Show that $f(x) = x^3 - 6x^2 + 15x - 7$ has no tangent line with slope $m = -2$

Suppose it does then §2.3 II #7

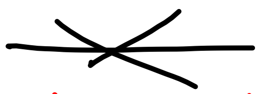
$f'(x) = 3x^2 - 12x + 15 = -2$ has a solution

$\Rightarrow 3x^2 - 12x + 17 = 0$

$$\begin{array}{r} 17 \\ 12 \\ \hline 34 \\ 170 \\ \hline 204 \end{array}$$

Discriminant $= b^2 - 4ac = (-12)^2 - 4(3)(17)$

$= 144 - 204 < 0$



$\therefore f'(x) \neq -2 \;\; \forall x \in \mathbb{R}$

§3.3 I #s 3, 5, 8, 9, 11, 13, 15, 17

§3.3 II #s 29–39

Increasing/Decreasing Test

(a) If $f'(x) > 0$ on an interval, then f is increasing on that interval. ↗

(b) If $f'(x) < 0$ on an interval, then f is decreasing on that interval. ↘

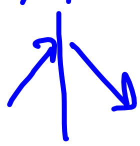
The First Derivative Test Suppose that c is a critical number of a continuous function f.

(a) If f' changes from positive to negative at c, then f has a local maximum at c.

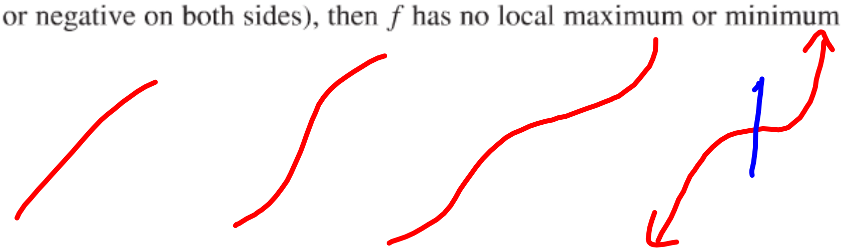
(b) If f' changes from negative to positive at c, then f has a local minimum at c.

(c) If f' does not change sign at c (for example, if f' is positive on both sides of c or negative on both sides), then f has no local maximum or minimum at c.

MAX

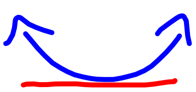


c

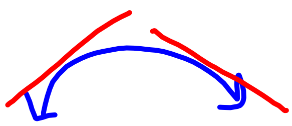


Definition If the graph of f lies above all of its tangents on an interval I, then it is called **concave upward** on I. If the graph of f lies below all of its tangents on I, it is called **concave downward** on I.

C-up



$f'' > 0$

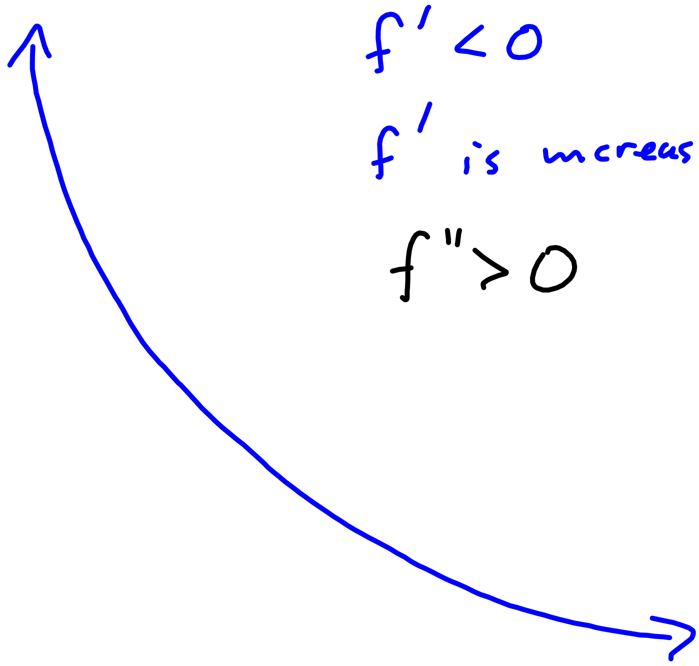


$f'' < 0$

$f' < 0$

f' is increasing

$f'' > 0$



f

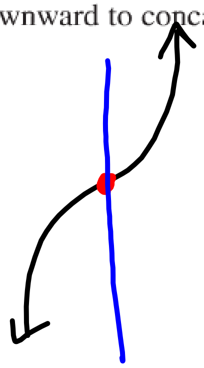
Concavity Test

(a) If $f''(x) > 0$ for all x in I, then the graph of f is concave upward on I.

