201 §3.3 II #s 29-39

#s 29-40 Graph it! Show all key features

29) $x^3-12x+2$ $\quad \mathcal{D} = \mathbb{R} = \mathbb{R}$

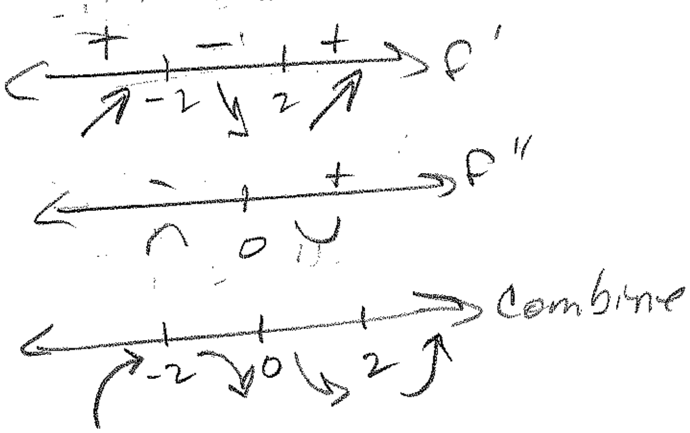
$f'(x) = 3x^2 - 12 \overset{SET}{=} 0$

$x^2 = 4$

$x = \pm 2$

$f''(x) = 6x \overset{SET}{=} 0$

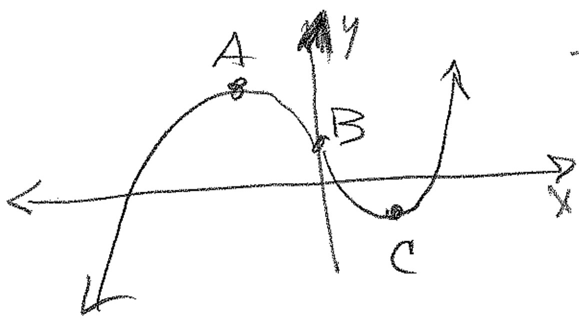
$x = 0$



$$\begin{array}{r|rrrr} -2 & 1 & 0 & -12 & 2 \\ & & -2 & 4 & 16 \\ \hline & 1 & -2 & -8 & 18 \end{array}$$

$\rightsquigarrow$ A: $\boxed{(-2,18) \text{ MAX}}$

$f(0) = 2 \rightsquigarrow$ B: $\boxed{(0,2) \text{ I.P.}}$



$$\begin{array}{r|rrrr} 2 & 1 & 0 & -12 & 2 \\ & & 2 & 4 & -16 \\ \hline & 1 & 2 & -8 & -14 \end{array}$$

$-14 = $ C

$\rightsquigarrow \boxed{(2,-14) \text{ MIN}}$

201 §3.3 #s 31–39

31) $f(x) = -x^4 + 2x^2 + 2$ $\boxed{\text{EVEN!}}$

$f'(x) = -4x^3 + 4x \overset{\text{SET}}{=} 0$

$-4x(x^2 - 1) = 0$

$x = 0, \pm 1$

$f''(x) = -12x^2 + 4 \overset{\text{SET}}{=} 0$

$12x^2 = 4$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}} = \pm \frac{\sqrt{3}}{3}$

