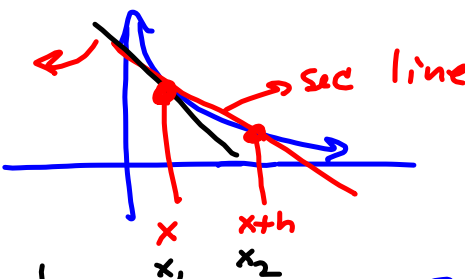


Bring Scientific Calculator to the test.  
No graphing calculator or Smartphone Apps!

$$\begin{aligned}
 & 3x^2 - 9x + 7 \\
 & = 3(x^2 - 3x \quad ) + 7 && 7 - 3\left(\frac{9}{4}\right) \\
 & \quad \downarrow \frac{3}{2} \rightsquigarrow \left(\frac{3}{2}\right)^2 = \frac{9}{4} && = 7 - \frac{27}{4} \\
 & = 3\left(x^2 - 3x + \left(\frac{3}{2}\right)^2\right) + 7 - 3\left(\frac{9}{4}\right) && = \frac{7}{1} \cdot \frac{4}{4} - \frac{27}{4} \\
 & \quad \leftarrow \text{red bracket under } x^2 - 3x + \left(\frac{3}{2}\right)^2 \text{ and } 7 - 3\left(\frac{9}{4}\right) && = \frac{28 - 27}{4} = \frac{1}{4} \\
 & = 3\left(x - \frac{3}{2}\right)^2 + \frac{1}{4} \\
 & (h, k) = \left(\frac{3}{2}, \frac{1}{4}\right), \text{ from } ax^2 + bx + c = a(x - h)^2 + k
 \end{aligned}$$

$$\frac{f(x+h) - f(x)}{h} = m_{\text{sec}}$$


LCD =  $\frac{1}{\sqrt{x+h} \sqrt{x}}$

$$f(x) = \frac{1}{\sqrt{x}} \Rightarrow$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} = \frac{1}{h} \left[ \frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}} \right]$$

See Board Shot.

$$= \frac{1}{h} \left[ \frac{1}{\sqrt{x+h}} \cdot \frac{\sqrt{x}}{\sqrt{x}} - \frac{1}{\sqrt{x}} \cdot \frac{\sqrt{x+h}}{\sqrt{x+h}} \right]$$

$$= \frac{1}{h} \left[ \frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x} \sqrt{x+h}} \right] \left[ \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}} \right]$$

$$= \frac{1}{h} \left[ \frac{(\sqrt{x})^2 - (\sqrt{x+h})^2}{\sqrt{x} \sqrt{x+h} (\sqrt{x} + \sqrt{x+h})} \right]$$

$a^2 - b^2 = (a-b)(a+b)$

$$= \frac{1}{h} \left[ \frac{x - (x+h)}{\sqrt{x} \sqrt{x+h} (\sqrt{x} + \sqrt{x+h})} \right]$$

$$= \frac{1}{h} \left[ \frac{-h}{\sqrt{x} \sqrt{x+h} (\sqrt{x} + \sqrt{x+h})} \right]$$

$\left( \frac{a+bi}{c+di} \right) \left( \frac{c-di}{c-di} \right) = \frac{ac+bd}{c^2+d^2}$

$$= \frac{-1}{\sqrt{x} \sqrt{x+h} (\sqrt{x} + \sqrt{x+h})} \xrightarrow{h \rightarrow 0} \frac{-1}{\sqrt{x} \sqrt{x} (\sqrt{x} + \sqrt{x})}$$

$$= \frac{-1}{x (2\sqrt{x})} = \boxed{\frac{-1}{2x\sqrt{x}}} = \frac{-1}{2x^1 x^{\frac{1}{2}}} = \frac{-1}{2x^{\frac{3}{2}}} = -\frac{1}{2} x^{-\frac{3}{2}}$$

$$\sqrt{x} \sqrt{x} = x$$

$$(\sqrt{x})^2 = x$$

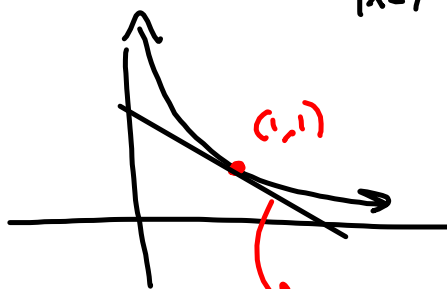
$$\sqrt{x^2} = |x|$$

$$\text{So, } m_{\text{tan}} =$$

$$= f'(x) = \frac{-1}{2x\sqrt{x}}$$

So, the slope of  $\frac{1}{\sqrt{x}}$  @  $x=1$  is

$$f'(1) = m_{\text{tan}} \Big|_{x=1} = \frac{-1}{2(1)(\sqrt{1})} = -\frac{1}{2} = m$$



$$y = m(x - x_1) + y_1$$

$y = -\frac{1}{2}(x-1) + 1$  is  
equation of tangent  
line to  $f(x) = \frac{1}{\sqrt{x}}$   
@  $x=1$  (i.e., @  $(1,1)$ )

