Last little piece from Section 1.7: Infinite limit, defined precisely.

$\boxed{6}$ **Definition** Let f be a function defined on some open interval that contains the number a, except possibly at a itself. Then

$$\lim_{x \to a} f(x) = \infty$$

means that for every positive number M there is a positive number δ such that

$$\text{if} \quad 0 < |x - a| < \delta \quad \text{then} \quad f(x) > M$$

So you can make the function bigger than any real number by getting sufficiently close to the number *a*. Furthermore, it will *stay* bigger than that number, if you go closer than "sufficiently close" to *a*.

Claim: $\lim_{x \to 3} \frac{7}{(x-3)^2} = \infty$

$$\sqrt{R^2} = |R|$$

Bull. Make it bigger than 1000!

Method should be to get one fraction on one side & do a sign pattern on the frac.

$(x-3)^2 > 0$ so I can cross-multiply with impunity.

$$\frac{7}{(x-3)^2} > 1000$$

$$7 > 1000(x-3)^2$$

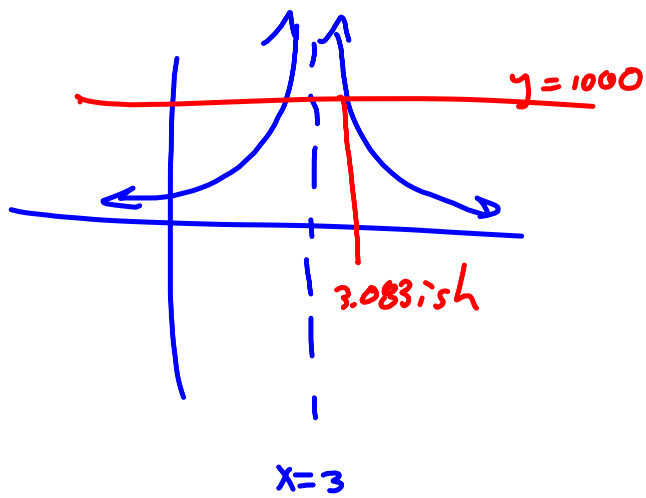
$$(x-3)^2 < \frac{7}{1000}$$

$$\sqrt{(x-3)^2} < \sqrt{\frac{7}{1000}}$$

$$|x-3| < \frac{\sqrt{7}}{10\sqrt{10}}$$

$$-\frac{\sqrt{7}}{10\sqrt{10}} < x-3 < \frac{\sqrt{7}}{10\sqrt{10}}$$

$$3-\frac{\sqrt{7}}{10\sqrt{10}} < x < \frac{\sqrt{7}}{10\sqrt{10}}+3$$



Let $M>0$ be given

Want $\frac{7}{(x-3)^2} > M$ whenever $|x-3| < \delta$

$$M < \frac{7}{(x-3)^2}$$

$$(x-3)^2 M < 7$$

$$(x-3)^2 < \frac{7}{M}$$

$$|x-3| < \sqrt{\frac{7}{M}}$$

$$\frac{7}{(x-3)^2} > M$$

Let $M > 0$ be given

Define $\delta = \sqrt{\frac{7}{M}}$

Then if $0 < |x-3| < \delta$, we have

$$\frac{7}{(x-3)^2} > \frac{7}{\delta^2} = \frac{7}{\left(\sqrt{\frac{7}{M}}\right)^2} = \frac{7}{\frac{7}{M}} = \frac{7}{1}\cdot\frac{M}{7} = M \quad \blacksquare$$

$$|(x-3)| < \delta$$

$$\frac{\text{[illegible]}}{|x-3|} > \frac{\text{[illegible]}}{\delta}$$

$$\lim_{x \to 3} \left(\frac{x}{5}\right) = \frac{3}{5}$$

.... $\delta = 5\varepsilon$

Let $\varepsilon > 0$ be given

define $\delta = 5\varepsilon$

Then, if $|x-3| < \delta$, we have

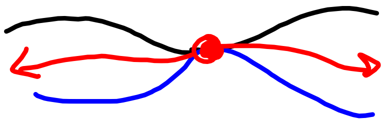
$$|f(x) - L| = \left|\frac{x}{5} - \frac{3}{5}\right| = \frac{1}{5}|x-3| < \frac{1}{5}\delta = \frac{1}{5} \cdot 5\varepsilon = \varepsilon \blacksquare$$