f': + | -1 | − | 0 | + | 1 | −

↗ -1 ↘ 0 ↗ 1 ↘

f'': − | $-\frac{\sqrt{3}}{3}$ | + | $\frac{\sqrt{3}}{3}$ | −

∩ $-\frac{\sqrt{3}}{3}$ ∪ $\frac{\sqrt{3}}{3}$ ∩

combine

-1, $-\frac{\sqrt{3}}{3}$, 0, $\frac{\sqrt{3}}{3}$, 1

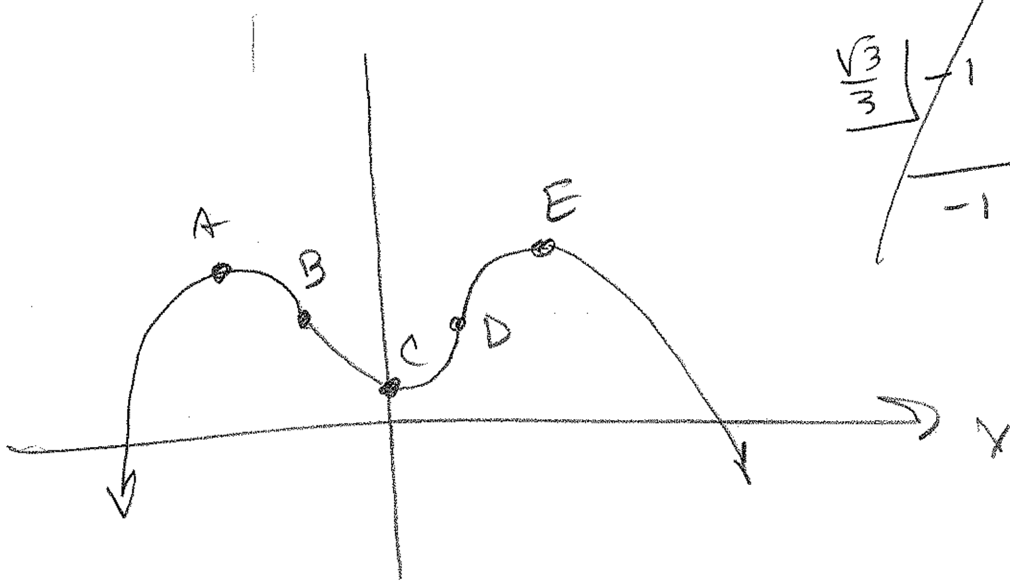
$f(0) = 2$

~~-1~~	-1	0	2	2
		1	-1	-1
	-1	1	1	1

(−1, 1) MAX

→ (1, 1) MAX

$\frac{\sqrt{3}}{3}$	-1	0	2	2
		$-\frac{\sqrt{3}}{3}$	$-\frac{1}{3}$	$\frac{5\sqrt{3}}{9}$
	-1	$-\frac{\sqrt{3}}{3}$	$\frac{5}{3}$	$2+\frac{5\sqrt{3}}{9}$



A = (−1, 3) MAX

B = $\left(-\frac{\sqrt{3}}{3}, \frac{23}{9}\right)$ IP

C = (0, 2)

D = (1, 3) MAX

E = $\left(\frac{\sqrt{3}}{3}, \frac{23}{9}\right)$ IP

201 §3.3 II #s 31-39

31 Re-do f(x)'s

$$\begin{array}{r|rrrrr} -1 & -1 & 0 & 2 & 0 & 2 \\ & & 1 & -1 & -1 & 1 \\ \hline & -1 & 1 & 1 & -1 & 3 \end{array} \quad (-1, 3) \Rightarrow (1, 3)$$

$$\begin{array}{r|rrrrr} \frac{\sqrt{3}}{3} & -1 & 0 & 2 & 0 & 2 \\ & & -\frac{\sqrt{3}}{3} & -\frac{1}{3} & \frac{5\sqrt{3}}{9} & \frac{5}{9} \\ \hline & -1 & -\frac{\sqrt{3}}{3} & \frac{5}{3} & \frac{5\sqrt{3}}{9} & \frac{23}{9} \end{array} \quad \left(\frac{\sqrt{3}}{3}, \frac{23}{9}\right)$$

$$\Rightarrow \left(-\frac{\sqrt{3}}{3}, \frac{23}{9}\right)$$

201 §3.3 #s 33–39

33 $h(x) = (x+1)^5 - 5x - 2$

$h'(x) = 5(x+1)^4 - 5 \overset{\text{SET}}{=} 0$

$(x+1)^4 = 1$

$x+1 = \pm 1$

$x = -1 \pm 1 \to 0, -2$

$h''(x) = 20(x+1)^3 \overset{\text{SET}}{=} 0$

$x = -1$

$h(0) = 1 - 2 = -1$ $\boxed{(0,-1)}$

$h(-2) = (-1) + 10 - 2 = 7$ $\boxed{(-2,7)}$

$h(-1) = 5 - 2 = 3$ $\boxed{(-1,3)}$

f': $+$ on $(-\infty,-2)$ ↗, $-$ on $(-2,0)$ ↘, $+$ on $(0,\infty)$ ↗

f'': $-$ on $(-\infty,-1)$ ∩, $+$ on $(-1,\infty)$ ∪

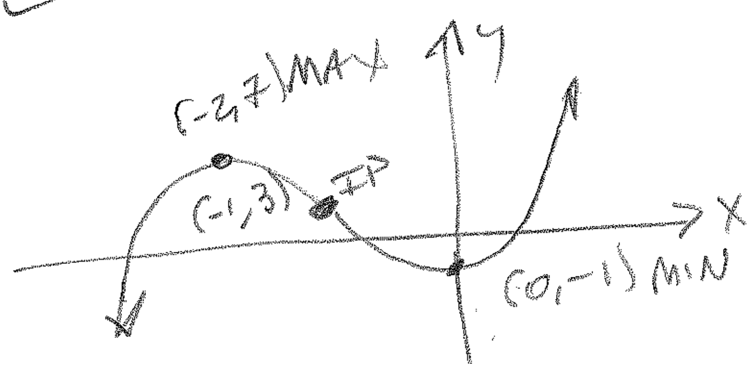
Combine: -2, -1, 0

Check: $(x+1)^4 - 1$

$= ((x+1)^2 - 1)((x+1)^2 + 1)$

$= (x+1-1)(x+1+1)((x+1)^2+1)$

$x = 0, -2$ ✓



201 §3.3 II #s 35–39

35

$F(x) = x(6-x)^{\frac{1}{2}}$

$F'(x) = (6-x)^{\frac{1}{2}} + x\left(\frac{1}{2}(6-x)^{-\frac{1}{2}}(-1)\right)$

$= \left(\sqrt{6-x}\right)\left(\frac{2\sqrt{6-x}}{2\sqrt{6-x}}\right) - \frac{x}{2\sqrt{6-x}}$

$= \frac{2(6-x) - x}{2\sqrt{6-x}} = \frac{12 - 2x - x}{2\sqrt{6-x}} = \frac{12 - 3x}{2\sqrt{6-x}}$

SET $= 0 \implies x = 4$

$+ \quad - \quad |$

$\nearrow \quad 4 \quad \searrow \quad$ 6 = End of domain.

