§2.1 Questions?

TODAY §2.2 At *last*! Stewart lets me talk about $f'(x)$ as a function! Woo-hoo!

2.2 says $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \frac{d}{dx}[f(x)]$

$= \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = D(f(x)) = D_x(f(x)) = \frac{df}{dx}$

Liebniz

To make sure it's w.r.t. x.

= Derivative of f with respect to x

= Instantaneous change in y with incremental change in x

$\frac{d}{dx} = D = $ Differential Operator

Think of differentiation as acting on a function to get another function.

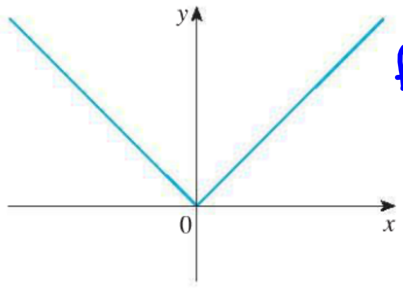
$f(x) = |x|$ acts on a real #, like $x = -5$ & gives you another real #, like $y = 5$.

$|x|$ "acts on" real numbers, producing real #s.

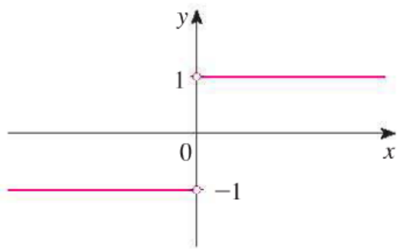
$f : \mathbb{R} \longrightarrow \mathbb{R}$

D: Differentiable funcs $\longrightarrow$ Some other family of funcs.

$C^1(\mathbb{R}) \longrightarrow ?$ $C(\mathbb{R})$



(a) $y = f(x) = |x|$



(b) $y = f'(x)$

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$x > 0$:

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h) - x}{h} = \frac{h}{h} = 1 \xrightarrow{h \to 0} 1$$

$x < 0$:

$$\frac{f(x+h) - f(x)}{h} = \frac{-(x+h) - (-x)}{h}$$

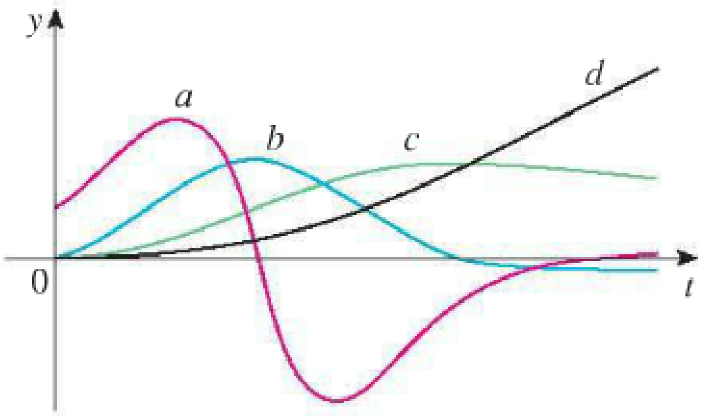
$$= \frac{-x - h + x}{h} = \frac{-h}{h} = -1 \xrightarrow{h \to 0} -1$$

what about @ $x = 0$?

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \nexists$$

$$\left.\begin{aligned} \lim_{h \to 0^-} \frac{f(x+h) - f(x)}{h} &= -1 \\ \lim_{h \to 0^+} \frac{f(x+h) - f(x)}{h} &= +1 \end{aligned}\right\} \therefore f'(0) \nexists$$

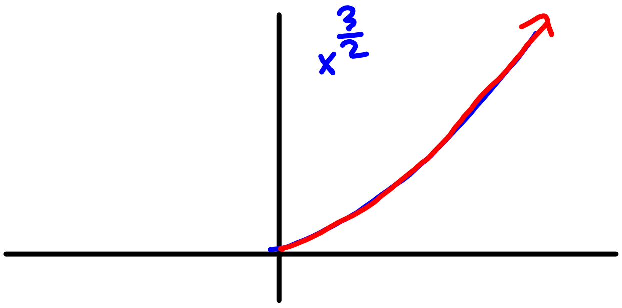
The figure shows the graphs of four functions. One is the position function of a car, one is the velocity of the car, one is its acceleration, and one is its jerk. Identify each curve, and explain your choices.



19–29 Find the derivative of the function using the definition of derivative. State the domain of the function and the domain of its derivative.

$g(x) = \sqrt{9 - x}$ **28.** $f(x) = x^{3/2}$

You guys try to find the derivative of this one: $f(x) = x + \sqrt{|x|}$



$x^{\frac{odd}{even}}$? $x^{\frac{5}{4}}$

$x^{\frac{even}{odd}}$? $x^{\frac{2}{3}}$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^{\frac{3}{2}} - x^{\frac{3}{2}}}{h} = \frac{1}{h}\left[(x+h)^{\frac{3}{2}} - x^{\frac{3}{2}}\right]$$

$$= \left(\frac{(x+h)^{\frac{3}{2}} - x^{\frac{3}{2}}}{h}\right)\left(\frac{(x+h)^{\frac{3}{2}} + x^{\frac{3}{2}}}{(x+h)^{3/2} + x^{\frac{3}{2}}}\right) = \frac{(x+h)^3 - x^3}{h\left((x+h)^{3/2} + x^{3/2}\right)}$$

$$= \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h\left((x+h)^{3/2} + x^{3/2}\right)} = \frac{3x^2h + 3xh^2 + h^3}{h\left(\cancel{(x+h)^{3/2} + x^{3/2}}\right)}$$

$$= \frac{3x^2 + 3xh + h^2}{\left((x+h)^{3/2} + x^{3/2}\right)} \xrightarrow{h \to 0} \frac{3x^2}{2x^{3/2}} = \frac{3}{2}x^{\frac{1}{2}}$$

$$= \frac{3\sqrt{x}}{2}$$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

Pascal's Triangle

Binomial Theorem