We made decent progress, today, but we didn't *really* finish 1.3. There are a couple problems on working a composite function from a graph of two functions or a table of two functions.

Perhaps we should talk about that, Tuesday, briefly. I had the example ready to go, but we ran out of time. But I'm glad we had the comments/questions we *did* have, which slowed us, a little.

What we've covered is highlighted.:

1. What's differential calculus? Extension of simple concept of slope of a straight line, with *limits*.
    a. Quick mention of integral calculus as extension of concept of area of a rectangle, with *limits*.
2. 4 ways to represent a function
3. Evaluating Functions
4. Domain of a Function
5. Families of Functions
    a. Power Functions
    b. Lines
    c. Trig functions
6. Moving functions around (Graphing by transforming). (Started)
7. Piecewise-Defined Functions
8. Domain of sum/difference/product of two functions
9. Domain of quotient of two functions
10. Domain of the Composition of two functions.
11. Evaluating a composite function from table or graph of two functions
12. Numerical approximations of average slope and the two versions of average slope (i.e., slope of the secant line.)

Covered these, today.

$$(x_1, y_1) = (x, f(x)) = (x_1, f(x_1)) \text{ and } (x_2, y_2) = (x+h, f(x+h)) = (x_2, f(x_2))$$
$$\Rightarrow$$
$$m_{avg} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x+h) - f(x)}{h} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

When we're done with piecewise-defined, I think that finishes 1.1, and pretty much all of 1.2. Section 1.3 is basically #s 8 - 11, above. Section 1.4 is #12.

Sum Difference Product

$$\mathcal{D}(f+g) = \mathcal{D}(f-g) = \mathcal{D}(fg) =$$

$$= \{x \mid x \in \mathcal{D}(f) \text{ and } x \in \mathcal{D}(g)\}$$

Both must eat x

Quotient

Same, but $g(x) \neq 0$, too.

$$\mathcal{D}\left(\frac{f}{g}\right) = \{x \mid x \in \mathcal{D}(f) \text{ and } x \in \mathcal{D}(g) \text{ and } g(x) \neq 0\}$$

Composition

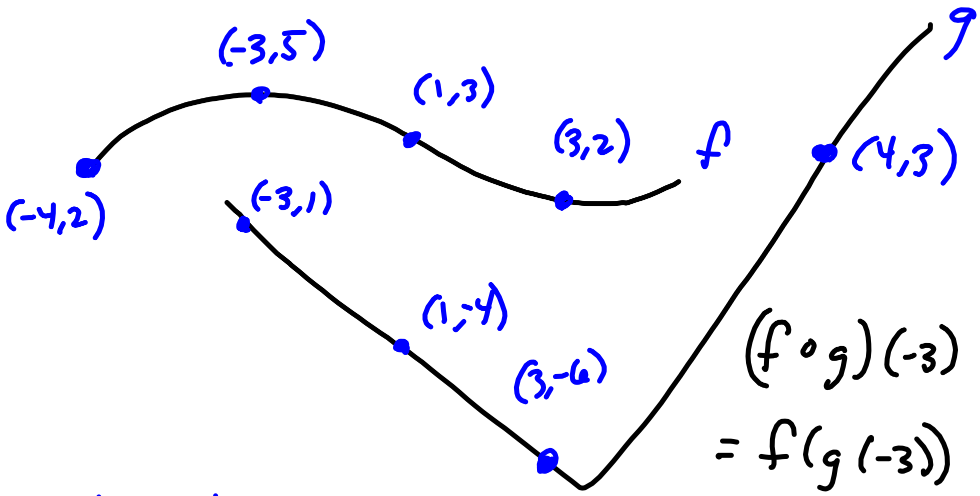
$$\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$$

g eats x, f eats g.

$$(f \circ g)(x) = f(g(x))$$

Today – wipe out 1.3 & 1.4.

§1.3 Evaluating Composite from graph/table.




| x | -4 | -3 | 1 | 3 | 4 |
|---|---|---|---|---|---|
| f | 2 | 5 | 3 | 2 | ∄ |
| g | ∄ | 1 | -4 | -6 | 3 |

$\mathcal{D}(f \circ g) = \{$ Hand $\}$

$(f \circ g)(-3)$

$= f(g(-3))$

$= f(1)$

$= 3$

$f(g(-4))$ ∄

$f(g(1)) = f(-4) = 2$

$f(g(3)) = f(-6)$ ∄

$f(g(4)) = f(3) = 2$

$g \circ g$ :