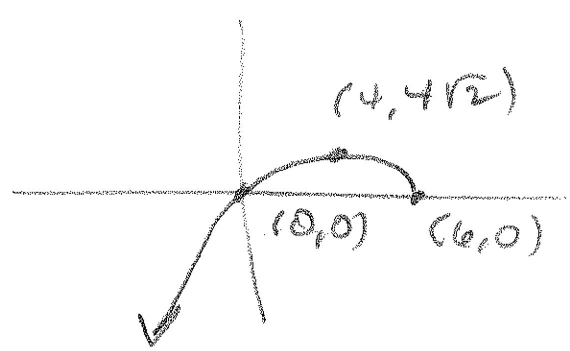
$F''(x) = \frac{-3(2\sqrt{6-x}) - (12-3x)(2)\left(\frac{1}{2}(6-x)^{-\frac{1}{2}}(-1)\right)}{4(6-x)}$

=

wow!

$f'' < 0$ on its domain

max of $4(2^{\frac{1}{2}}) = 4\sqrt{2}$ @ $x = 4$



201 §3.3 II #s 35-39

(35) $F(x) = x\sqrt{6-x} = x(6-x)^{\frac{1}{2}}$ $\quad \mathcal{D} = \{x \mid 6-x \geq 0\}$

$= \{x \mid 6 \geq x\}$

$= (-\infty, 6]$

$F'(x) = (6-x)^{\frac{1}{2}} + x\left(\frac{1}{2}(6-x)^{-\frac{1}{2}}(-1)\right)$

$= (6-x)^{\frac{1}{2}} - \dfrac{x}{2(6-x)^{1/2}}$

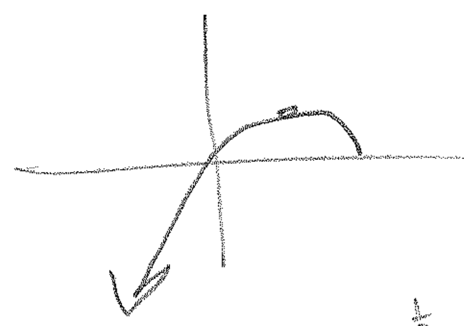
$= \dfrac{6-x-x}{2\sqrt{6-x}} = \dfrac{6-2x}{2\sqrt{6-x}}$

Goes vertical @ edge of its domain $(x=6)$

$6-2x=0$

$x=3$ $\quad 3\sqrt{6-3} = 3\sqrt{3}$

$(3, 3\sqrt{3})$ MAX



$+ \quad - \quad F'$

$\nearrow \; 3 \; \searrow$

$F''(x) = \dfrac{-2(2\sqrt{6-x}) - (6-2x)\left[2\left(\frac{1}{2}(6-2x)^{-\frac{1}{2}}(-2)\right)\right]}{4(6-x)}$

$= \dfrac{\left(-4\sqrt{6-x}\right)\left(\frac{\sqrt{6-x}}{\sqrt{6-x}}\right) - (6-2x)\left(\frac{-2}{\sqrt{6-x}}\right)}{4(6-x)}$

$= \dfrac{-4(6-x)+12-4x}{4(6-x)^{3/2}} = \dfrac{-24+4x+12-4x}{4(6-x)^{3/2}} = \dfrac{-12}{4(6-x)^{3/2}}$

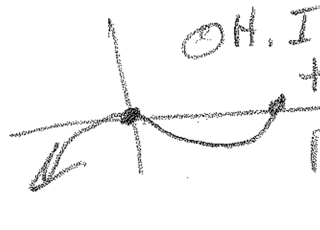
$f'' < 0$ on its domain. ✓

Missing something

$F(0) = F(6) = 0$

Didn't find a min!

OH. I mis-interpreted the sign pattern on f!



201 §3.3 II #s 37, 39

(37) $c(x) = x^{\frac{1}{3}}(x+4) = 0$ @ $x = 0, -4$

$\cancel{c'(x)} = x^{\frac{4}{3}} + 4x^{\frac{1}{3}}$

$\Longrightarrow c'(x) = \frac{4}{3}x^{\frac{1}{3}} + \frac{4}{3}x^{-\frac{2}{3}} = \frac{4}{3}x^{-\frac{2}{3}}(x+1)$

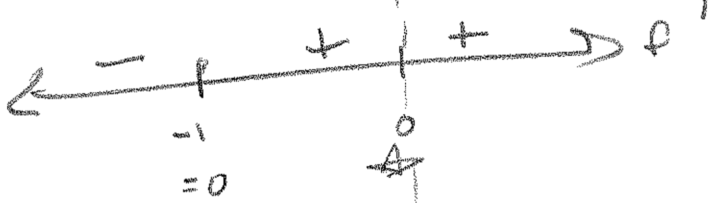
$\nexists$ @ $x = 0$

$= 0$ @ $x = -1$

No sign change f' @ $x = 0$

$x^{-\frac{2}{3}} = \frac{1}{x^{\frac{2}{3}}} = \frac{1}{\left(x^{\frac{1}{3}}\right)^2}$

See the 2? It's even.



