16. $\lim_{x \to \infty} \frac{x^2}{\sqrt{x^4 + 1}}$

18. $\lim_{x \to -\infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1}$

22. $\lim_{x \to \infty} \cos x$

§3.4

2 videos up!

Test 2
2 Thursdays
ago
(Not
counting
Spring Break.

⑯ $\frac{x^2}{\sqrt{x^4+1}} = \frac{x^2}{\sqrt{x^4\left(1+\frac{1}{x^4}\right)}}$

$\sqrt{x^4}\sqrt{1+\frac{1}{x^4}}$

$= \frac{x^2}{x^2\sqrt{1+\frac{1}{x^4}}} = \frac{1}{\sqrt{1+\frac{1}{x^4}}}$

$\xrightarrow{x \to \infty} 1$

$$|x^2| = |x|^2$$

$$\lim_{x \to -\infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1} = \lim_{x \to -\infty} \frac{\sqrt{x^6}\sqrt{9 - \frac{1}{x^5}}}{x^3\left(1 + \frac{1}{x^3}\right)} = \lim_{x \to -\infty} \frac{|x|^3\sqrt{9 - \frac{1}{x^5}}}{x^3\left(1 + \frac{1}{x^3}\right)}$$

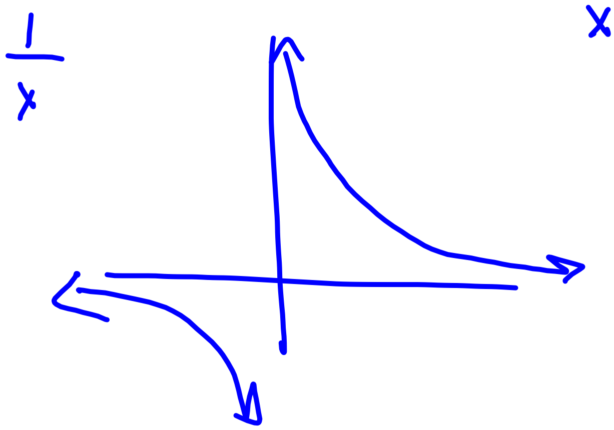
$$= \lim_{x \to -\infty} \frac{-x^3\sqrt{9 - \frac{1}{x^5}}}{x^3\left(1 + \frac{1}{x^3}\right)} = \lim_{x \to -\infty} \frac{-\sqrt{9 - \frac{1}{x^5}}}{1 + \frac{1}{x^3}} = -3$$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$\boxed{\lim_{x\to\infty} \cos x}$ $\nexists$

$\lim_{x\to\infty} f(x) = L$

means that if $\varepsilon > 0$ is given, then there is an $M > 0$ such that $|f(x) - L| < \varepsilon$ whenever $x > M$



Claim $\lim_{x\to\infty} \frac{1}{x} = 0$

Let $\varepsilon = \frac{1}{12}$. Let $M=12$. Then if $x > 12$, $\frac{1}{x} < \frac{1}{12} = \varepsilon$

Scratch: want $\left|\frac{1}{x} - 0\right| < \frac{1}{12}$

$\left|\frac{1}{x}\right| < \frac{1}{12}$

$x >$ Big $\longrightarrow$ $\frac{1}{x} < \frac{1}{12}$

$M = 12 < x$

Let $\varepsilon > 0$.

Define $M = \frac{1}{\varepsilon}$

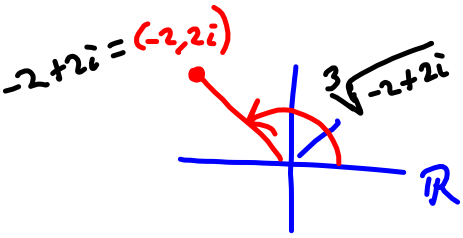
Then

$x > M \Rightarrow$

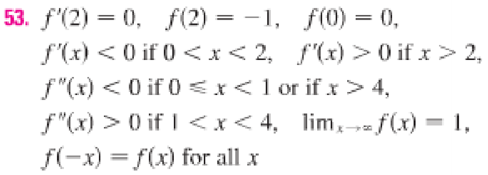
$\frac{1}{x} < \frac{1}{M} = \varepsilon$

$$\sqrt{x^4} = |x^2| = x^2$$

$$\sqrt{25} = 5$$



Principal Root gets the radical sign.



53. $f'(2) = 0, \quad f(2) = -1, \quad f(0) = 0,$
$f'(x) < 0$ if $0 < x < 2, \quad f'(x) > 0$ if $x > 2,$
$f''(x) < 0$ if $0 \leqslant x < 1$ or if $x > 4,$
$f''(x) > 0$ if $1 < x < 4, \quad \lim_{x \to \infty} f(x) = 1,$
$f(-x) = f(x)$ for all x

A. Domain It's often useful to start by determining the domain D of f, that is, the set of values of x for which $f(x)$ is defined.

B. Intercepts The y-intercept is $f(0)$ and this tells us where the curve intersects the y-axis. To find the x-intercepts, we set $y = 0$ and solve for x. (You can omit this step if the equation is difficult to solve.)

