§2.3, 2.5 questions and solutions are up.

http://harryzaims.com/201/201-spring-15/homework/homework-questions/chapter-02/

Click here

<u>Recall</u> $f(5x)$ grows 5 times faster than $f(x)$, from college algebra, at least in a sense.

$\sqrt{5x}$ achieves a height of $y=2$, when $x=\frac{4}{5}$;

whereas,

$\sqrt{x}$ achieves $y=2$ @ $x=4$.

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Slope of straight line

$$= \frac{\text{Change in } Y}{\text{corresponding change in } X} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$ Algebra →

Calc $\xrightarrow{\Delta x \to 0}$ $\frac{dy}{dx} = \frac{\text{instantaneous change in } y}{\text{instantaneous change in } x}$

This is differentiating with respect to x.

$y = f(x) = x^{16} \Rightarrow \frac{dy}{dx} = 16x^{15}$

$y = f(☺) = ☺^{16} \Rightarrow \frac{dy}{dx} = 0$

But $\frac{dy}{d☺} = 16☺^{15}$

$y = f(27x^2 - 13x) = (27x^2 - 13x)^{16}$

$\Rightarrow \frac{dy}{dx} =$ the topic for §2.5.

But $\frac{dy}{d(27x^2-13x)} = 16(27x^2 - 13x)^{15}$

The Chain Rule says

$$\frac{dy}{dx} = \frac{dy}{d(27x^2-13x)} \cdot \frac{d(27x^2-13x)}{dx}$$

$27x^2 - 13x$ is like the $5x$ in $\sqrt{5x}$

The 5 was $\frac{d}{dx}[5x]$

Your book would say

$y = (27x^2 - 13x)^{16}$ is of the form

$f(g(x))$, where $y = f(u) = u^{16}$ outside

and $u = g(x) = 27x^2 - 13x$.

$\frac{dy}{dx}$ = how fast u changes wrt x

times

how fast f(u) changes wrt (an incremental change) in u(x), itself.

Chain Rule says $\frac{d}{dx}\left[(27x^2 - 13x)^{16}\right]$

$= 16(27x^2 - 13x)^{15}(54x - 13)$

Differentiate

$$f(x) = \sqrt[3]{(x^{-3} + 2x^5)} = (x^{-3}+2x^5)^{\frac{1}{3}}$$

$y = f(u) = u^{\frac{1}{3}}$, $u = x^{-3} + 2x^5$

$\frac{dy}{du} = \frac{1}{3}u^{-\frac{2}{3}}$ $\frac{du}{dx} = -3x^{2} + 10x$

$$f'(x) = \frac{1}{3}(x^{-3}+2x^5)^{-\frac{2}{3}}(-3x^{-4} + 10x^{4})$$

Differentiate

$$y = (x^3 + a^3)^5$$

We'd have to assume one was variable & one was constant.

$$5(x^3+a^3)^4(3x^2) \qquad 5(x^3+a^3)^4(3a^2)$$

which?

In the sequel,

$$\frac{dy}{dx} = 5(x^3+a^3)^4\left(3x^2 + 3a^2 \cdot \frac{da}{dx}\right)$$

OR $3a^2 a'$

Chain rule on a^3, assuming a is a function of x.

See Implicit Differentiation, which sorta does this

Differentiate

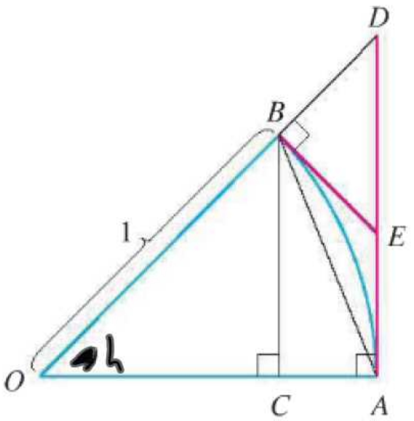
$$y = (x^2+5x)^{13}$$

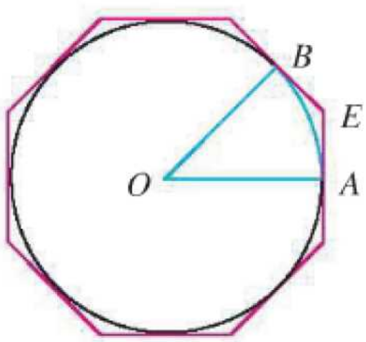
$$y = (x^2+5x)^{13}(2x^3-5)^{28}$$

$$y = \frac{(x^2+5x)^{13}}{(2x^3-5)^{28}}$$

$$y = \sqrt[11]{\frac{(x^2+5x)^{13}}{(2x^3-5)^{28}}}$$

Recall arc length $= r\theta = h$, since
$r = 1$, $\theta = h$ = arc AB



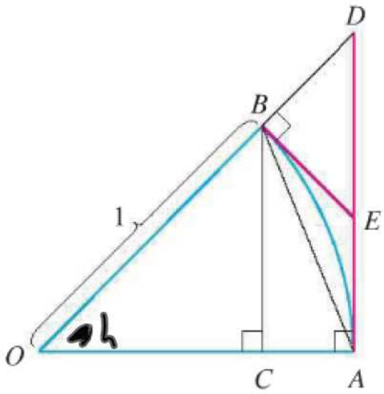


$|BC| < |AB| < \text{arc } AB = h$

$\sin h < h$

$\dfrac{\sin h}{h} < 1$

... if we assume $h > 0$. The case $h < 0$ is an equivalent argument, so we'll let it go with the $h > 0$ case, because it's easier.



$h = \text{arc}AB < |AE| + |EB|$

$< |AE| + |ED|$

$= |AD|$, i.e.

$h < |AD|$

Notice $\dfrac{|AD|}{|OA|} = \tan h$

i.e., $h < \tan h$, since $|OA| = 1$

$h < \tan h$

$h < \dfrac{\sin h}{\cos h}$

$\boxed{\cos h < \dfrac{\sin h}{h}}$

Put it together:

$$\cos h < \frac{\sin h}{h} < 1$$

$$\begin{array}{ccc} \cos h & & 1 \\ \Big\downarrow{\scriptstyle h \to 0} & & \Big\downarrow{\scriptstyle h \to 0} \\ 1 & & 1 \end{array}$$