§3.2

6. Show that $(x-3)^{-2}$ does *not* yield a c in $(1,4)$ such that $f'(c)=m_{avg}=\dfrac{f(b)-f(a)}{b-a}$ on the interval.

Why does this *not* violate the Mean Value Theorem?

$= m_{AVG}$ on $[a,b]$

RAA means Reductio Ad Absurdum

Assume it's true. If it's NOT, then you'll arrive at an absurdity, a contradiction —X

$$m_{avg}=\frac{f(b)-f(a)}{b-a}=\frac{f(4)-f(1)}{4-1}=\frac{(4-3)^{-2}-(1-3)^{-2}}{3}$$

$$=\frac{\frac{1}{(1)^2}-\frac{1}{(-2)^2}}{3}=\frac{1-\frac{1}{4}}{3}=\frac{\frac{3}{4}}{3}=\frac{1}{4}$$

Suppose there IS such a c. Find it.

$f(x)=(x-3)^{-2}\implies f'(x)=-2(x-3)^{-3}\overset{SET}{=}m_{AVG}=\frac{1}{4}$

$$\implies \frac{-2}{(x-3)^3}=\frac{1}{4}$$

$$-8=(x-3)^3$$

$$(x-3)^3=-8$$

$$\sqrt[3]{(x-3)^3}=\sqrt[3]{-8}$$

$$x-3=-2$$

$x=1=c$ & it's the ONLY real sol'n to $f'(x)=m_{avg}$.

But $1\notin(1,4)$, so $\nexists\ c\in(1,4)\ni f'(c)=m_{AVG}$.

This does NOT violate MVT, b/c it doesn't satisfy the hypotheses of MVT:

① cnt^s on $[1,4]$? Nope
Blows up @ $x=3$! $f(3)\ \nexists$

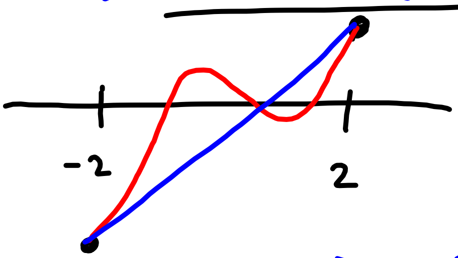
② No dif^{ble} on $(1,4)$, same reason.
$f'(3)\ \nexists$.

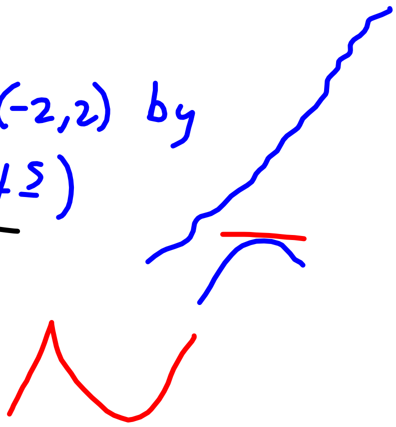
7. Show that $f(x) = 2x + \cos(x)$ has *exactly* one root in the interval $[-2, 2]$.

$f(-2) = 2(-2) + \cos(-2) < 0$

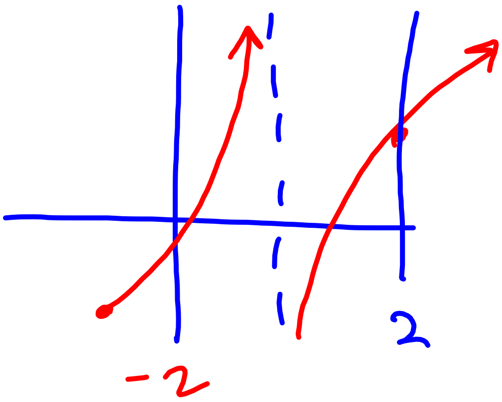
$f(2) = 2(2) + \cos(2) > 0$

$\therefore \exists$ at least one root in $(-2,2)$ by IVT (cosine & polys are cnt^{s})





$f'(x) = 2 - \sin x = -\sin x + 2 > 0$
and it exists on $[-2,2]$ (actually $\forall x \in \mathbb{R}$)

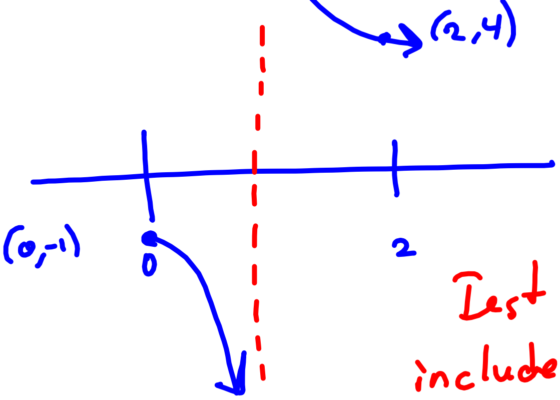


Can't happen!
$2 - \sin x$ is continuous

9. Does there exist a function f such that $f(0)=-1$, $f(2)=4$, and $\boxed{f'(x)\le 2}$ for all x?

Implies it's dif'ble

Angela thought yes, at first, b/c



Here's her Counterexample.

Test question, I'd include "Does there exist a differentiable function such that blah blah blah...

$$\frac{f(b)-f(a)}{b-a} = \frac{4+1}{2} = \frac{5}{2} = 2.5$$

m avg. not x-value.

