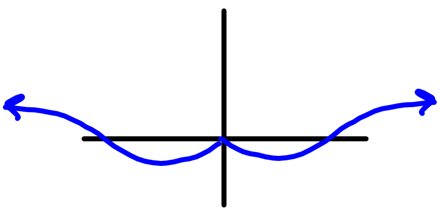
$f(-x) = f(x)$



$x^2, |x|, x^4, \ldots$

$g(|x|)$

even + even

(even)(even)

$\frac{\text{even}}{\text{even}}$ $\frac{\pm}{+}$

(odd)(odd)

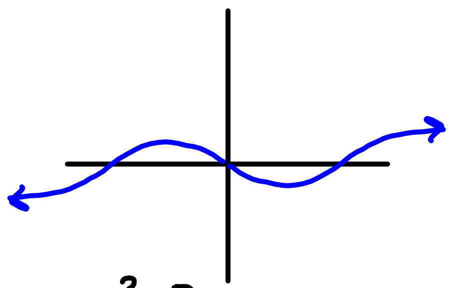
$(-)\ (-)$

$\frac{\text{odd}}{\text{odd}}$ $\frac{-}{-}$

$\frac{x^3 - 7x}{\sin x}$

$\cos x$

$f(-x) = -f(x)$



$x, x^3, x^5, \ldots$

odd + odd

(odd)(even)

$\frac{\text{odd}}{\text{even}}$

$\sin x$

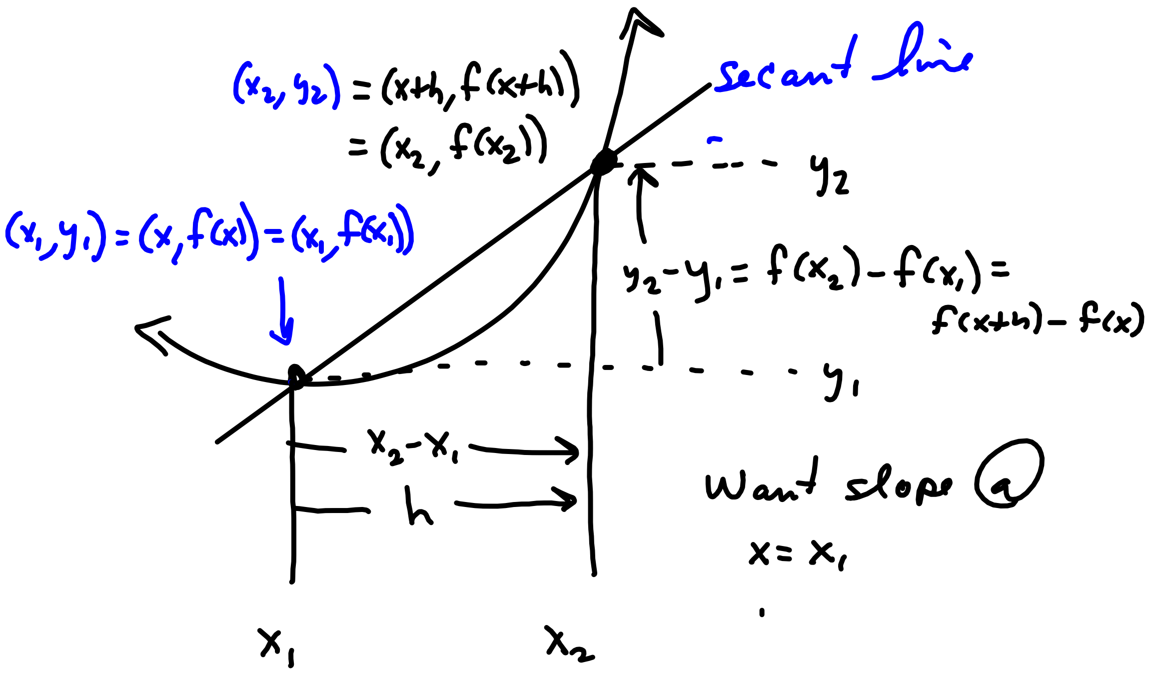
$\tan x = \frac{\sin x}{\cos x}$

$\tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin x}{\cos x}$

$= -\tan x$

Did not get to the following, today (Friday). This is essentially where the nuts and bolts of 1.4 begins.

$$(x_1, y_1) = (x, f(x)) = (x_1, f(x_1)) \text{ and } (x_2, y_2) = (x+h, f(x+h)) = (x_2, f(x_2))$$
$$\Rightarrow$$
$$m_{avg} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x+h) - f(x)}{h} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \text{secant slope}$$



The idea is, $x_2 \longrightarrow x_1$

$h \longrightarrow 0$

Numerical (& Graphical Methods)

Let h = very small is how you approach it, intuitively.

