Today Finish $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Rep. Functions < Verbal, Numerical, Graphical, Algebraically

Evaluating

$\mathcal{D}$ & $\mathcal{R}$

1.1 $\frac{\text{www}}{0}$ Bad $\sqrt{\text{negative}}$ Bad

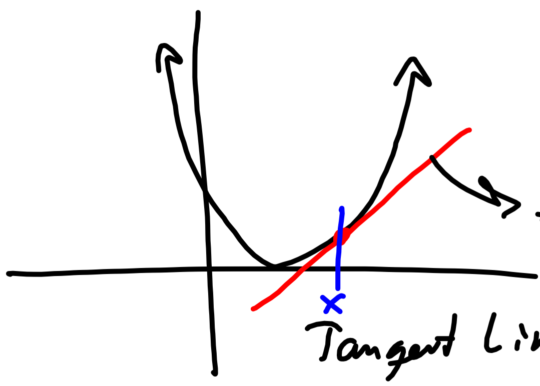
Piecewise-Defined Funcs

Line segment. Lines.

Families of Functions.

1.2 Lines

Power Funcs



$\cancel{3x^2-5x+4}$

$f(x) = (x-1)^2 = x^2-2x+1$

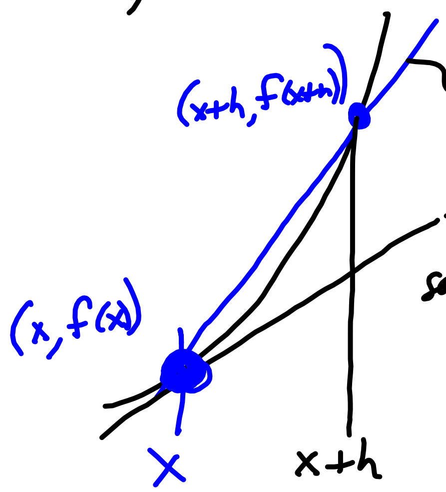
How steep is the curve?

Tangent Line Prob

$$m = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{f(x+h)-f(x)}{x+h-x}$$

$$= \frac{f(x+h)-f(x)}{h}$$



Secant Line

tangent line has same slope as f(x),

The smaller h is, the closer the slope of the secant line is to the slope of the tangent line

To evaluate $\lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h}$, write

$f(x) = x^2 - 2x + 1$

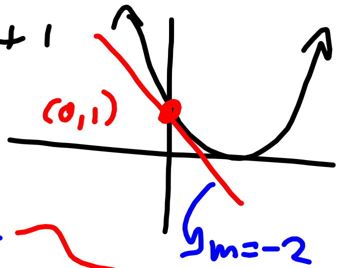
$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2 - 2(x+h) + 1 - (x^2 - 2x + 1)}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 2x - 2h + 1 - x^2 + 2x - 1}{h}$$

$$= \frac{2xh + h^2 - 2h}{h} = \frac{\cancel{h}(2x + h - 2)}{\cancel{h}} = 2x + h - 2 \xrightarrow{h \to 0} 2x - 2$$

Tells you the slope of $x^2 - 2x + 1$ @ any point.

$(x-1)^2 = x^2 - 2x + 1$



@ (0,1)

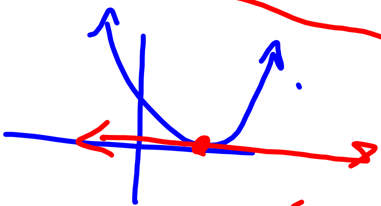
$m = (2x-2)\Big|_{x=0} = -2$

Evaluation!

@ (1,0)

$m = (2x-2)\Big|_{x=1}$

$= 2(1) - 2 = 0$



$(2x-2)\Big|_{x=0} = -2$

$f(x)$

$f(2)$

Evaluating

$f(x) = x^3 - 2x \Longrightarrow$ "f of '2'"

$f(2) = 2^3 - 2(2) = 8 - 4 = \boxed{4 = f(2)}$

$f(a) = a^3 - 2a$ $\qquad x$

$f(x+h) = (x+h)^3 - 2(x+h)$ $\qquad x$

$f(☺) = ☺^3 - 2☺$ $\qquad ☺$

Find Domain $\mathcal{D}$.

$= \{ x \mid f(x) \text{ exists!} \}$

$= \{ x \mid f(x) \text{ is defined} \}$

2 things.

(1) Division by zero is bad.

$$\frac{\text{numerator}}{\text{denominator}}$$

$$\sqrt{9} = 3$$

Need Statement for Domain:

Need denominator $\neq 0$

(2) $\sqrt{\text{radicand}}$ ≥ 0 always.

Need radicand ≥ 0

Find $\mathcal{D}$:

$\pi x^{23} + 25x^2 - x$ $\quad \mathcal{D} = \mathbb{R} = (-\infty, \infty) = \{x \mid x \text{ is real}\}$

$= \{x \mid x \in \mathbb{R}\}$

$$\frac{\pi x^{23} + 25x^2 - x}{x^2 - 3x + 2} = f(x)$$

$(x-1)(x-2) = 0$

$x - 1 = 0$ OR $x - 2 = 0$

Need $x^2 - 3x + 2 \neq 0$

$(x-2)(x-1) \neq 0$

$x - 2 \neq 0$ AND $x - 1 \neq 0$

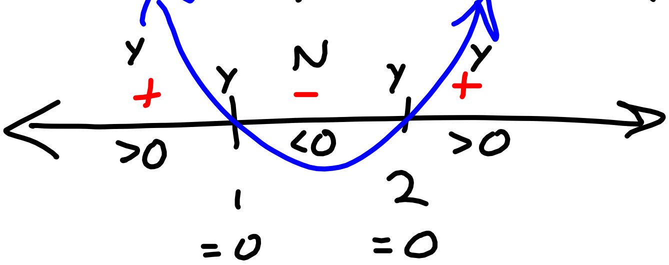
$\mathcal{D}(f) = \{x \mid x \neq 2 \text{ \& } x \neq 1\} = \mathbb{R} \setminus \{1, 2\}$

$f(x) = \sqrt{x^2-3x+2}$

Need $x^2-3x+2 \geq 0$

$\vdots$

$x=1,\ x=2$ are "critical"



$\longrightarrow \mathcal{D}(f) = (-\infty, 1] \cup [2, \infty)$

Start 1.1, 1.2
Print Questions

$f(x+h) \neq f(x)+h$

$(x+h)^2 = x^2+2xh+h^2$

$(x-3)^2$

$(a+b)^2 = a^2+2ab+b^2$

$(2x+5)^2$

$(2x-7)^2$