1. (15 pts) Find $f'(x)$ by the definition of the derivative (The long way!) for $f(x) = 2x^2 - 5x - 1$

$$\frac{f(x+h)-f(x)}{h} = \frac{2(x+h)^2 - 5(x+h) - 1 - (2x^2 - 5x - 1)}{h}$$

$$= \frac{2(x^2+2xh+h^2) - 5x - 5h - 1 - 2x^2 + 5x + 1}{h}$$

$$= \frac{2x^2 + 4xh + 2h^2 - 5x - 5h - 1 - 2x^2 + 5x + 1}{h}$$

$$= \frac{4xh + 2h^2 - 5h}{h} = \frac{h(4x+2h-5)}{h} = 4x + 2h - 5 \xrightarrow{h \to 0} \boxed{\begin{array}{l} 4x-5 \\ = f'(x) \end{array}}$$

$(h \neq 0)$

2. Find the first derivatives (5 pts each). Do not simplify!

a. $f(x) = x^2 - x^{-3} + 2\sqrt[3]{x} + \frac{2}{\sqrt{x}} + 111.234 \Rightarrow$

$2\sqrt[3]{x} = 2x^{\frac{1}{3}}$

$\frac{2}{\sqrt{x}} = 2x^{-\frac{1}{2}}$

$$\boxed{f'(x) = 2x + 3x^{-4} + \frac{2}{3}x^{-2/3} - x^{-3/2}}$$

b. $g(x)=\dfrac{6x^5+2x^3-5x}{x^3-1}$ $\Longrightarrow$ $\dfrac{f'g-fg'}{g^2}$

$$g'(x)=\frac{(30x^4+6x^2-5)(x^3-1)-(6x^5+2x^3-5)(3x^2)}{(x^3-1)^2}$$

c. $h(x) = \sin^2\left(x^2 + \cos(x)\right) = \left(\sin\left(x^2+\cos(x)\right)\right)^2$

$$2\left(\sin\left(x^2+\cos(x)\right)\right)\left(\cos\left(x^2+\cos(x)\right)\right)\left(2x-\sin(x)\right)$$

d. $(x^2 - 7x)\sin(2x) = y' \Rightarrow$ $f'g + fg'$

$y' = (2x-7)\sin(2x) + (x^2 - 7x)(\cos(2x))(2)$

3. (10 pts) Find an equation of the tangent line to $f(x) = \sqrt[3]{x^2}$ at the point $P = (1,1)$

$= x^{2/3}$

$\Rightarrow f'(x) = \frac{2}{3}x^{-\frac{1}{3}} \Rightarrow f'(1) = \frac{2}{3}(1)^{-\frac{1}{3}} = \frac{2}{3} = m_{tan}$

$y = m(x - x_1) + y_1$ $\qquad (x_1, y_1) = (1,1)$

$$\boxed{y = \frac{2}{3}(x-1) + 1}$$

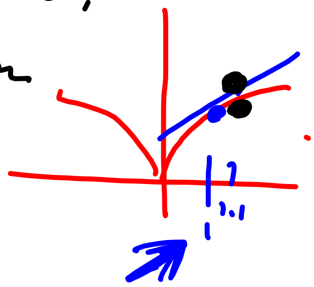
4. (5 pts) Estimate $\sqrt[3]{(1.1)^2}$ using the Linearization of a particular function f at a handy value of x.

$f(x) = \sqrt[3]{x^2}$

$f(1.1)$ from #3 !

$L_1(x) = $ tangent line $=$ linearization

$= \frac{2}{3}(x-1)+1 \implies$



$f(1.1) \approx L_1(1.1) = \frac{2}{3}(1.1-1)+1$

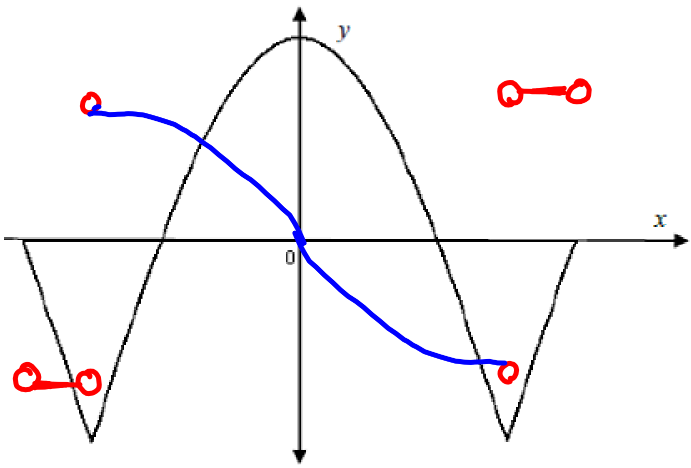
$\boxed{= \frac{2}{3}(.1)+1}$

$\approx (.66667)(.1)+1$

$= .066667 + 1$

$= 1.066667 \approx \sqrt[3]{(1.1)^2}$

5. (10 pts) The graph of a function f is shown. Sketch the graph of f' on the same set of axes.



