More Algebra Review

$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$ Difference of two cubes.

$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

Sum of two cubes.

$x^3 - 8 = 0$

$\Rightarrow x^3 = 8$ has $x=2$ as solution.

Then $x-2$ is a factor.

Divide x^3-8 by $x-2$, synthetically.

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 0 & -8 \\ & & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & 0 \\ & x^2 & x & c & r \end{array}$$

Sweet!

This says Dividend

$x^3 - 8 = (x-2)(x^2 + 2x + 4)$

$$x-2\,\overline{\big)\,x^3-8}^{\;x^2+2x+4}$$

Divisor quotient

$$3\,\overline{\big)\,27}^{\;9}$$

quotient

Dividend

Divisor

3 is a factor of 27

$\frac{27}{3} = 9$ means

$27 = 3 \cdot 9$

§1.7 questions?

Yes, I know #7 says $\lim_{x\to 2}$ & solutions do $\lim_{x\to 3}$.

Here's a version of #7:

$\lim_{x\to 3} x^3 = 27$ ← Not '8', silly.

Scratch

Want $|x^3-27| < \varepsilon$

$|x-3||x^2+3x+9| < |x^2+3x+9|\delta$

Make x^2+3x+9 as bad as it can get, assuming x is relatively close to $x=3$.

Assume $\delta \le 1$, then $|x-3| \le 1$

$-1 \le x-3 \le 1$

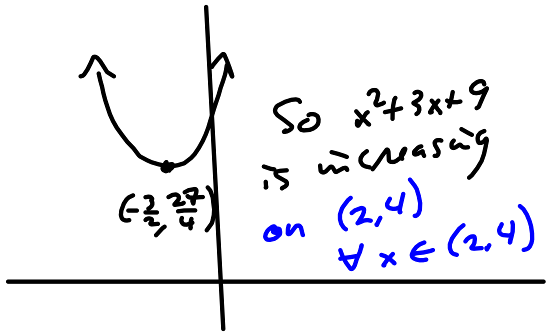
$2 \le x \le 4$

x^2+3x+9

$= x^2+3x+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+9$

$= \left(x+\frac{3}{2}\right)^2 - \frac{9}{4} + \frac{36}{4}$

$= \left(x+\frac{3}{2}\right)^2 + \frac{27}{4}$



So x^2+3x+9 is increasing on $(2,4)$ $\forall x \in (2,4)$

Minimalist for our purpose

M1 x^2+3x+9

$= x^2+3x+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+9$

$= \left(x+\frac{3}{2}\right)^2$

$x=-\frac{3}{2}$ is vertex x-coord

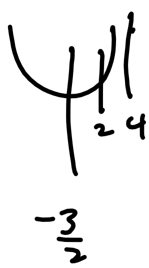
M2 $a=1, b=3, c=9$

$-\frac{b}{2a} = -\frac{3}{2(1)} = -\frac{3}{2} =$ x-coord. of vertex.

M3 $f(x) = x^2+3x+9$

$\Rightarrow f'(x) = 2x+3$

is increasing for $x \in (2,4)$



Increasing

So, if $\delta \le 1$, then $|x^2+3x+9| \le |4^2+3(4)+9|$

$= 37$ is as BIG as we'll let $|x^2+3x+9|$ get, so....

$$|x-3||x^2+3x+9| < |x^2+3x+9|\,\delta$$

$$\le 37\delta \overset{\text{want}}{<} \varepsilon$$

$\delta < \frac{\varepsilon}{37}$ is the idea

$\delta \le 1$, $\delta \le \frac{\varepsilon}{37}$

Combine in proof:

Let $\varepsilon > 0$. Define $\delta = \min\{1, \frac{\varepsilon}{37}\}$. Then

$0 < |x-3| < \delta \implies$

$$|f(x) - L| = |x^3-27| = |x-3||x^2+3x+9|$$

$$< \delta|x^2+3x+9| \le \delta \cdot 37 \le \frac{\varepsilon}{37}\cdot 37 = \varepsilon \;\blacksquare$$