MVT says $\exists\, c \in (1,4) \ni f'(c)=2.5$

But $f'(c)\le 2$ makes that impossible.

$$\boxed{\begin{array}{c} f'(c)\le 2 < 2.5 \Rightarrow \\ f'(c) \ne 2.5, \text{ ever.}\end{array}}$$

$F(x) = x(6-x)^{\frac{1}{2}}$ $\mathcal{D} = \{x \mid 6-x \geq 0\} = \{x \mid 6 \geq x\}$

$= (-\infty, 6]$

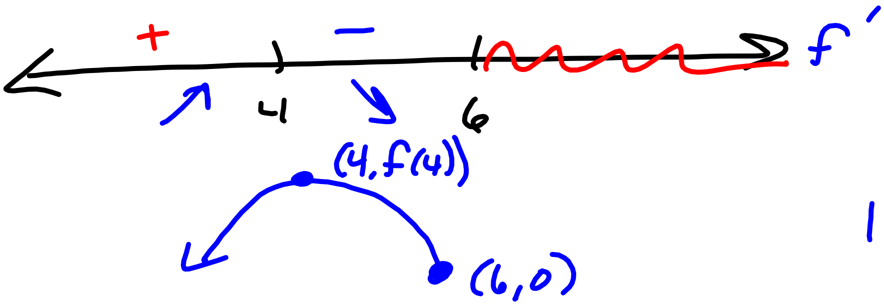
$\Rightarrow F'(x) = (6-x)^{\frac{1}{2}} + x\left(\frac{1}{2}(6-x)^{-\frac{1}{2}}\right)(-1)$

$= \frac{(6-x)^{\frac{1}{2}}}{1} \cdot \frac{2(6-x)^{\frac{1}{2}}}{2(6-x)^{\frac{1}{2}}} - \frac{x}{2(6-x)^{\frac{1}{2}}}$

$= \frac{2(6-x) - x}{2(6-x)^{1/2}} = \frac{12-2x-x}{\sim\sim} = \frac{12-3x}{\sim\sim}$

$= \boxed{\frac{3(4-x)}{2(6-x)^{1/2}}} \overset{\text{SET}}{=} 0 \Rightarrow 4-x=0 \Rightarrow \boxed{x=4}$

set $(6-x)^{\frac{1}{2}} = 0 \Rightarrow \boxed{x=6}$



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$$\frac{\frac{1}{4}}{\frac{3}{1}} = \frac{1}{12}$$

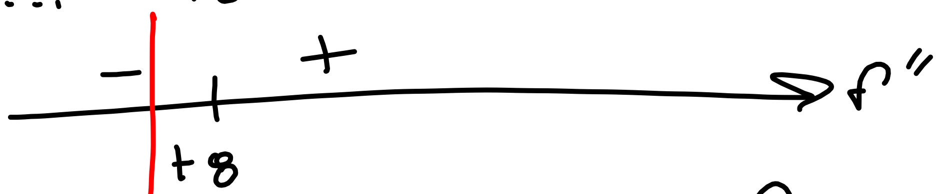
$$f''(x) = \frac{d}{dx}\left[\frac{3}{2}\cdot\frac{4-x}{(6-x)^{\frac{1}{2}}}\right]$$

$$= \frac{3}{2}\left[\frac{-1(6-x)^{\frac{1}{2}} + (4-x)\left(\frac{1}{2}(6-x)^{-\frac{1}{2}}\right)}{(6-x)}\right]$$

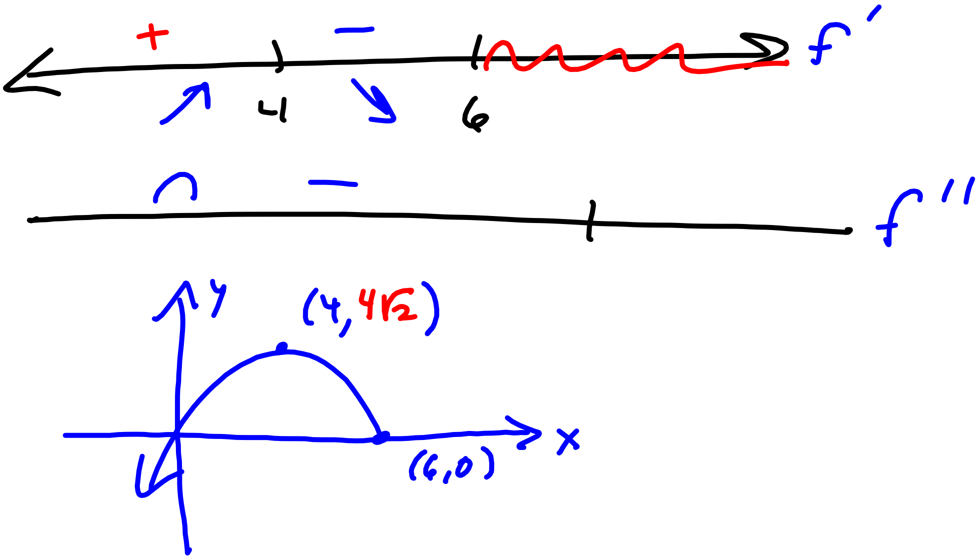
$$= \frac{3}{2}\left[\frac{-\frac{(6-x)^{\frac{1}{2}}}{1}\cdot\frac{2(6-x)^{\frac{1}{2}}}{2(6-x)^{\frac{1}{2}}} + \frac{4-x}{2(6-x)^{1/2}}}{6-x}\right]$$

$$= \frac{3}{2}\left[\frac{-2(6-x)+4-x}{2(6-x)(6-x)^{1/2}}\right] = \frac{-12+2x+4-x}{2(6-x)^{3/2}} = \frac{-8+x}{2(6-x)^{3/2}}$$

.... $x = +8$



Based on this, $f'' < 0$ on $\mathcal{D}$


+
−
4
6
f'
∩
−
f''
y
(4, 4√2)
(6,0)
x