(b) If $f''(x) < 0$ for all x in I, then the graph of f is concave downward on I.

Definition A point P on a curve $y = f(x)$ is called an **inflection point** if f is continuous there and the curve changes from concave upward to concave downward or from concave downward to concave upward at P.

ex x^3

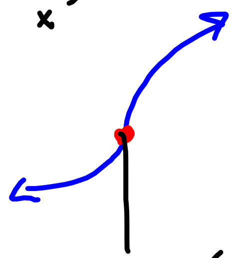


$f''=0$

$x^{\frac{1}{3}}$

$\frac{1}{3}x^{-2/3} = y'$

$-\frac{2}{9}x^{-5/3} = y''$



$f'' \nexists$

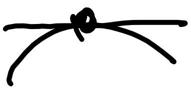
$f' \nexists$

The Second Derivative Test Suppose f'' is continuous near c.

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

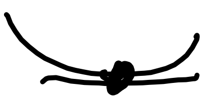
(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

$f'=0, f''<0$



MAX

$f'=0 \quad f''>0$



MIN

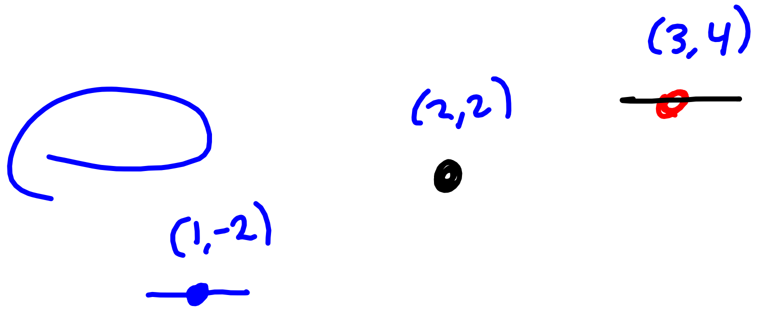
4. (10 pts) Suppose a function g satisfies all of the following properties. Sketch a graph of g that incorporates all of the following properties into it:

