201 §2.1 The derivative as a function.

FINALLY!

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx}\left[f(x)\right] = Df(x) = D_x f(x)$$

D & $\frac{d}{dx}$ are called "Differential Operators"

Think of them as acting on a function to obtain a new function.

Liebniz NOTATION

$$f'(6) = \left.\frac{dy}{dx}\right|_{x=6}$$

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f is differentiable at $x = a$ if $f'(a)$ exists

f is differentiable on an interval if it's differentiable at a for all a in that interval.

201 §2.2

$\boxed{\text{T4}}$ pg 119

Differentiable $\Longrightarrow$ Continuous

but not all continuous functions are diff$^{\text{ble}}$

WTS $\lim_{x \to a} f(x) = f(a)$

$\boxed{\text{Pf}}$ The setup:

$$f(x) - f(a) = \left(\frac{f(x) - f(a)}{x - a}\right)(x - a)$$

$$\boxed{\lim_{x \to a} \left(f(x) - f(a)\right)} = \lim_{x \to a} \left(\frac{f(x) - f(a)}{x - a}\right)(x - a)$$

$$= f'(a) \cdot 0 \boxed{= 0}$$

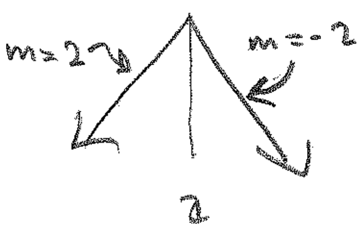
$$\text{So } \lim_{x \to a} f(x) = \lim_{x \to a} \left(f(x) - f(a) + f(a)\right)$$

$$= \lim_{x \to a} \left(f(x) - f(a)\right) + \lim_{x \to a} f(a)$$

$$= 0 + \lim_{x \to a} f(a) = f(a),$$

since $f(a)$ is a constant!

20' §2.2



Not dibl @ $x = a$

$$\lim_{h\to 0^-} \frac{f(a+h)-f(a)}{h} = 2$$

$$\lim_{h\to 0^+} \frac{f(a+h)-f(a)}{h} = -2$$

$$\therefore \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} \nexists$$

Higher derivatives

$$f'(x) = \frac{d}{dx}\left[f(x)\right]$$ Speed

$$f^{(2)}(x) = f''(x) = \frac{d}{dx}\left[\frac{d}{dx}\left[f(x)\right]\right]$$ accel. =

$$= \frac{d^2}{dx^2}\left[f(x)\right] = \frac{d^2y}{dx^2}$$

$f^{(3)}(x)$ you jerk!

Graph $f(x), f'(x), \ldots$ on same graph.

$$f(x) = (x-1)(x-2)(x-3)$$