①

$$f(-1) = 2$$

$P(-1, 2)$  :  $x_1 = -1, y_1 = 2$

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = m_{sec} = \frac{f(1.001) - f(-1)}{1.001 - (-1)}$$

$$= \frac{f(1.001) - 2}{-1.001 - (-1)} = \frac{\quad}{-.001}$$

②

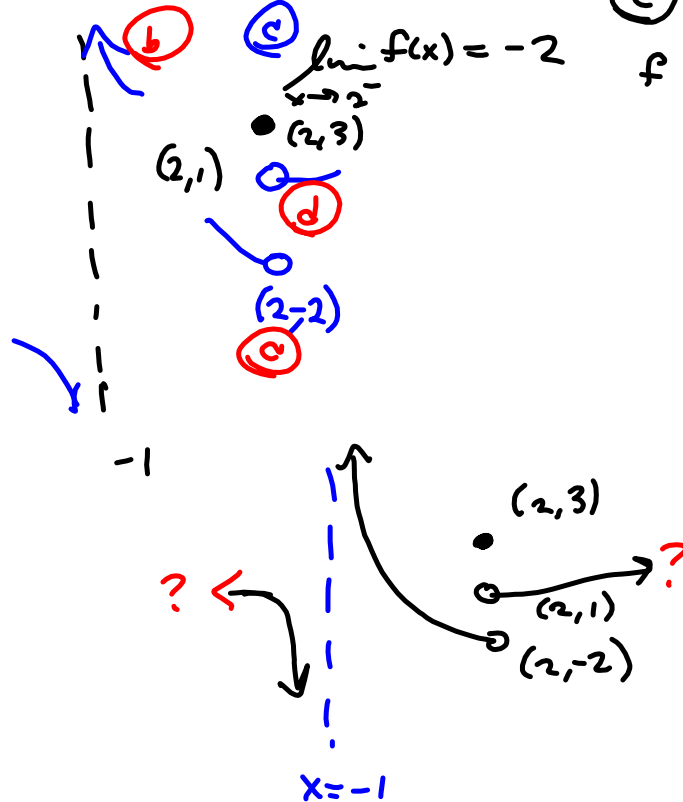
Ⓐ  $\lim_{x \rightarrow -1^-} f(x) = -\infty$

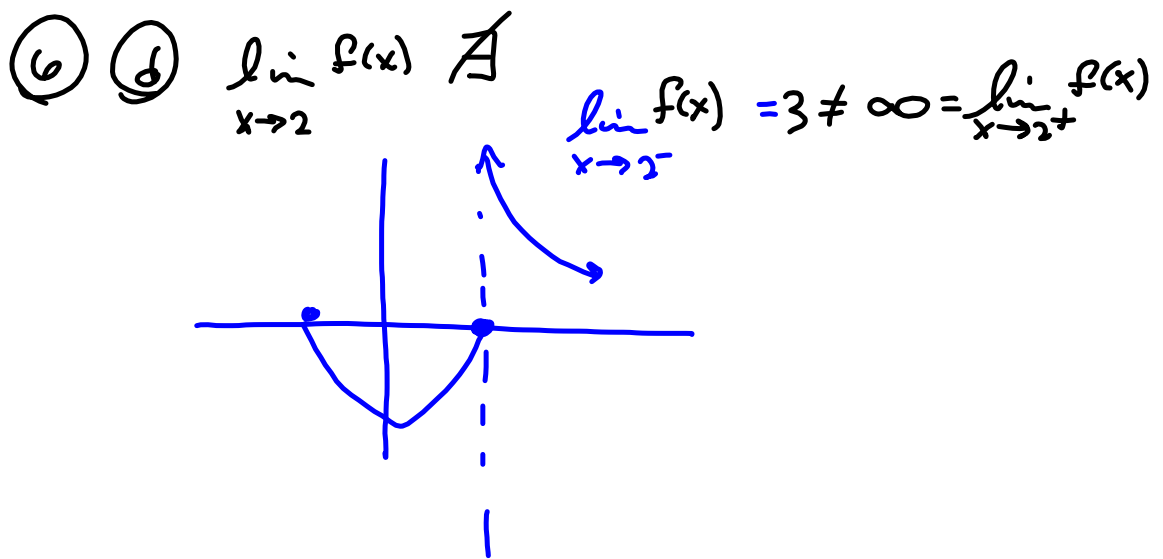
Ⓑ  $\lim_{x \rightarrow -1^+} f(x) = +\infty$

Ⓓ  $\lim_{x \rightarrow 2^+} f(x) = 1$

Ⓔ  $f(2) = 3$   
f

Ⓒ  $\lim_{x \rightarrow 2^-} f(x) = -2$





Alternate version of difference quotient.

$$\frac{f(x+h) - f(x)}{h} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x) - f(c)}{x - c}$$

#7 is saying  $x_1 = c = 3$  & wants the slope (a)  
 $x = 3$

$$\begin{aligned} \frac{f(x) - f(3)}{x - 3} &= \frac{x^2 + x + 2 - (3^2 + 3 + 2)}{x - 3} \\ &= \frac{x^2 + x + 2 - 14}{x - 3} = \frac{x^2 + x - 12}{x - 3} = \frac{(x-3)(x+4)}{x-3} = x+4 \quad (x \neq 3) \end{aligned}$$

$$\begin{aligned} \frac{f(x) - f(c)}{x - c} &= \frac{x^2 + x + 2 - c^2 - c - 2}{x - c} \\ &= \frac{x^2 - c^2 + x - c}{x - c} = \frac{(x-c)(x+c) + (x-c)}{x-c} \\ &= \frac{(x-c)[x+c+1]}{x-c} = \begin{matrix} x+c+1 \\ x \neq c \end{matrix} \xrightarrow{x \rightarrow c} \begin{matrix} c+c+1 \\ = 2c+1 = f'(c) \end{matrix} \\ & \qquad \qquad \qquad c=3 \rightarrow f'(3) = 7 \end{aligned}$$