$g(1) = -2 \quad g(2) = 2 \quad g(3) = 4$

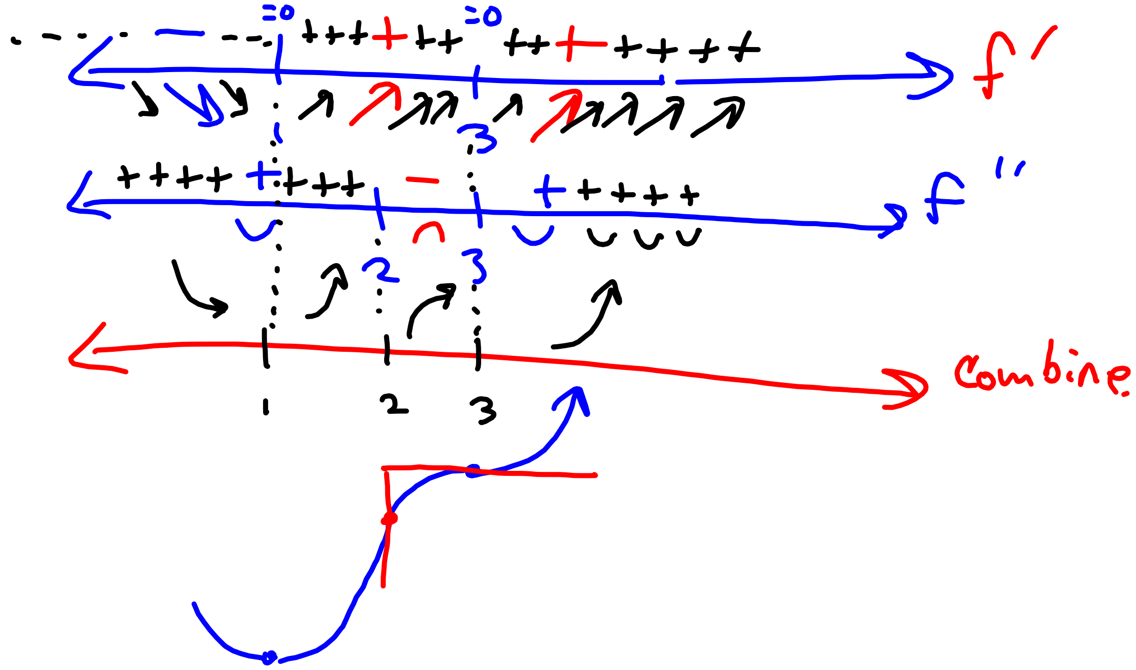
$g'(1) = 0 \quad g'(3) = 0$

$g'(x) > 0$ on $(1,3) \cup (3,\infty)$, $\quad g'(x) < 0$ on $(-\infty,1)$

$g''(x) > 0$ on $(-\infty,2) \cup (3,\infty)$, $\quad g''(x) < 0$ on $(2,3)$



$g'(x) > 0$ on $(1,3) \cup (3,\infty)$



3. (10 pts) Let $f(x) = -2\sin(x)\cos(x) - x$. Find all local extrema in the interval $[0, 2\pi]$.

$$f'(x) = -2\cos^2 x + \cancel{+}2\sin^2 x - 1$$

$$= 2\sin^2 x - 2\cos^2 x - 1$$

$$f''(x) = 4\sin x \cos x + 4\cos x \sin x$$

$f'(x) \overset{\text{set}}{=} 0 \implies$

$$2\sin^2 x - 2\cos^2 x - 1 = 0$$

$$2\sin^2 x - 2(1 - \sin^2 x) - 1 = 0$$

$$2\sin^2 x - 2 + 2\sin^2 x - 1 = 0$$

$$4\sin^2 x - 3 = 0 \qquad ax^2 + bx + c = 0$$

$$4u^2 - 3 = 0$$

$$a = 4, b = 0, c = 3$$

$$b^2 - 4ac = -4(4)(-3)$$

$$= 48$$

$$x = \frac{0 \pm \sqrt{48}}{2(4)} = \frac{\pm 4\sqrt{3}}{8} = \pm\frac{\sqrt{3}}{2}$$

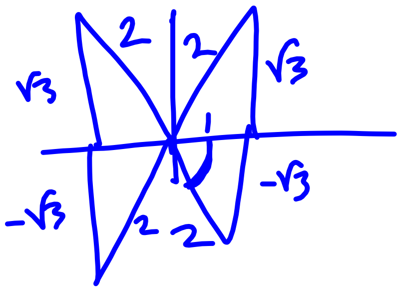
$4u^2 - 3 = 0$

$4u^2 = 3$

$u^2 = \frac{3}{4}$

$u = \pm\frac{\sqrt{3}}{2}$

$\sin x = \pm\frac{\sqrt{3}}{2}$



$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

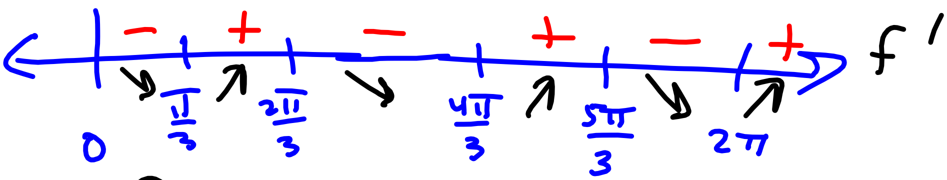
$$4\left(\sin x - \frac{\sqrt{3}}{2}\right)\left(\sin x + \frac{\sqrt{3}}{2}\right)$$

$x = c$ makes it zero

$x - c$ is a factor.

$x = 0 \implies \sin x = 0$

$$4\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$



Mins @ $x = \frac{\pi}{3}, \frac{4\pi}{3}, 2\pi$

Max's @ $x = 0, \frac{2\pi}{3}, \frac{5\pi}{3}$

Suppose $f'(x) = (x-2)(x+5)^2(x-3)^3(x+11)$