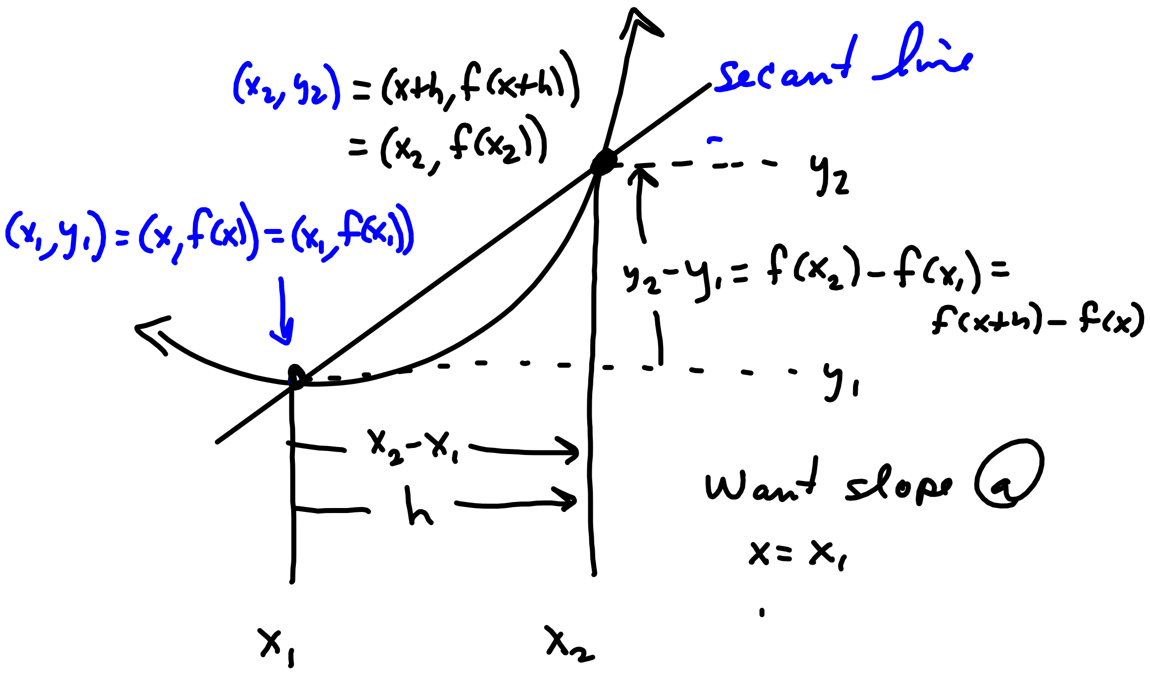
$g(g(-3)) = g(1) = -4$

Did not get to the following, today (Friday). This is essentially where the nuts and bolts of 1.4 begins.

$$(x_1, y_1) = (x, f(x)) = (x_1, f(x_1)) \text{ and } (x_2, y_2) = (x+h, f(x+h)) = (x_2, f(x_2))$$

$$\Rightarrow$$

$$m_{avg} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x+h) - f(x)}{h} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \text{secant slope}$$




The idea is, $x_2 \longrightarrow x_1$

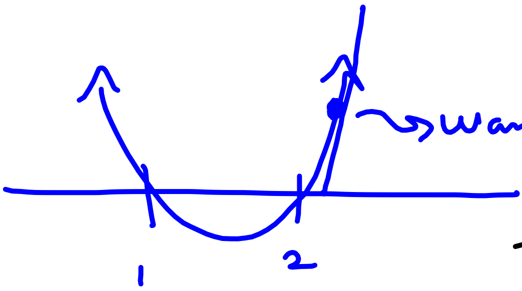
$h \longrightarrow 0$

Numerical (& Graphical Methods)

Let $h$ = very small is how you approach it, intuitively.

Find slope of $f(x) = x^2 - 3x + 2$ at $x = 3$

$$\frac{f(x) - f(3)}{x-3} \quad \text{OR} \quad \frac{f(3+h) - f(3)}{h}$$




want the slope @ $x=3$

Take $x$ close to 3

Take $h$ " " 0

$$\frac{(x^2-3x+2)-(3^2-3(3)+2)}{x-3}$$

$$= \frac{x^2-3x+2-2}{x-3}$$

$$\boxed{= \frac{x^2-3x}{x-3}}$$

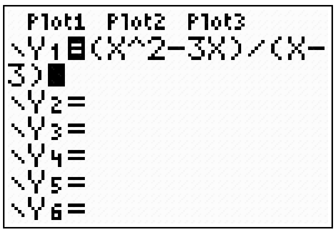
$x = 2.99, 2.9999$

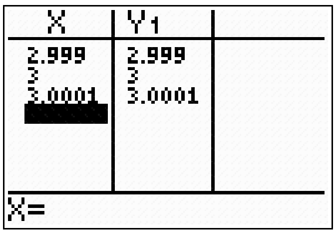
$x = 3.01, 3.0001$

$$= \frac{x(x-3)}{x-3}$$

$$= x \xrightarrow{x \to 3} 3$$

Very Deceptively Easy.