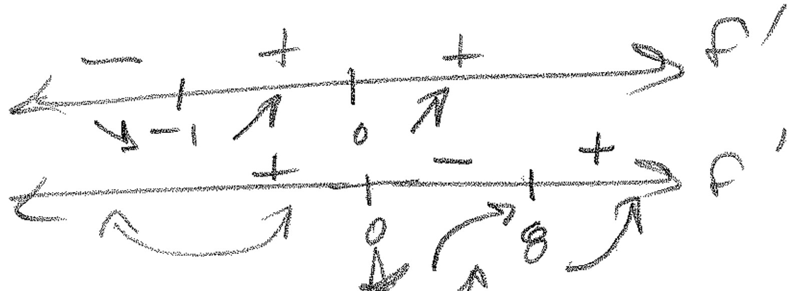
$f''(x) = \frac{4}{9}x^{-\frac{2}{3}} - \frac{8}{9}x^{-\frac{5}{3}}$

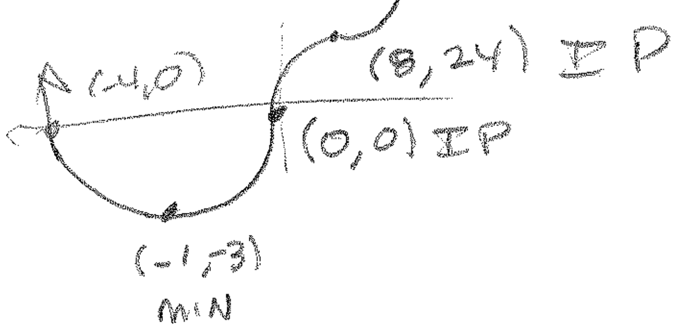
$= \frac{4}{9}x^{-\frac{5}{3}}\left(x^{\frac{1}{3}} - 2\right)$

$\nexists$ @ $x = 0$

$= 0$ @ $x^{\frac{1}{3}} = 2$

$x = 8$





201 S 3/3 IV #39

(39) $f(\theta) = 2\cos\theta + \cos^2\theta \quad 0 \le \theta \le 2\pi$

$f(\theta) = 0 \Longrightarrow$

$\cos\theta(\cos\theta + 2) = 0 \Longrightarrow$

$\cos\theta = 0 \Longrightarrow$ $\qquad \cos\theta + 2 = 0$ Never

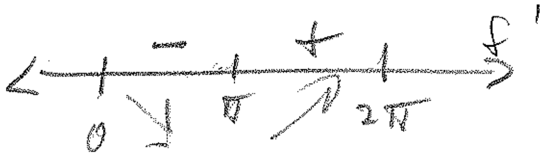
$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ x-ints

$f'(\theta) = -2\sin\theta + 2\cos\theta(-\sin\theta)$

$= -2\sin\theta(1 + \cos\theta)$

$-2\sin\theta = 0 \qquad \cos\theta = -1$

$\theta = 0, \pi, 2\pi \qquad \theta = \pi$



$-2\sin\frac{\pi}{2}\left(1 + \cos\frac{\pi}{2}\right)$

$= -2(1) = -2$ $\quad -$

$-2\sin\left(\frac{3\pi}{2}\right)\left(1 + \cos\frac{3\pi}{2}\right)$

$+2(1) = 2$ $\quad +$

$f''(\theta) = -2\cos\theta - 2\left[-\sin\theta\sin\theta + \cos\theta\cos\theta\right]$

$= -2\cos\theta - 2\left[\cos^2\theta - \sin^2\theta\right]$

$= -2\cos\theta - 2\cos^2\theta + 2(1 - \cos^2\theta)$

$= -2\cos\theta - 2\cos^2\theta + 2 - 2\cos^2\theta$

$= -4\cos^2\theta - 2\cos\theta + 2 \overset{SET}{=} 0$

$b^2 - 4ac$
$1^2 - 4(2)(1)$
$= -7$

$-2(2\cos^2\theta - \cos\theta + 1) = 0$

$\Longrightarrow 2u^2 - u + 1 = 0 \Longrightarrow u = 1 \pm$

No real zeros $\therefore$ ALWAYS NEGATIVE

Not making sense

201 §3.3 III #39

(39) $f'(\theta) = -2\sin\theta + 2\cos\theta(-\sin\theta)$

$= -2\sin\theta - 2\sin\theta\cos\theta = -2\sin\theta(1+\cos\theta) \overset{SET}{=} 0 \longrightarrow$