Section 1.8

1 Definition A function f is **continuous at a number a** if

$$\lim_{x \to a} f(x) = f(a)$$

① $\lim$ $\exists$
② $f(a)$ $\exists$ ③ limit & actual value agree

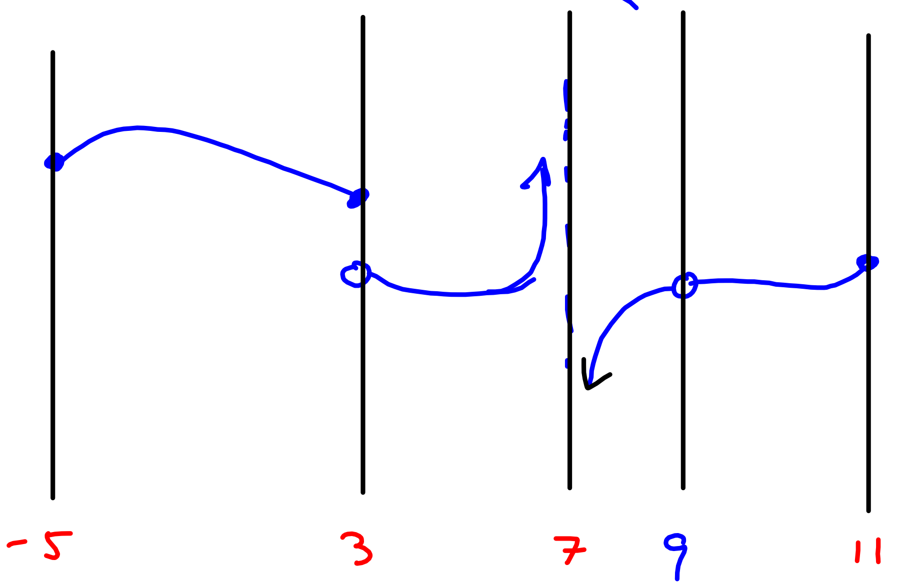


2 Definition A function f is **continuous from the right at a number a** if

$$\lim_{x \to a^+} f(x) = f(a)$$

and f is **continuous from the left at a** if

$$\lim_{x \to a^-} f(x) = f(a)$$



Continuous $\forall\, x \in [-5, 3) \cup (3, 7) \cup (7, 9) \cup (9, 11]$

cnt's from the right

"Jump" discontinuity @ $x = 3$
"Infinite" " @ $x = 7$
"Removable" " @ $x = 9$ (hole)

Patching the hole:

MAKE $f(9) = \lim\limits_{x \to 9} f(x)$

$\boxed{E}$ $\dfrac{x^2 - 3x + 2}{x - 1} = \dfrac{(x-2)(x-1)}{x-1} = x - 2$ (if $x \neq 1$)

$$f(x) = \begin{cases} \dfrac{x^2 - 3x + 2}{x - 1} & \text{if } x \neq 1 \\ -1 & \text{if } x = 1 \end{cases}$$

Make $x^2 \sin(\frac{1}{x})$ a continuous func.

we cool, except @ $x=0$

$$\lim_{x\to 0} x^2 \sin(\tfrac{1}{x}) = 0$$

Define $f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

$\boxed{3}$ **Definition** A function f is **continuous on an interval** if it is continuous at every number in the interval. (If f is defined only on one side of an endpoint of the interval, we understand *continuous* at the endpoint to mean *continuous from the right* or *continuous from the left.*)

$\boxed{4}$ **Theorem** If f and g are continuous at a and c is a constant, then the following functions are also continuous at a:

1. $f + g$ **2.** $f - g$ **3.** cf

4. fg **5.** $\dfrac{f}{g}$ if $g(a) \neq 0$

$\boxed{5}$ **Theorem**

(a) Any polynomial is continuous everywhere; that is, it is continuous on $\mathbb{R} = (-\infty, \infty)$.

(b) Any rational function is continuous wherever it is defined; that is, it is continuous on its domain.

$\boxed{7}$ **Theorem** The following types of functions are continuous at every number in their domains:

polynomials	rational functions
root functions	trigonometric functions

Surprise! If you feed a continuous function to another continuous function, the resulting function is continuous. That's all that's being said, below. Only tricky part may be handling domain. The first theorem (#8), below, says that limits behave themselves with respect to function composition.

$\boxed{8}$ **Theorem** If f is continuous at b and $\lim_{x\to a} g(x) = b$, then $\lim_{x\to a} f(g(x)) = f(b)$. In other words,

$$\lim_{x\to a} f(g(x)) = f\left(\lim_{x\to a} g(x)\right)$$

The following theorem is a consequence of the one above. Since limits behave themselves, is it any wonder that continuity is an inherited trait?

$\boxed{9}$ **Theorem** If g is continuous at a and f is continuous at $g(a)$, then the composite function $f \circ g$ given by $(f \circ g)(x) = f(g(x))$ is continuous at a.

10 **The Intermediate Value Theorem** Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.

Show that f has a zero in the interval (2, 4).

$$f(x) = x^4 - 3\cdot x^3 + x^2 - 6\cdot x + 7$$

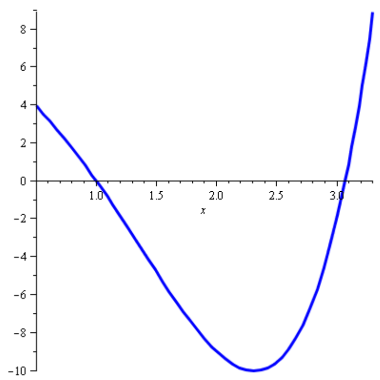
$f(2) = -9 < 0$

$f(4) = 63 > 0$

By IVT, there is a c in (2,4) such that $f(c) = 0$

$$\begin{array}{r|rrrrr} 2 & 1 & -3 & 1 & -6 & 7 \\ & & 2 & -2 & -2 & -16 \\ \hline & 1 & -1 & -1 & -8 & -9 \end{array}$$

$$\begin{array}{r|rrrrr} 4 & 1 & -3 & 1 & -6 & 7 \\ & & 4 & 4 & 20 & 56 \\ \hline & 1 & 1 & 5 & 14 & 63 \end{array}$$



From here on, we're reviewing, answering questions on the board from Test 1, Spring, 2011 and anything else we can think of.

Perhaps a smartphoney will take board shots.