What's the deal on all this "continuous on closed interval" and "differentiable on the open interval" stuff? How do we *know* if a function's continuous or not?

BY GETTING A HANDLE ON ALL THE DOMAIN STUFF FROM PREVIOUS CLASSES!!! BASICALLY, IF IT'S IN THE DOMAIN, THEN *F* IS CONTINUOUS THERE, FOR JUST ABOUT ANY FUNCTION WE CAN WRITE, DAGNABBIT!!!

Continuity & Differentiability

$\mathcal{D}(f)$ & $\mathcal{D}(f')$

$\mathbb{Z}$

$\mathbb{Q}$ $\frac{r}{s}$

$\mathbb{R}$

$\mathcal{D}$ = Domain = $\{x \mid f(x) \text{ is defined}\}$

Pretty much everything IS defined, except...

$\sqrt[2n]{\text{Negative}}$ $\qquad \frac{\text{Stuff}}{0}$ $\qquad \forall n \in \mathbb{Z}$

"for x such that x is an integer."

$\forall x \in \mathbb{Z}$

$\in$ is an element / member of

§3.1 # 14

I dropped a sign.

§3.1 & 3.2
Understanding the hypotheses of the theorems.

FINDING that pesky 'c'!

Some examples in MVT context from trig.

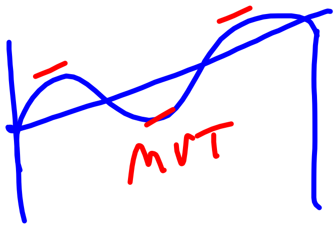
$f(\theta) = 2\cos\theta + \cos^2\theta, \quad 0 \leqslant \theta \leqslant 2\pi$ $f(0) = f(2\pi)$

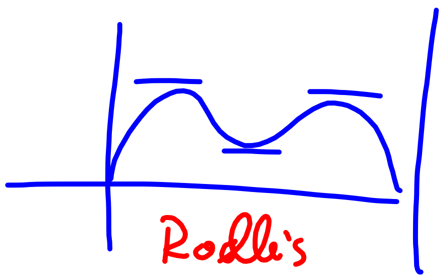
$S(x) = x - \sin x, \quad 0 \leqslant x \leqslant 4\pi$ $f(0) = 0, f(4\pi) = 4\pi$

MVT $\exists\, c \in (0, 4\pi) \ni$

$f'(c) = \dfrac{f(4\pi) - f(0)}{4\pi - 0} = 1 = m_{avg}$

$f(x) = 2\cos x + \sin^2 x$





$f(\theta) = 2\cos\theta + \cos^2\theta, \quad 0 \leqslant \theta \leqslant 2\pi$

f is cnt$^{\underline{s}}$ on $[0, 2\pi]$, because cosine is continuous on its domain, and all positive powers. Also, the sum of two cnt^s functions is cnt$^{\underline{s}}$

$f'(\theta) = -2\sin\theta + (2\cos\theta)(-\sin\theta) = -2\sin\theta - 2\sin\theta\cos\theta.$

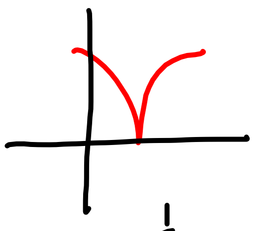
Defined, because sine & cosine are defined and so is their product.

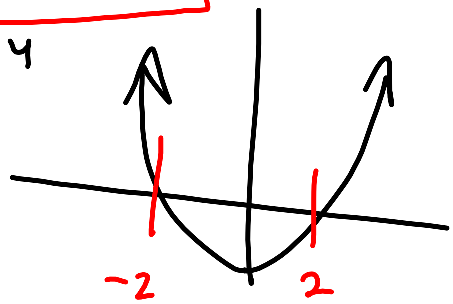
SUBTLE POINT: The Theorem does NOT say that f' has to be continuous (although it usually is.) It JUST has to be DEFINED!

$f(x)=\sqrt{x^2-4}$ on $[-1, 3]$

$\boxed{(x-2)^{2/3} \text{ on } [1,3]}$

x^2-4

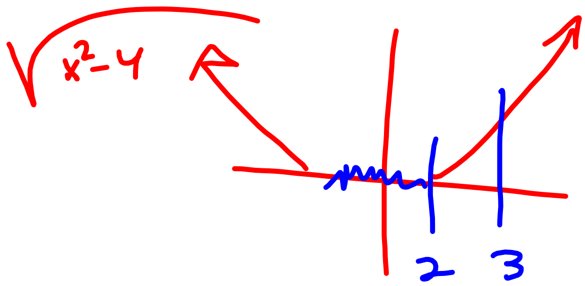




Does not satisfy hypo of MVT. Not Cnt's on $[-1,3]$!

$\frac{2}{3}(x-2)^{-\frac{1}{3}}$

$= \frac{2}{3\sqrt[3]{x-2}}$



$(x-2)^{2/3}$ on $[1,3]$

$f(1) = (-1)^{2/3} = \left((-1)^2\right)^{\frac{1}{3}} = (1)^{\frac{1}{3}} = 1$

$f(3) = 1^{2/3} = 1$

why no c in $(1,3) \ni f'(c) = 0$

B/c $f'(2) \nexists$.

$2 \in (1,3)$