$-2\sin\theta = 0$

$\sin\theta = 0$

$\theta \in \{0, \pi, 2\pi\}$

$\cos\theta + 1 = 0$

$\cos\theta = -1$

$\theta \in \{\pi\}$

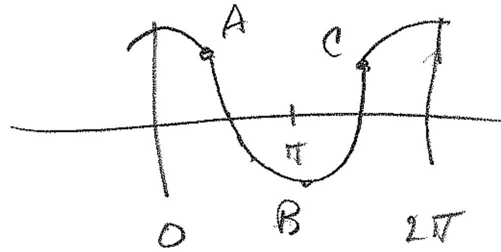
$(2\pi, 3)$ MAX

$(0, 3)$ MAX

$A = (\frac{\pi}{3}, \frac{5}{4})$ IP

$B = (\pi, -1)$ MIN

$C = (\frac{5\pi}{3}, \frac{5}{4})$ IP



$f''(\theta) = -2\cos\theta - 2\cos^2\theta + 2\sin^2\theta$

$= -2\cos\theta - 2(\cos^2\theta - \sin^2\theta)$

$= -2\cos\theta - 2(\cos^2\theta - (1-\cos^2\theta))$

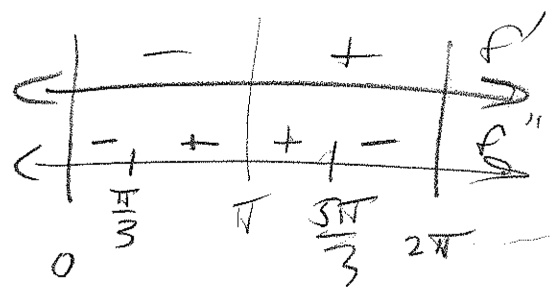
$= -2\cos\theta - 2(\cos^2\theta - 1 + \cos^2\theta)$

$= -2\cos\theta - 2(2\cos^2\theta - 1)$

$= -4\cos^2\theta - 2\cos\theta + 2$

$= -2(2\cos^2\theta + \cos\theta - 1)$

$= 2(2u^2 + u - 1) \overset{SET}{=} 0$



$\Rightarrow (2u-1)(u+1) = 0$

$2u = 1$

$u = \frac{1}{2}$

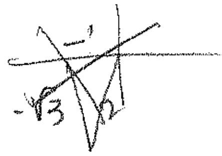
$\cos\theta = \frac{1}{2}$

$\boxed{\theta = \frac{\pi}{3}, \frac{5\pi}{3}}$

$u = -1$

$\cos\theta = -1$

$\boxed{\theta = \pi}$



$2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$(2\cos\theta - 1)(\cos\theta + 1)$

@ $\theta = \pi$, $f''(x)$ doesn't change sign.

201 Test 2 Take-Home

(1) $f(x) = -2\sin x \cos x - x$ on $[0, 2\pi]$

$f(0)=0,\ f(2\pi) = -2\pi \longrightarrow (0,0),\ (2\pi, -2\pi)$

$(2\pi, -6.283)$

$f'(x) = -2[\cos^2 x - \sin^2 x] - 1$

$= -2[\cos^2 x - 1 + \cos^2 x] - 1$

$= -4\cos^2 x + 2 - 1 = f'(x)$

$= -4\cos^2 x + 1 \overset{SET}{=} 0$

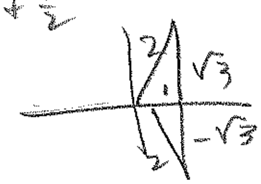
$\frac{9\pi}{6} = \frac{3\pi}{2} \geq \pi$

$\longrightarrow 4\cos^2 x = 1$

$\cos^2 x = \frac{1}{4}$

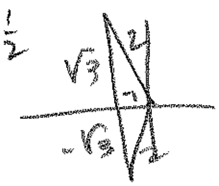
$\cos x = \pm \frac{1}{2}$

$+\frac{1}{2}$



$\frac{\pi}{3},\ 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$-\frac{1}{2}$



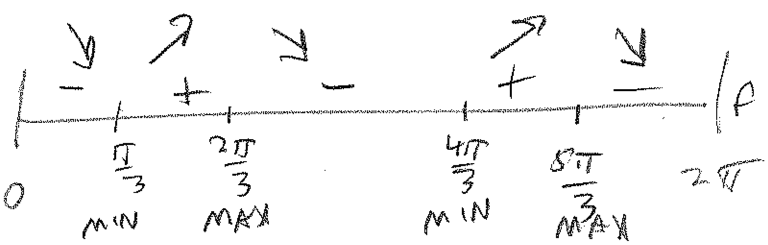
$\frac{2\pi}{3},\ \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$f\left(\frac{\pi}{3}\right) = -2\sin\frac{\pi}{3}\cos\frac{\pi}{3} - \frac{\pi}{3}$

