculus

$(x-3+\sqrt{2})(x-3-\sqrt{2}) =$

$(x-(3-\sqrt{2}))(x-(3+\sqrt{2}))$

$= x^2-(3+\sqrt{2})x-(3-\sqrt{2})x-(3-\sqrt{2})(-(3+\sqrt{2}))$

$= x^2-3x-\sqrt{2}x-3x+\sqrt{2}x+(3-\sqrt{2})(3+\sqrt{2})$

$= x^2-6x+7 \overset{SET}{=} 0$

$x^2-6x = -7$

$x^2-6x+3^2=-7+9$

$(x-3)^2=2$

$x-3=\pm\sqrt{2}$

$x=3\pm\sqrt{2}$

$\Rightarrow x^2-6x+7=(x-(3+\sqrt{2}))(x-(3-\sqrt{2}))$

$x=c$ makes $P(x)=0 \rightarrow$

$x-c$ is a factor of $P(x)$

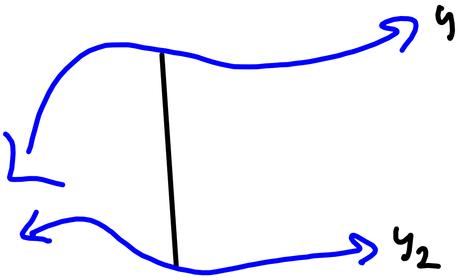
Maximize the vertical distance between

$y = x+2$ & $y = x^2$

between $x=-1$ and $x=2$

$y_1 - y_2$ if you know y_1's on top.

In general, it's $|y_1 - y_2|$



Maximize $f(N) = \dfrac{kN}{N^2+1}$ k = Konstant.

Maximize the vertical distance between

$y = x+2$ & $y = x^2$

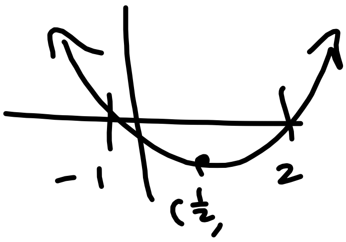
between $x=-1$ and $x=2$

Vertical Distance

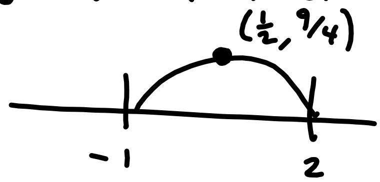
$|x+2-x^2| = |-x^2+x+2| = |x^2-x-2| =$

$x^2-x-2 = (x-2)(x+1)$

$\frac{2+(-1)}{2} = \frac{1}{2} = x$



So $|x^2-x-2|$ looks like

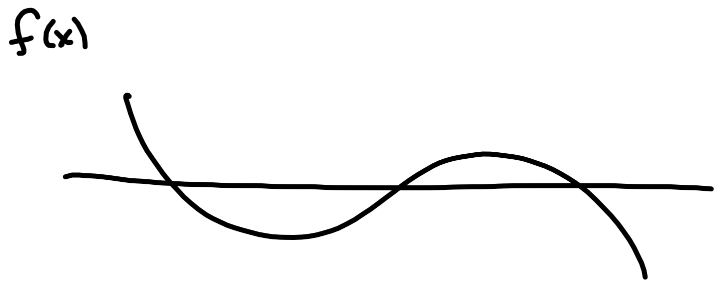


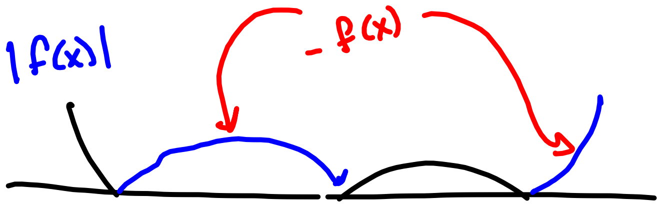
$|x^2-x-2|\Big|_{x=\frac{1}{2}}$

$= |(\tfrac{1}{2})^2 - (\tfrac{1}{2}) - 2|$

$= |\tfrac{1}{4} - \tfrac{2}{4} - \tfrac{8}{4}| = |-\tfrac{9}{4}| = \tfrac{9}{4}$

$\frac{9}{4}$ is max distance.