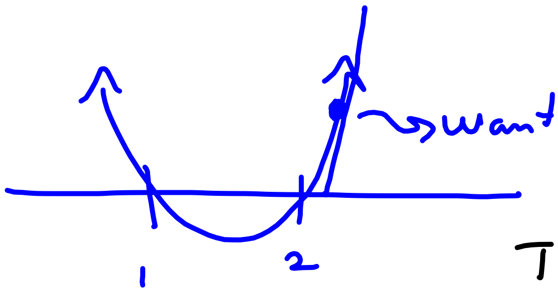
We did the x_2 ----> x_1 method. Today, a quick look at h ---> 0 formulation.

x_2 = 3.001 would correspond to h = 0.001, etc.

They look *very* different, but are equivalent.

Find slope of $f(x) = x^2 - 3x + 2$ at $x = 3$

$$\frac{f(x) - f(3)}{x - 3} \quad \text{OR} \quad \frac{f(3+h) - f(3)}{h}$$



want the slope @ $x=3$

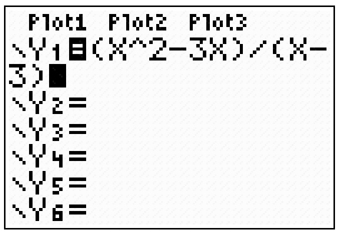
Take x close to 3

Take h " " 0

$$\frac{(x^2 - 3x + 2) - (3^2 - 3(3) + 2)}{x - 3}$$

$$= \frac{x^2 - 3x + 2 - 2}{x - 3}$$

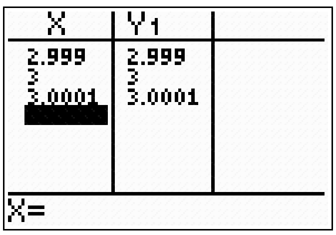
$$\boxed{= \frac{x^2 - 3x}{x - 3}} \rightarrow \begin{array}{l} x = 2.99, 2.9999 \\ x = 3.01, 3.0001 \end{array}$$



$$= \frac{x(x-3)}{x-3}$$

$$= x \xrightarrow{x \to 3} 3$$

Very Deceptively Easy.



Not all functions are this well-behaved.

$$f := x \rightarrow \sin\left(\frac{10 \cdot \mathrm{Pi}}{x}\right)$$

$$x \rightarrow \sin\left(\frac{10\,\pi}{x}\right)$$

$$ss := x \rightarrow \frac{(f(x) - f(1))}{x - 1}$$

$$x \rightarrow \frac{f(x) - f(1)}{x - 1} \qquad \frac{\sin\left(\frac{10\,\pi}{x}\right)}{x - 1}$$

$$limit(ss(x), x = 1)$$

$$-10\,\pi$$

$$evalf(\%)$$

$$-31.41592654$$

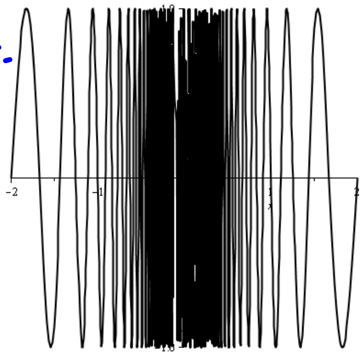
$$tanline := x \rightarrow -10 \cdot \mathrm{Pi} \cdot (x - 1) + f(1)$$

$$x \rightarrow -10\,\pi\,(x - 1) + f(1)$$

$plot(f(x), x = -2..2, color = black, thickness = 2)$

Wiggles too fast to be confident.

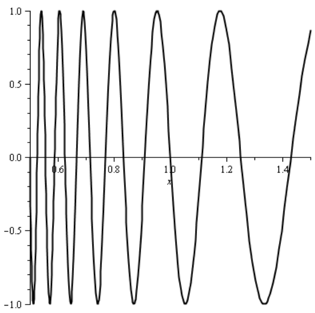
Non-example, cautionary tale.



