§2.1 Errata

#5 Answer is $\frac{1}{3}$, not $\frac{1}{4}$

#6 Moving to the left:

(2,3)

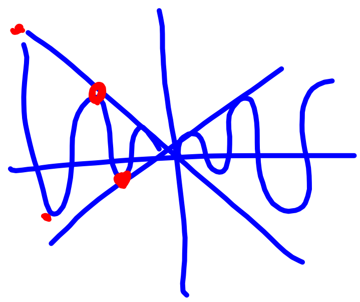
B1 Is $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

dif$b^{\underline{l}}$? NOTE, It is cnt$^{\underline{s}}$, since

$x\sin\left(\frac{1}{x}\right)$ is continuous on its domain $= \mathbb{R} \setminus \{0\}$

$f(x)$ has $f(0) \equiv 0$ &

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} x\sin\left(\tfrac{1}{x}\right)$$



Squeeze it:

$$-1 \le \sin\left(\tfrac{1}{x}\right) \le 1$$

$$-|x| \le x\sin\left(\tfrac{1}{x}\right) \le |x|$$

$$\xrightarrow{x\to 0} \qquad\qquad \xrightarrow{x\to 0}$$

$$0 \le \lim_{x\to 0} x\sin\left(\tfrac{1}{x}\right) \le 0$$

$$\lim_{x\to 0} f(x) = f(0)$$

$f'(x) \; \exists \; @ \; x=0 ?$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)\sin\left(\frac{1}{x+h}\right) - x\sin\left(\frac{1}{x}\right)}{h}$$

@ $x=0$:

$$\frac{f(0+h)-f(0)}{h} = \frac{h\sin\left(\frac{1}{h}\right)-0}{h}$$

$$= \sin\left(\frac{1}{h}\right) \xrightarrow{h\to 0} \nexists$$

$$g(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & \end{cases} \qquad \text{W} \qquad \Rightarrow$$

$$\frac{g(0+h) - g(0)}{h} = \frac{h^2 \sin\left(\frac{1}{h}\right) - 0}{h}$$

$$= h \sin\left(\frac{1}{h}\right) \xrightarrow{h \to 0} 0$$

how small do you want $h \sin\left(\frac{1}{h}\right)$ to be?
less than 0.1?

$$-1 \leq \sin(\text{x}) \leq 1$$

Let $|h| < .1$

Then $-.1 < .1 \sin \theta < .1$

so I've shown

$$\left| h \sin\left(\frac{1}{h}\right) - 0 \right| < .1 \to \varepsilon$$

whenever $|h| < .1$

$|h| \to \delta$