C. Symmetry

(i) If $f(-x) = f(x)$ for all x in D, that is, the equation of the curve is unchanged when x is replaced by $-x$, then f is an **even function** and the curve is symmetric about the y-axis. This means that our work is cut in half. If we know what the curve looks like for $x \geqslant 0$, then we need only reflect about the y-axis to obtain the complete curve [see Figure 3(a)]. Here are some examples: $y = x^2$, $y = x^4$, $y = |x|$, and $y = \cos x$.

(ii) If $f(-x) = -f(x)$ for all x in D, then f is an **odd function** and the curve is symmetric about the origin. Again we can obtain the complete curve if we know what it looks like for $x \geqslant 0$. [Rotate 180° about the origin; see Figure 3(b).] Some simple examples of odd functions are $y = x$, $y = x^3$, $y = x^5$, and $y = \sin x$.

(iii) If $f(x + p) = f(x)$ for all x in D, where p is a positive constant, then f is called a **periodic function** and the smallest such number p is called the **period.** For instance, $y = \sin x$ has period 2π and $y = \tan x$ has period π. If we know what the graph looks like in an interval of length p, then we can use translation to sketch the entire graph (see Figure 4).

D. Asymptotes

(i) *Horizontal Asymptotes.* Recall from Section 3.4 that if either $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$, then the line $y = L$ is a horizontal asymptote of the curve $y = f(x)$. If it turns out that $\lim_{x\to\infty} f(x) = \infty$ (or $-\infty$), then we do not have an asymptote to the right, but that is still useful information for sketching the curve.

(ii) *Vertical Asymptotes.* Recall from Section 1.5 that the line $x = a$ is a vertical asymptote if at least one of the following statements is true:

$$\boxed{1} \qquad \lim_{x\to a^+} f(x) = \infty \qquad \lim_{x\to a^-} f(x) = \infty$$

$$\lim_{x\to a^+} f(x) = -\infty \qquad \lim_{x\to a^-} f(x) = -\infty$$

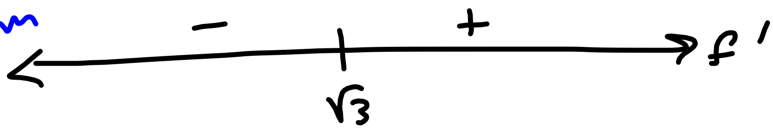
(For rational functions you can locate the vertical asymptotes by equating the denominator to 0 after canceling any common factors. But for other functions this method does not apply.) Furthermore, in sketching the curve it is very useful to know exactly which of the statements in $\boxed{1}$ is true. If $f(a)$ is not defined but a is an endpoint of the domain of f, then you should compute $\lim_{x\to a^-} f(x)$ or $\lim_{x\to a^+} f(x)$, whether or not this limit is infinite.

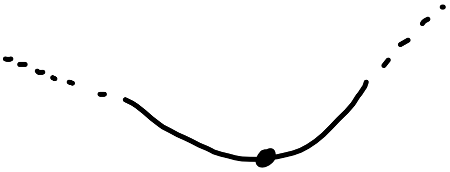
(iii) *Slant Asymptotes.* These are discussed at the end of this section.

E. Intervals of Increase or Decrease Use the I/D Test. Compute $f'(x)$ and find the intervals on which $f'(x)$ is positive (f is increasing) and the intervals on which $f'(x)$ is negative (f is decreasing).

3.5 Suppose I know $f'(x)=0$ @ $x=\sqrt{3}$

Sign pattern for f'!





$(\sqrt{3}, f(\sqrt{3}))$ MIN

Finding the exact value here is not time-effective on a test.

§3.5 #s 4, 11, 26, 34

3.5 probs:

30. $y = x^{5/3} - 5x^{2/3}$ **27.** $y = \frac{\sqrt{1 - x^2}}{x}$

22. $y = 2\sqrt{x} - x$

$\mathcal{D}(f) = \{x \mid x \neq 0 \text{ and } -1 \leq x \leq 1\}$

$= [-1,0) \cup (0,1]$

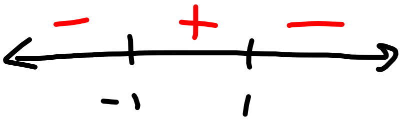
Domain

Need

$x \neq 0$

$1 - x^2 \geq 0$

$(1-x)(1+x) \geq 0$



Need $-1 \leq x \leq 1$

(27) $y = \frac{(1-x^2)^{\frac{1}{2}}}{x} \longrightarrow$

$$y' = \frac{\frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) \times - (1-x^2)^{\frac{1}{2}}(1)}{x^2}$$

$$\frac{\frac{-x^2}{(1-x^2)^{1/2}} - \frac{(1-x^2)^{\frac{1}{2}}}{1} \cdot \frac{(1-x^2)^{\frac{1}{2}}}{(1-x^2)^{\frac{1}{2}}}}{x^2}$$

$$= \frac{\frac{-x^2 - (1-x^2)}{(1-x^2)^{\frac{1}{2}}}}{\frac{x^2}{1}} = \frac{-x^2 - 1 + x^2}{(1-x^2)^{\frac{1}{2}}} \cdot \frac{1}{x^2}$$

$1 - x^2 = 0$

$x = \pm 1$

$$= \frac{-1}{x^2(1-x^2)^{\frac{1}{2}}} = 0 \longrightarrow$$ Nevah!

C.P.: $x = 0$, $x = \pm 1$