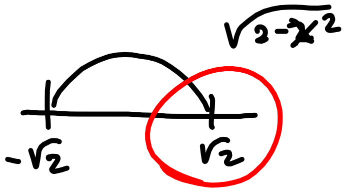




$$|f(x)| = \begin{cases} f(x) & \text{if } f(x) \geq 0 \\ -f(x) & \text{if } f(x) < 0 \end{cases}$$

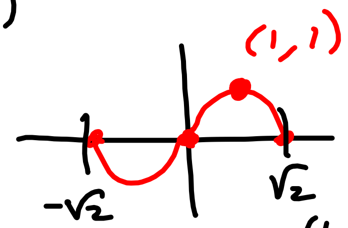
§3.5 #26

$$f(x) = x\sqrt{2-x^2} = x(2-x^2)^{\frac{1}{2}}$$

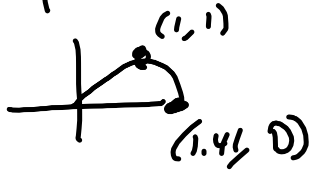


$$f'(x) = (2-x^2)^{\frac{1}{2}} + x\left(\frac{1}{2}(2-x^2)^{-\frac{1}{2}}(-2x)\right)$$

$$= (2-x^2)^{\frac{1}{2}} + \frac{-x^2}{(2-x^2)^{1/2}}$$



$$= \frac{(2-x^2)^{\frac{1}{2}}}{1} \cdot \frac{(2-x^2)^{\frac{1}{2}}}{(2-x^2)^{\frac{1}{2}}} - \frac{x^2}{(2-x^2)^{1/2}}$$



$$= \frac{2-x^2-x^2}{(2-x^2)^{1/2}} = \frac{2-2x^2}{(2-x^2)^{1/2}} \overset{SET}{=} 0 \implies x = \pm 1$$

$$\overset{SET}{=} \nexists \implies x = \pm\sqrt{2}$$

Set denominator $= 0$

$$x + \cos x \overset{SET}{=} 0$$

That's a 3.8 or grapher question

Newton's Method

$$f'(x) = \frac{2-2x^2}{(2-x^2)^{1/2}} \Rightarrow$$

$$f''(x) = \frac{-4x(2-x^2)^{\frac{1}{2}} - (2-2x^2)\frac{1}{2}(2-x^2)^{-\frac{1}{2}}(-2x)}{2-x^2}$$

$$= \frac{\dfrac{-4x(2-x^2)^{\frac{1}{2}}}{1} \cdot \dfrac{(2-x^2)^{\frac{1}{2}}}{(2-x^2)^{1/2}} + \dfrac{(2-2x^2)(x)}{(2-x^2)^{1/2}}}{2-x^2}$$

$$= \frac{-4x(2-x^2)^{\frac{1}{2}+\frac{1}{2}} + 2x-2x^3}{(2-x^2)(2-x^2)^{1/2}}$$

$$= \frac{-4x(2-x^2) + 2x - 2x^3}{\sim\sim} = \frac{-8x+4x^3+2x-2x^3}{\sim\sim}$$

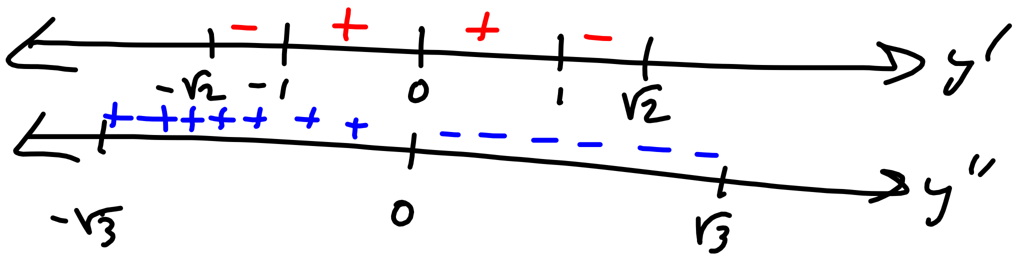
$$= \frac{2x^3-6x}{(2-x^2)(2-x^2)^{1/2}} \overset{\text{SET}}{=} 0 \Rightarrow 2x(x^2-3)=0$$

$$\Rightarrow x=0 \text{ or } x=\pm\sqrt{3}$$

Not in Domain

Set denom $=0$

$x = \pm\sqrt{2}$



Test $x=1$ in y'': $\frac{2-6}{(2-1)^{3/2}}$ = neg.

$$\frac{1}{0}$$

$$\frac{1}{\sqrt{3}}$$

$2-x^2=0$

$2=x^2$

$\sqrt{2}=\sqrt{x^2}$

$\sqrt{2}=|x|$

$\pm\sqrt{2}=x$

$(-\sqrt{2})^2=2$