$= -2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \frac{\pi}{3}$

$= -\frac{\sqrt{3}}{2} - \frac{\pi}{3}$

$\rightsquigarrow \left(\frac{\pi}{3}, -\frac{\sqrt{3}}{2} - \frac{\pi}{3}\right) \approx \left(\frac{\pi}{3}, -1.913\right)$



$f\left(\frac{2\pi}{3}\right) = -2\sin\frac{2\pi}{3}\cos\frac{2\pi}{3} - \frac{2\pi}{3}$

$= -2\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right) - \frac{2\pi}{3}$

$= \frac{\sqrt{3}}{2} - \frac{2\pi}{3}$

$\rightsquigarrow \left(\frac{2\pi}{3}, \frac{\sqrt{3}}{2} - \frac{2\pi}{3}\right) \approx \left(\frac{2\pi}{3}, -1.228\right)$

$f'\left(\frac{\pi}{4}\right) = -4\cos^2\frac{\pi}{4} + 1 = -1$

$f'\left(\frac{\pi}{2}\right) = 0 + 1 = 1$

$f'(\pi) = -4\cos^2(\pi) + 1 = -3$

$f'\left(\frac{3\pi}{2}\right) = 1$

$f'\left(\frac{11\pi}{6}\right) = -2$

$f\left(\frac{4\pi}{3}\right) = -2\sin\frac{4\pi}{3}\cos\frac{4\pi}{3} - \frac{4\pi}{3}$

$= -2\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right) - \frac{4\pi}{3} = -\frac{\sqrt{3}}{2} - \frac{4\pi}{3}$