From $\delta \le 1$

$\delta \le 1$ & $\delta \le \frac{\varepsilon}{37}$

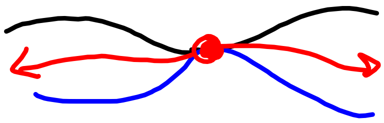
Section 1.8

1 Definition A function f is **continuous at a number a** if

$$\lim_{x \to a} f(x) = f(a)$$

① lim ∃

② $f(a)$ ∃ ③ limit & actual value agree

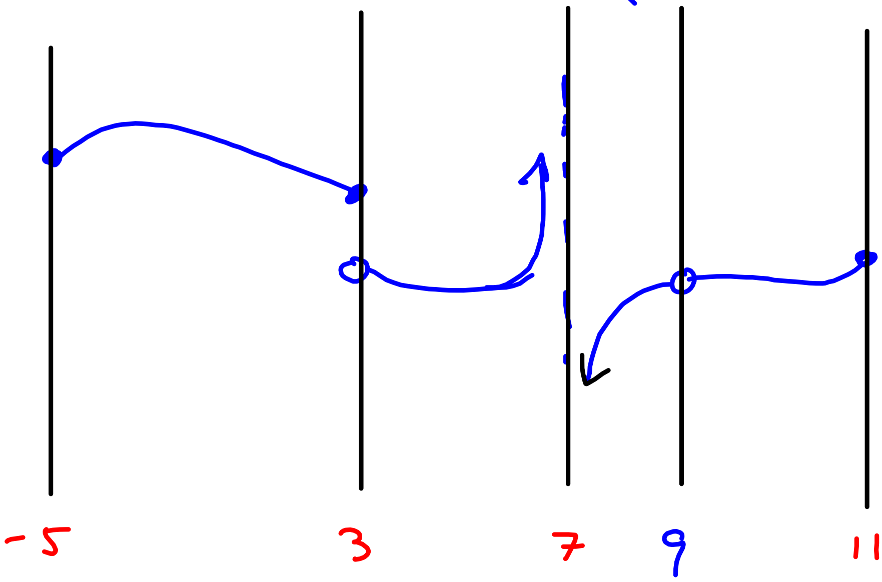


2 Definition A function f is **continuous from the right at a number a** if

$$\lim_{x \to a^+} f(x) = f(a)$$

and f is **continuous from the left at a** if

$$\lim_{x \to a^-} f(x) = f(a)$$



Continuous $\forall\, x \in [-5, 3) \cup (3, 7) \cup (7, 9) \cup (9, 11]$

cnt's from the right

"Jump" discontinuity @ $x = 3$

"Infinite" " @ $x = 7$

"Removable" " @ $x = 9$ (hole)

Patching the hole:

MAKE $f(9) = \lim_{x \to 9} f(x)$

$\boxed{E}$ $\dfrac{x^2 - 3x + 2}{x - 1} = \dfrac{(x-2)(x-1)}{x-1} = x - 2$ (if $x \neq 1$)

$$f(x) = \begin{cases} \dfrac{x^2 - 3x + 2}{x - 1} & \text{if } x \neq 1 \\ -1 & \text{if } x = 1 \end{cases}$$

Make $x^2\sin\left(\frac{1}{x}\right)$ a continuous func.

we cool, except @ $x=0$

$$\lim_{x\to 0} x^2\sin\left(\frac{1}{x}\right) = 0$$

Define $f(x) = \begin{cases} x^2\sin\left(\frac{1}{x}\right) & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$

3 Definition A function f is **continuous on an interval** if it is continuous at every number in the interval. (If f is defined only on one side of an endpoint of the interval, we understand *continuous* at the endpoint to mean *continuous from the right* or *continuous from the left*.)

4 Theorem If f and g are continuous at a and c is a constant, then the following functions are also continuous at a:

1. $f + g$
2. $f - g$
3. cf
4. fg
5. $\dfrac{f}{g}$ if $g(a) \neq 0$

$\boxed{6}$ **Definition** Let f be a function defined on some open interval that contains the number a, except possibly at a itself. Then

$$\lim_{x \to a} f(x) = \infty$$

means that for every positive number M there is a positive number δ such that

$$\text{if} \qquad 0 < |x - a| < \delta \qquad \text{then} \qquad f(x) > M$$

5 Theorem

(a) Any polynomial is continuous everywhere; that is, it is continuous on $\mathbb{R} = (-\infty, \infty)$.

(b) Any rational function is continuous wherever it is defined; that is, it is continuous on its domain.

7 Theorem The following types of functions are continuous at every number in their domains:

polynomials	rational functions
root functions	trigonometric functions

8 Theorem If f is continuous at b and $\lim_{x \to a} g(x) = b$, then $\lim_{x \to a} f(g(x)) = f(b)$. In other words,

$$\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right)$$

9 Theorem If g is continuous at a and f is continuous at $g(a)$, then the composite function $f \circ g$ given by $(f \circ g)(x) = f(g(x))$ is continuous at a.

10 The Intermediate Value Theorem Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.