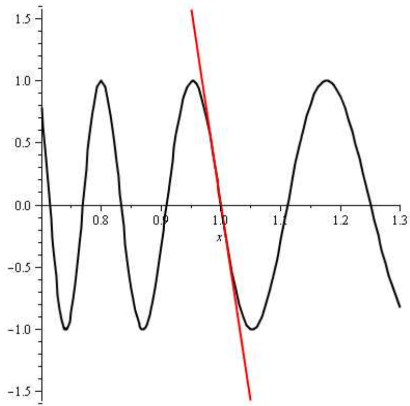
Numerical Methods Suck.

$plot(f(x), x = 0.5..1.5, color = black, thickness = 2)$

Motivation for learning Limit Laws in Later section.



$plot1 := plot(f(x), x = 0.7..1.3, color = black, thickness = 2) : \% :$
$plot2 := plot(tanline(x), x = 0.95..1.05, color = red, thickness = 2) : \% :$
$display([plot1, plot2])$



$$\frac{\sin\left(\frac{10\pi}{x}\right) - \sin\left(\frac{10\pi}{1}\right)}{x-1}$$

(1,0) on graph

$$= \frac{\sin\left(\frac{10\pi}{x}\right) - 0}{x-1} \xrightarrow{x \to 1} -10\pi$$

Tan line $y = m(x - x_1) + y_1$

$= -10\pi(x-1) + 0$

$\left(x, \sin\left(\frac{10\pi}{x}\right)\right)$

$(1, 0)$

Needs a minus sign in front of the 10 Pi.

1.1

1.01

1.001

⋮

~~lead to misleading~~

misleading.

A typical test question on this numerical approach to limits, I'd just plug in one or two values close to the limiting value. On test, time is of the essence and you won't have a graphing calculator.

Final 1.4 comment: velocity.

$$\frac{h(x_2)-h(x_1)}{x_2-x_1} \qquad \frac{\text{feet}}{\text{seconds}}$$

Slope of position is rate.

$h(t) = 40t - 16t^2$

Height (distance from ground) of object thrown upward from ground level at 40 miles per hour. We can find the velocity of the object by finding the (instantaneous) slope of the position function. Analyze units...

5 Beers in 1 hour

$$\frac{5 \text{ Beers}}{1 \text{ hr}} = 5 \frac{\text{Beers}}{\text{hr}}$$

1.1 Due Friday
1.2 ~~Monday~~ Tuesday
1.3 ~~Monday~~ Tuesday
1.4 ~~Tuesday~~ Wed or Thursday
1.5 ~~Tuesday or~~ Wednesday, and that's as far out as I'm going.

		Classes (50 minutes)	
CHAPTER 1 - FUNCTIONS AND LIMITS			
1.1	Four Ways to Represent a Function	1	
1.2	Mathematical Models	2	Week 1
1.3	New Functions from Old Functions	1	
1.4	The Tangent and Velocity Problems	1	
1.5	The Limit of a Function	2	
1.6	Calculating Limits Using the Limit Laws	2	Week 2
1.7	The Precise Definition of a Limit	2	
1.8	Continuity	1	
Review		1	
TEST 1 – Chapter 1		1	FRIDAY Week 3 is target date for Test 1

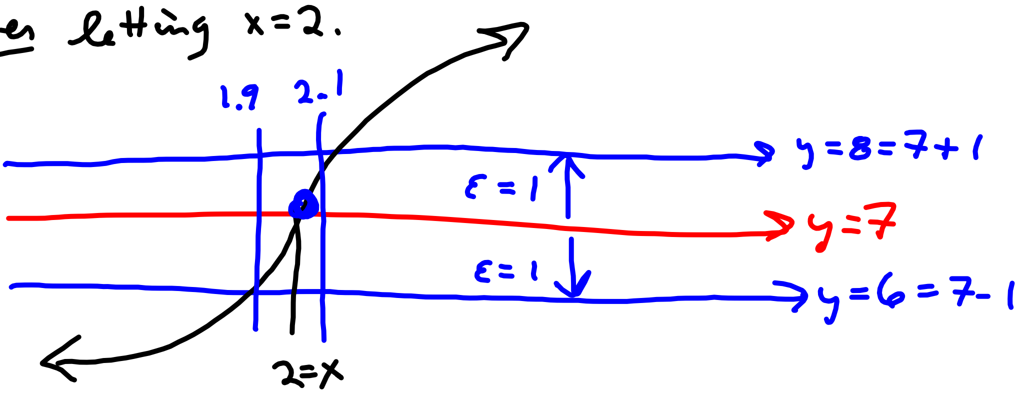
§1.5

$\lim_{x \to 2} f(x) = 7$

$f(2)$ is irrelevant.