$\approx \left(\frac{4\pi}{3}, -5.055\right)$

$f\left(\frac{5\pi}{3}\right) \approx -4.370$

$\left(\frac{5\pi}{3}, -4.370\right)$

201 TEST 2 TAKE-HOME

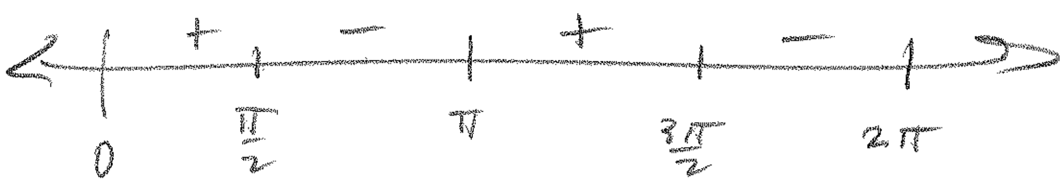
① cont'd

$$f'(x) = -4\cos^2 x + 1 \Longrightarrow$$

$$f''(x) = (-8\cos x)(-\sin x)$$

$$= 8\sin x \cos x \overset{\text{set}}{=} 0$$

$$\Longrightarrow x = 0, \frac{\pi}{2}, \frac{\pi}{1}, \frac{3\pi}{2}, 2\pi$$

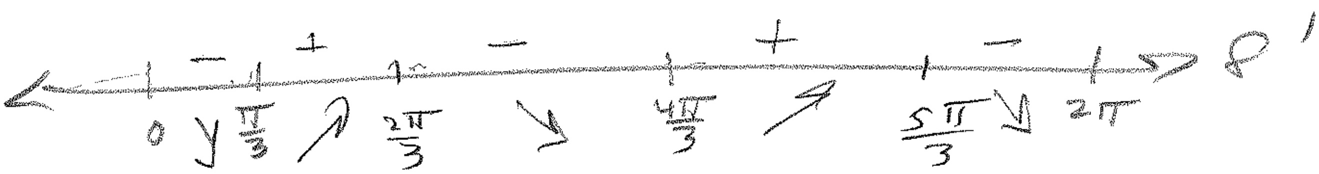


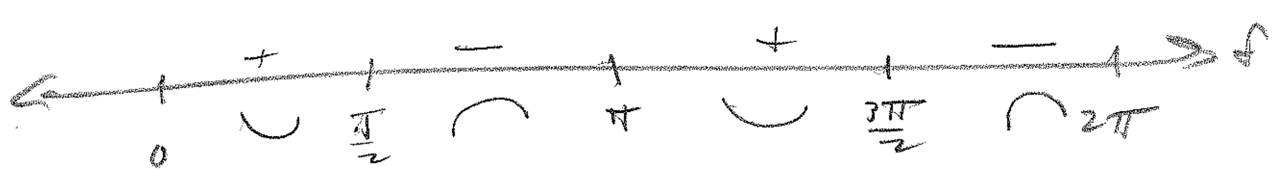
Test $\frac{\pi}{4}$: +

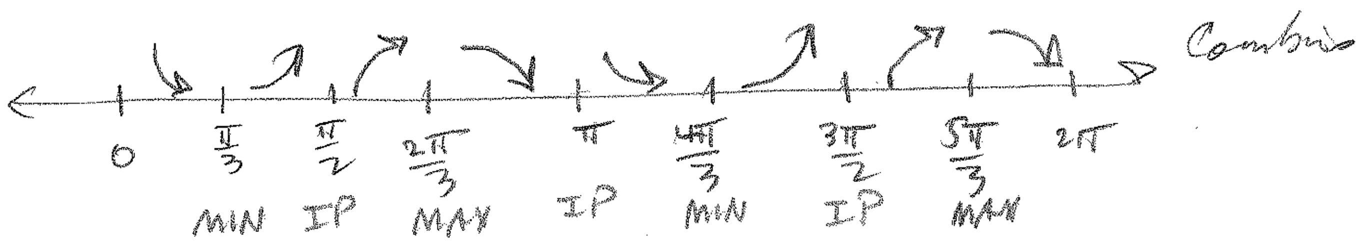
$\frac{2\pi}{3}$: −

$\frac{4\pi}{3}$: +

$\frac{7\pi}{4}$: −







201 TEST 2 TAKE-HOME

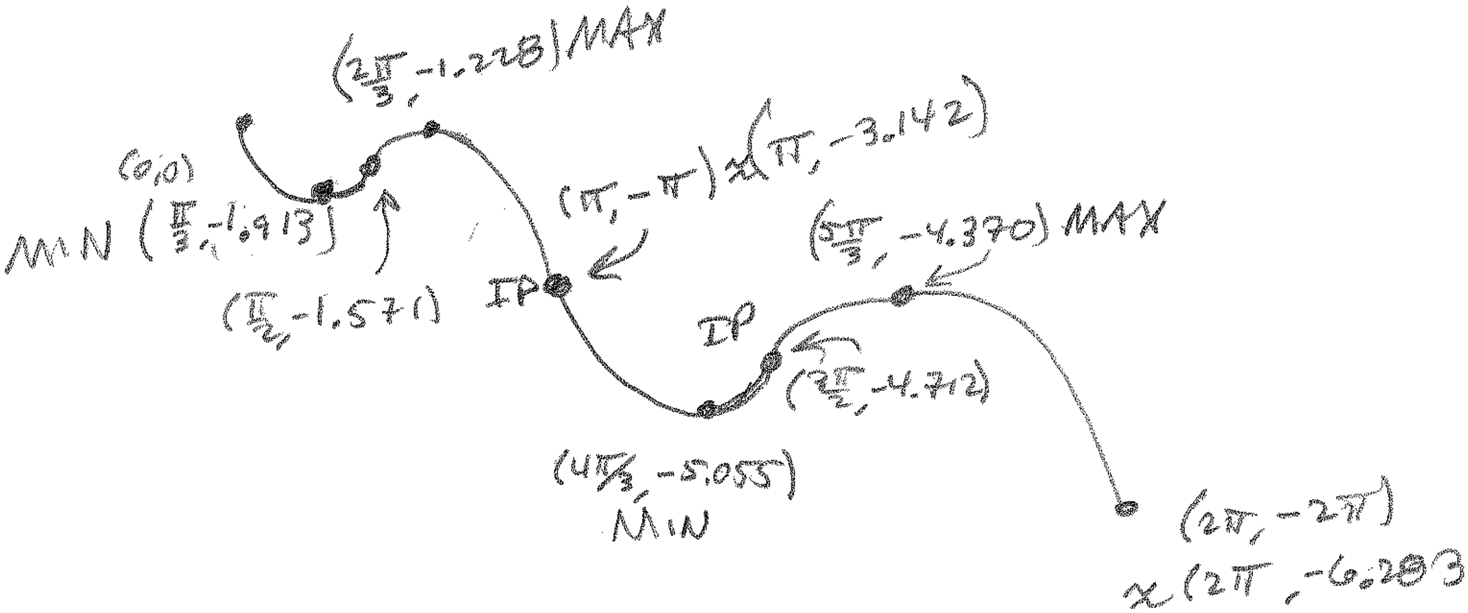
(1) $f(\frac{\pi}{2}) = -\frac{\pi}{2} \rightarrow (\frac{\pi}{2}, -1.571)$ $\quad -2\sin\frac{3\pi}{2}\cos\frac{3\pi}{2} - \frac{3\pi}{2}$

$f(\pi) = -\pi \rightarrow (\pi, -3.142)$

$f(\frac{3\pi}{2}) = -\frac{3\pi}{2} \rightarrow (\frac{3\pi}{2}, -4.712)$

$f(2\pi) = -2\pi \rightarrow (2\pi, -2\pi)$

$f(0) = 0 \rightarrow (0,0)$



201 TEST 2 TAKE-HOME

(2) Sketch $f(x) = \frac{2x^2 - x - 10}{x-2}$ $\mathcal{D} = \mathbb{R} \setminus \{2\}$

$b^2 - 4ac = (-1)^2 - 4(2)(-10)$

$= 1 + 80$

$= 81 \rightsquigarrow \sqrt{81} = 9$

$x = \frac{1 \pm 9}{2(2)}$ $\to \frac{10}{4} = \frac{5}{2}$, $\to -\frac{8}{4} = -2$

