MAT 201 Test 1
Spring, 2011 Name___________________________
100 Points

1. The point $P=(4,2)$ lies on the graph of $f(x)=\sqrt{x}$. Let $Q=(x,\sqrt{x})$ be another point on the graph of f.

a. (5 pts) Find the slope m_{PQ} to 4 decimal places for the following values of x:

- $x = 3.999$

- $x = 4.001$

b. (5 pts) Based on your work for part a., estimate the slope of the tangent line $m_{\tan}$ at $x=4$.

c. (5 pts) Based on your work for part b., construct an equation for the tangent line to $f(x)=\sqrt{x}$. (Point-slope form is just fine: $y=m(x-x_1)+y_1$.)

2. (10 pts) Sketch the graph of a function that meets all of the following requirements:

a. $\lim_{x\to 3^-} f(x) = -\infty$

b. $\lim_{x\to 3+} f(x) = \infty$

c. $\lim_{x\to -2^-} f(x) = 3$

d. $\lim_{x\to -2^+} f(x) = 2$

e. $f(-2) = 4$

3. Find the limit, if it exists. If it doesn't, say so (in writing, of course).

a. (5 pts) $\lim_{x \to -5} \frac{x^2 + 2x - 15}{x + 5}$

b. (5 pts) $\lim_{x \to 1} \frac{x^2 - 2x + 1}{x^2 + 3}$

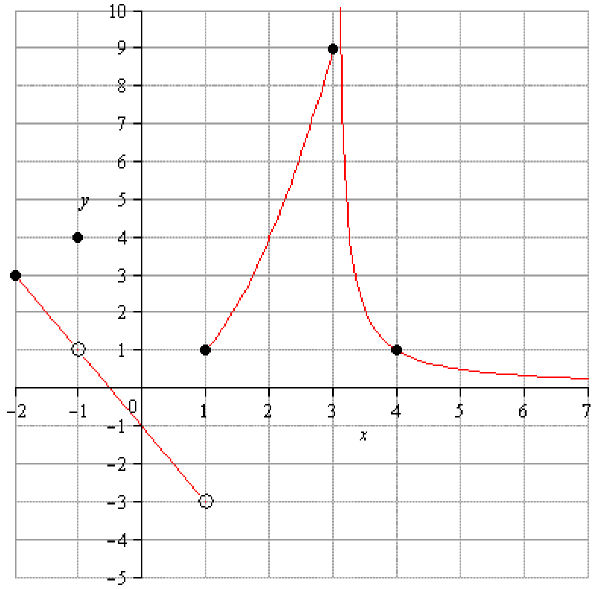
c. (5 pts) $\lim_{x \to 3} \frac{x^2 - 2x + 1}{x^2 - 9}$

d. (5 pts) $\lim_{x \to -4^-} \frac{x^2 - 16}{|x + 4|}$

4. (10 pts) Use the precise definition of a limit to prove that $\lim_{x\to 3}\left(\frac{1}{3}x+5\right)=6$.

5. (**Bonus 5 pts**) Use the precise definition of a limit to prove that $\lim_{x\to 2}\left(x^2+5x-6\right)=8$.

6. The graph of a function f is given below. Evaluate each limit, if it exists. If the limit does *not* exist, explain why. Employ the language (shorthand) of limits in your explanation(s), as needed.



a. (5 pts) $\lim_{x\to -1} f(x)$

b. (5 pts) $\lim_{x\to 3^-} f(x)$

c. (5 pts) $\lim_{x\to 3^+} f(x)$

d. (5 pts) $\lim_{x\to 3} f(x)$

e. (5 pts) Use the limit definition of continuity to explain why f is *not* continuous at $x = -1$.

f. (5 pts) Use the limit definition of continuity to explain why f *is* continuous at $x = 4$

7. Let $f(x) = \sqrt{x}$. Then the slope of the secant line between two points, $(x, f(x))$ and $(4, f(4))$, is given by the difference quotient:

$$m_{\text{sec}} = \frac{f(x) - f(4)}{x - 4}.$$

a. (5 pts) Write the difference quotient for f at $x = 4$. (Don't overthink this one.)

b. (5 pts) Based on your work in part a., compute the slope of the tangent line to f at $x = 4$, by computing the limit $m_{\tan} = \lim_{x \to 4} \frac{f(x) - f(4)}{x - 4}$. Hint: Rationalize the numerator.

c. (5 pts) Based on your work in part b., construct an equation of the tangent line to f at $x = 4$. If you didn't *get* b., then ***make up*** *a number for part* b. *and use it* ! Leave your answer in point-slope form.