A neighborhood of $x=2$.

I can make $f(x)$ as close to 7 as you command, by choosing x close enough to 2, but without Even letting $x=2$.



Response: Keep x within, say, $\delta = .1$ of $x=2$

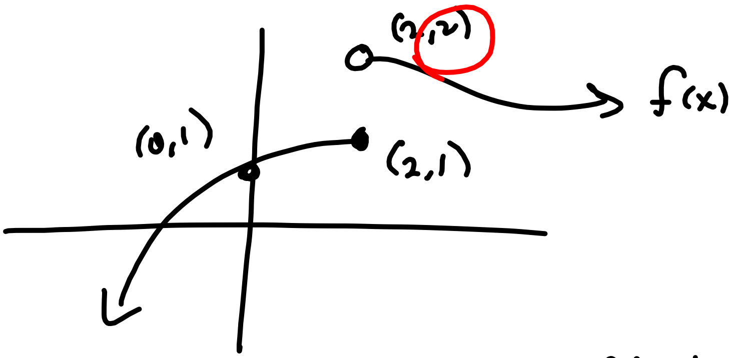
I can find a neighborhood of $x=2$ that keeps the height of $f(x)$ in your neighborhood of $y=7$

How big's the neighborhood of $y=7$? $\varepsilon = 1$ is its radius.

How big's the neighborhood of $x=2$? $\delta = 0.1$ is its radius.

We're within 1 of $y=7$: $|f(x)-7| < 1 = \varepsilon$

We're within 0.1 of $x=2$: $|x-2| < 0.1 = \delta$



$\lim_{x \to 2^-}$ means nothing w/o f(x) as argument

Left- & Right-sided limits

$\lim_{x \to 2^-} f(x) = 1$ $\lim_{x \to 2^+} f(x) = 2$

$\lim_{x \to 2} f(x) \nexists$, because $\lim_{x \to 2^-} f(x) = 1 \neq 2 = \lim_{x \to 2^+} f(x)$

Numerical limits Beware!

$$\lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2} = \frac{1}{6}$$

X	Y1
.1	.16662
.1	.16662
-.1	.16662
.01	.16667
1E-4	.16667
1E-6	0

X=

$(.000001)^2$ isn't quite zero

but

$\sqrt{(.000001)^2+9}\ -3$ is zero to my calculator.

§1.6 skill:

Section 1.6 Limit Laws: Basically they work the way you'd hope and expect. Not a lot of memorization required. Just common sense.

Limit Laws Suppose that c is a constant and the limits

$$\lim_{x \to a} f(x) \qquad \text{and} \qquad \lim_{x \to a} g(x)$$

exist. Then

1. $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

2. $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$

3. $\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$

4. $\lim_{x \to a} [f(x)\, g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

5. $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ if $\lim_{x \to a} g(x) \neq 0$

6. $\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$ where n is a positive integer

7. $\lim_{x \to a} c = c$

8. $\lim_{x \to a} x = a$

9. $\lim_{x \to a} x^n = a^n$ where n is a positive integer

10. $\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$ where n is a positive integer
    (If n is even, we assume that $a > 0$.)

11. $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$ where n is a positive integer
    $\left[\text{If } n \text{ is even, we assume that } \lim_{x \to a} f(x) > 0.\right]$

Only a handful, tops, of homework where you cite the specific rule being used. Otherwise, follow your instincts and seldom will you go wrong. Never ask this on a test, although failing to apply them appropriately in other context will be a problem.

Direct Substitution Property If f is a polynomial or a rational function and a is in the domain of f, then

$$\lim_{x \to a} f(x) = f(a)$$

If $f(x) = g(x)$ when $x \neq a$, then $\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$, provided the limits exist.