§2.6 $\overset{f\,g}{x^3y^4} - 7\overset{f\,g}{x^2y^3} + 11\overset{fg}{xy} + 12\overset{f}{x^2} = \pi$

Find $\frac{dy}{dx} = y'$

says y is assumed, implicitly, to be a function of x, so y^2 derivative entails chain rule. Product & chain rule happenin'!

$f'g + fg'$
$= (fg)'$

$$\boxed{\frac{d}{dx}\left[f(x)^4\right] = 4f(x)^3 \cdot \frac{df}{dx} \quad \text{chain.}}$$

$$3x^2y^4 + (x^3)\boxed{(4y^3y')} - 14xy^3 - 21x^2y^2y' + 11y + 11xy' + 24x = 0$$

$$-7(2x)y^3 - 7x^2(3y^2y')$$

$$y'\left[4x^3y^3 - 21x^2y^2 + 11x\right] = -3x^2y^4 + 14xy^3 - 11y - 24x$$

$$y' = \frac{-3x^2y^4 + 14xy^3 - 11y - 24x}{4x^3y^3 - 21x^2y^2 + 11x}$$

$$\frac{d}{dx}\left[(x^2+7)^5\right] = 5(x^2+7)^4(2x)$$

$$\frac{d}{dx}\left[y^5\right] = 5y^4y'$$

$$x^2 + y^2 = 5 \longrightarrow y^2 = 5 - x^2$$

$$y = \pm\sqrt{5-x^2}$$

$$y = (5-x^2)^{\frac{1}{2}} \text{ is top } \tfrac{1}{2}$$

$$y' = \tfrac{1}{2}(5-x^2)^{-\frac{1}{2}}(-2x)$$

$$= \frac{-x}{\sqrt{5-x^2}} = -\frac{x}{y},$$

since $y = \sqrt{5-x^2}$

$$2x + 2yy' = 0$$

$$2yy' = -2x$$

$$y' = -\frac{x}{y}$$

Section 2.7

The cost, in dollars, of producing x yards of a certain fabric is

$$C(x) = 1200 + 12x - 0.1x^2 + 0.0005x^3$$

(a) Find the marginal cost function.
(b) Find $C'(200)$ and explain its meaning. What does it predict?
(c) Compare $C'(200)$ with the cost of manufacturing the 201st yard of fabric.

$$C := x \to 1200 + 12 \cdot x - .1 \cdot x^2 + .0005 \cdot x^3$$

$$x \to 1200 + 12\,x + (-1) \cdot 0.1\,x^2 + 0.0005\,x^3$$

$$sort(C(x))$$

$$0.0005\,x^3 - 0.1\,x^2 + 12\,x + 1200$$

$$CP := \mathrm{D}(C)$$

$$x \to 12 - 0.2\,x + 0.0015\,x^2$$

$$C(201) - C(200)$$

$$32.2005$$

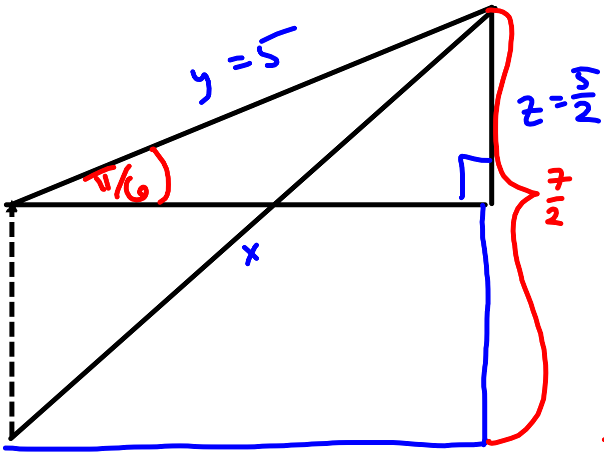
$$CP(200)$$

$$32.0000$$

A plane flying with a constant speed of 300 km/h passes over a ground radar station at an altitude of 1 km and climbs at an angle of 30°. At what rate is the distance from the plane to the radar station increasing a minute later?

@ $t = \frac{1}{60}$,

$y = 5$ Km



$\frac{dy}{dt} = \frac{300 \text{ Km}}{h}$

want $\left.\frac{dx}{dt}\right|_{t=\frac{1}{60}hr}$

$\frac{dz}{dt}$

$z = 5\sin 30$

$z = y\sin 30$

$\frac{dz}{dt} = \frac{dy}{dt}\sin 30 = \frac{1}{2}y'$

A television camera is positioned 4000 ft from the base of a rocket launching pad. The angle of elevation of the camera has to change at the correct rate in order to keep the rocket in sight. Also, the mechanism for focusing the camera has to take into account the increasing distance from the camera to the rising rocket. Let's assume the rocket rises vertically and its speed is 600 ft/s when it has risen 3000 ft.

(a) How fast is the distance from the television camera to the rocket changing at that moment?

(b) If the television camera is always kept aimed at the rocket, how fast is the camera's angle of elevation changing at that same moment?

A lighthouse is located on a small island 3 km away from the nearest point P on a straight shoreline and its light makes four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from P?