Not all functions are this well-behaved.

$$f := x \rightarrow \sin\left(\frac{10 \cdot \mathrm{Pi}}{x}\right)$$

$$x \rightarrow \sin\left(\frac{10\,\pi}{x}\right)$$

$$ss := x \rightarrow \frac{(f(x) - f(1))}{x - 1}$$

$$x \rightarrow \frac{f(x) - f(1)}{x - 1} \qquad \frac{\sin\left(\frac{10\,\pi}{x}\right)}{x - 1}$$

$$limit(ss(x), x = 1)$$

$$-10\,\pi$$

$$evalf(\%)$$

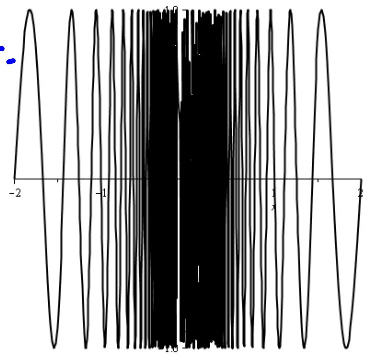
$$-31.41592654$$

$$tanline := x \rightarrow -10 \cdot \mathrm{Pi} \cdot (x - 1) + f(1)$$

$$x \rightarrow -10\,\pi\,(x - 1) + f(1)$$

$plot(f(x), x = -2..2, color = black, thickness = 2)$

Wiggles too fast to be confident.

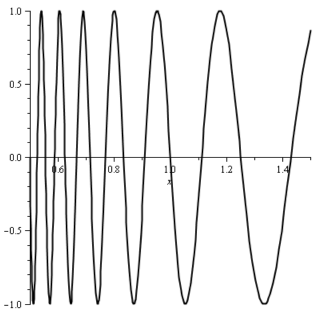


Numerical Methods Suck.

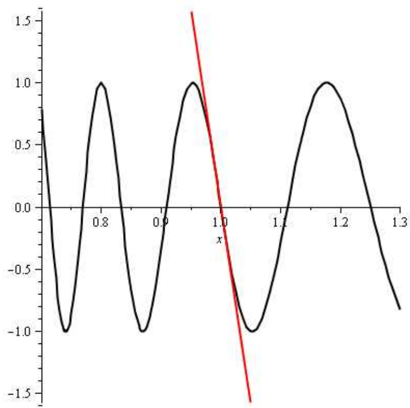
Non-example, cautionary tale.

$plot(f(x), x = 0.5..1.5, color = black, thickness = 2)$

Motivation for learning Limit Laws in Later section.



$plot1 := plot(f(x), x = 0.7..1.3, color = black, thickness = 2) : \% :$
$plot2 := plot(tanline(x), x = 0.95..1.05, color = red, thickness = 2) : \% :$
$display([plot1, plot2])$



$$\frac{\sin\left(\frac{10\pi}{x}\right) - \sin\left(\frac{10\pi}{1}\right)}{x-1}$$

(1,0) on graph

$$= \frac{\sin\left(\frac{10\pi}{x}\right) - 0}{x-1} \xrightarrow{x \to 1} 10\pi$$

Tan line $y = m(x - x_1) + y_1$

$$= 10\pi(x-1) + 0$$

$\left(x, \sin\left(\frac{10\pi}{x}\right)\right)$

$(1, 0)$

1.1 Due Friday
1.2 ~~Monday~~ } Tuesday
1.3 ~~Monday~~
1.4 ~~Tuesday~~ wed or Thursday
1.5 ~~Tuesday or~~ Wednesday, and that's as far out as I'm going.

| | | Classes (50 minutes) | |
|---|---|---|---|
| **CHAPTER 1 - FUNCTIONS AND LIMITS** | | | |
| 1.1 | Four Ways to Represent a Function | 1 | Week 1 |
| 1.2 | Mathematical Models | 2 | |
| 1.3 | New Functions from Old Functions | 1 | |
| 1.4 | The Tangent and Velocity Problems | 1 | |
| 1.5 | The Limit of a Function | 2 | Week 2 |
| 1.6 | Calculating Limits Using the Limit Laws | 2 | |
| 1.7 | The Precise Definition of a Limit | 2 | |
| 1.8 | Continuity | 1 | |
| Review | | 1 | |
| **TEST 1 – Chapter 1** | | 1 | FRIDAY Week 3 is target date for Test 1 |