6. (15 pts) Find $\frac{dy}{dx}$, given $x^2y^3 - 5x^2 - 5y^2 = y^3 + 11.3$ $\Rightarrow$

$$2xy^3 + x^2(3y^2y') - 10x - 10yy' = 3y^2y'$$

$$3x^2y^2y' - 10yy' - 3y^2y' = -2xy^3 + 10x$$

$$y'(3x^2y^2 - 10y - 3y^2) = -2xy^3 + 10x$$

$$\boxed{y' = \frac{-2xy^3 + 10x}{3x^2y^2 - 10y - 3y^2}}$$

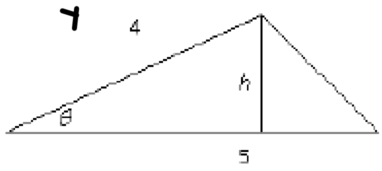
$$\frac{d}{dx}\left[f(x)^7\right] = 7f(x)^6 \cdot f'(x)$$

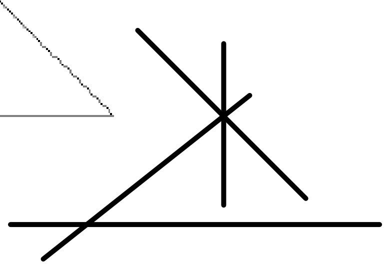
$$\frac{dy}{dx}$$

CHAPTER 2 - DERIVATIVES

2.1	Derivatives and Rates of Change	2	T W	
2.2	The Derivative as a Function	2	R M	
2.3	Differentiation Formulas	2	T W	Last week
2.4	Derivatives of Trigonometric Functions	3	R F	
2.5	The Chain Rule	2	M T	
2.6	Implicit Differentiation	2	W R	This week
2.7	Rates of Change in the Natural and Social Sciences	1	F	
2.8	Related Rates	1	M	
2.9	Linear Approximations and Differentials	1(optional)	T	Next week
Review		1	W	
TEST 2 – Chapter 2		1	R	

7. (15 pts) Two sides of a triangle are 4 cm and 5 cm, respectively. The 3rd side keeps changing, as the angle between the other two sides increases at a rate of 0.5 radians per second. Find the rate at which the *area* of the triangle is changing when the angle between the sides of fixed length is $\frac{\pi}{3}$.





$$\frac{d\theta}{dt} = .5 \frac{\text{radians}}{\text{sec}}$$

want $\left.\frac{dA}{dt}\right|_{\theta=\frac{\pi}{3}}$, where area $= A = \frac{1}{2}bh = \frac{5}{2}h$

$h = ?$

$$\frac{h}{4} = \sin\theta$$

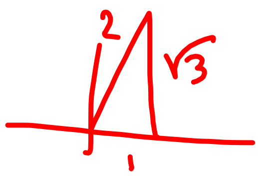
$$h = 4\sin\theta$$

$$A = 10\sin\theta$$

$$\frac{dA}{dt} = 10\cos\theta \cdot \frac{d\theta}{dt} = 10\cos\theta \cdot .5 = 5\cos\theta$$

$$\frac{dA}{d\theta} \cdot \frac{d\theta}{dt}$$

$$\left.\frac{dA}{dt}\right|_{\theta=\frac{\pi}{3}} = 5\cos\frac{\pi}{3} = \frac{5}{2}$$



The radius of an oil spill is increasing at a rate of $5 \frac{m}{min}$. How fast is the area of the spill growing when the radius is 100 m?

I want to paint a ball of radius 10 m with a 1-cm coat of TAR. Estimate the amount of paint needed with (a differential) the tangent line approximation. Hint $V = \frac{4}{3}\pi r^3$.

$y = m(x - x_1) + y_1$

New Volume   old Volume.

Amt Added $= y - y_1 = m(x - x_1)$ 10   10.01

$dy \approx \Delta y = \boxed{\frac{4}{3}\pi(10)^3 - \frac{4}{3}\pi(10.01)^3}$ = Vol of TAR

Let $x - x_1 = \Delta x = dx$

$\frac{dr}{dt}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ $m = m_{tan} = 4\pi(10^2)\frac{dr}{dt}$?

$\Delta r = dr = 1\,cm = .01\,m$

Δ

$\Delta V \approx dV = 4\pi r^2 dr$

$= \boxed{4\pi(10)^2(.01)}$

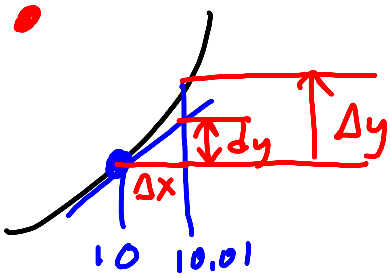
100(.01)

$= 4\pi$

$\frac{4\pi}{3}(10^3) - \frac{4\pi}{3}(10.01)^3$

$= \frac{4\pi}{3}\left[10^3 - 10.01^3\right] = \frac{4\pi}{3}\left[(10 - 10.01)(10^2 + 10 \cdot 10.01 + 10.01^2)\right.$

New y-value $= f(x+\Delta x)$



$$f(x+\Delta x) \approx y = m(x-x_1)+y_1$$

$$f(x+\Delta x) - f(x) = \Delta y = y - y_1$$

$$y - y_1 = f(x+\Delta x) - f(x) \approx m(x-x_1)$$

$$\Delta y \approx m(x-x_1) \qquad m = m_{tan}$$

$$x - x_1 = \Delta x = dx$$

$$dy = m(\Delta x)$$

$$\Delta y \approx dy = f'(x)\,\Delta x = f'(x)\,dx$$

$$\Delta V \approx \left(\frac{dV}{dr}\right)(dr)$$

$$= (4\pi r^2)(.01)$$

$$= 4\pi(100)(.01) = 4\pi$$

$$\Delta V \text{ "=" } 12.578$$

$$4\pi \approx 12.566 =$$

8. (10 pts) The radius of a sphere is measured as 10 cm, with a possible error in measure of 0.15 cm. Use differentials to estimate the maximum possible error in the measurement of the volume of the sphere. Hint: The volume of a sphere is $\frac{4}{3}\pi r^3$.