

Test 2, Spring '09, looks pretty decent.

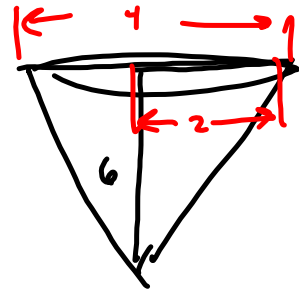
<http://www.harryzaims.com/201/201-spring-11/tests/old-tests/test-2-spr-09.pdf>

Solutions

<http://www.harryzaims.com/201/201-spring-11/tests/old-tests/test-2-spr-09-solns.pdf>

Out: $\frac{dV_o}{dt} = \frac{10000 \text{ cm}^3}{\text{min}}$

$\frac{1}{3}\pi r^2 h$



9. Water is leaking out of an inverted conical tank at a rate of 10,000 cm³/min at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4 m. If the water level is rising at a rate of 20 cm/min when the height of the water is 2 m, find the rate at which water is being pumped into the tank.

want IN: $\frac{dV_i}{dt} = ?$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\left(\frac{1}{3}h\right)^2 \pi h = \frac{1}{27}\pi h^3$

$\Rightarrow \frac{dV}{dt} = 3\left(\frac{1}{27}\right)\pi h^2 \frac{dh}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt}$

$\frac{130}{2} = 65$

$\left. \frac{dh}{dt} \right|_{h=200\text{cm}} = \frac{20 \text{ cm}}{\text{min}} \Rightarrow$

$\frac{dV}{dt} = \frac{\pi}{9} (200)^2 \cdot (20) = \frac{8\pi}{9} \times 10^5$

Want IN rate.

$= I_N - \text{Out}$

$= I_N - 10,000 = \frac{8\pi}{9} \times 10^5$

x-nought

$$\frac{1}{a^2}x^2 - \frac{1}{b^2}y^2 = 1$$

want tan line @ (x_0, y_0)

Assuming y is implicitly a function

Tangent line

$$y = f'(x)(x - x_0) + y_0$$

$$= m_{\text{tan}} = \left. \frac{dy}{dx} \right|_{x=x_0}$$

$$-\frac{yy'}{b^2} = -\frac{x}{a^2}$$

$$y' = \frac{b^2}{y} \cdot \frac{x}{a^2} = \frac{b^2}{a^2} \cdot \frac{x}{y}$$

$$\left. y' \right|_{(x_0, y_0)} = \frac{b^2 x_0}{a^2 y_0} \Rightarrow y = \frac{b^2 x_0}{a^2 y_0} (x - x_0) + y_0$$