$f(x) = \frac{2(x-\frac{5}{2})(x+2)}{x-2}$

$= \frac{(2x-5)(x+2)}{x-2}$

$\boxed{\text{V.A. } x = 2}$

$\boxed{\text{O.A. } y = 2x+3}$

$\boxed{\text{x-int's } (\tfrac{5}{2}, 0), (-2, 0)}$

$$\begin{array}{r|rrr} 2 & 2 & -1 & -10 \\ & & 4 & 6 \\ \hline & 2 & 3 & -4 \\ & x & c & r \end{array}$$

$\boxed{2x+3}$

$f(x) = 2x + 3 - \frac{4}{x-2}$ is easier for derivative

$f'(x) = \frac{(4x-1)(x-2) - (2x^2 - x - 10)}{(x-2)^2}$

Nah. $f(x) = 2x + 3 - \frac{4}{x-2} = 2x + 3 - 4(x-2)^{-1}$

$\Rightarrow f'(x) = 2 + 4(x-2)^{-2} = \frac{2(x-2)^2}{(x-2)^2} + \frac{4}{(x-2)^2}$

$= \frac{2(x^2 - 4x + 4) + 4}{(x-2)^2} = \frac{2x^2 - 8x + 8 + 4}{(x-2)^2}$ SET $0 \Rightarrow$

$2x^2 - 8x + 12 = 0$

$x^2 - 4x + 6 = 0$

$(x-3)(x-1) = 0 \Longleftrightarrow x \in \{1, 3\}$

$2(1) + 3 - \frac{4}{1-2} = 5 + 4 = 9$

$(1, 9)$ max/min?

$2(3) + 3 - \frac{4}{3-2} = 9 - 4 = 5$

$\boxed{(3, 5)}$ max/min?

201 TEST 2 Chapter 3

(2) cnt'd

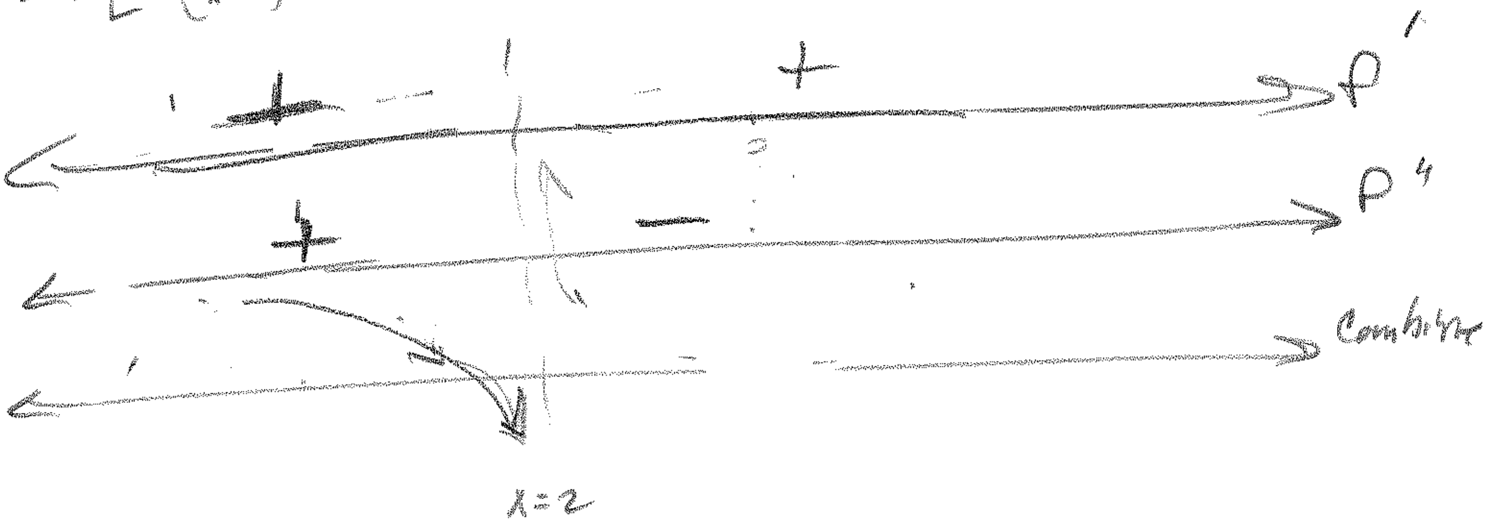
$$f''(x) = \frac{d}{dx}\left[\frac{2(x^2-4x+6)}{(x-2)^2}\right] = 2\frac{d}{dx}\left[\frac{x^2-4x+6}{(x-2)^2}\right]$$

$$= 2\left[\frac{(2x-4)(x-2)^2 - (x^2-4x+6)(2(x-2))}{(x-2)^4}\right] \rightarrow 2(x-2)$$

$$= 2\left[\frac{(x-2)[(2x-4)(x-2) - (2)(x^2-4x+6)]}{(x-2)^2}\right]$$

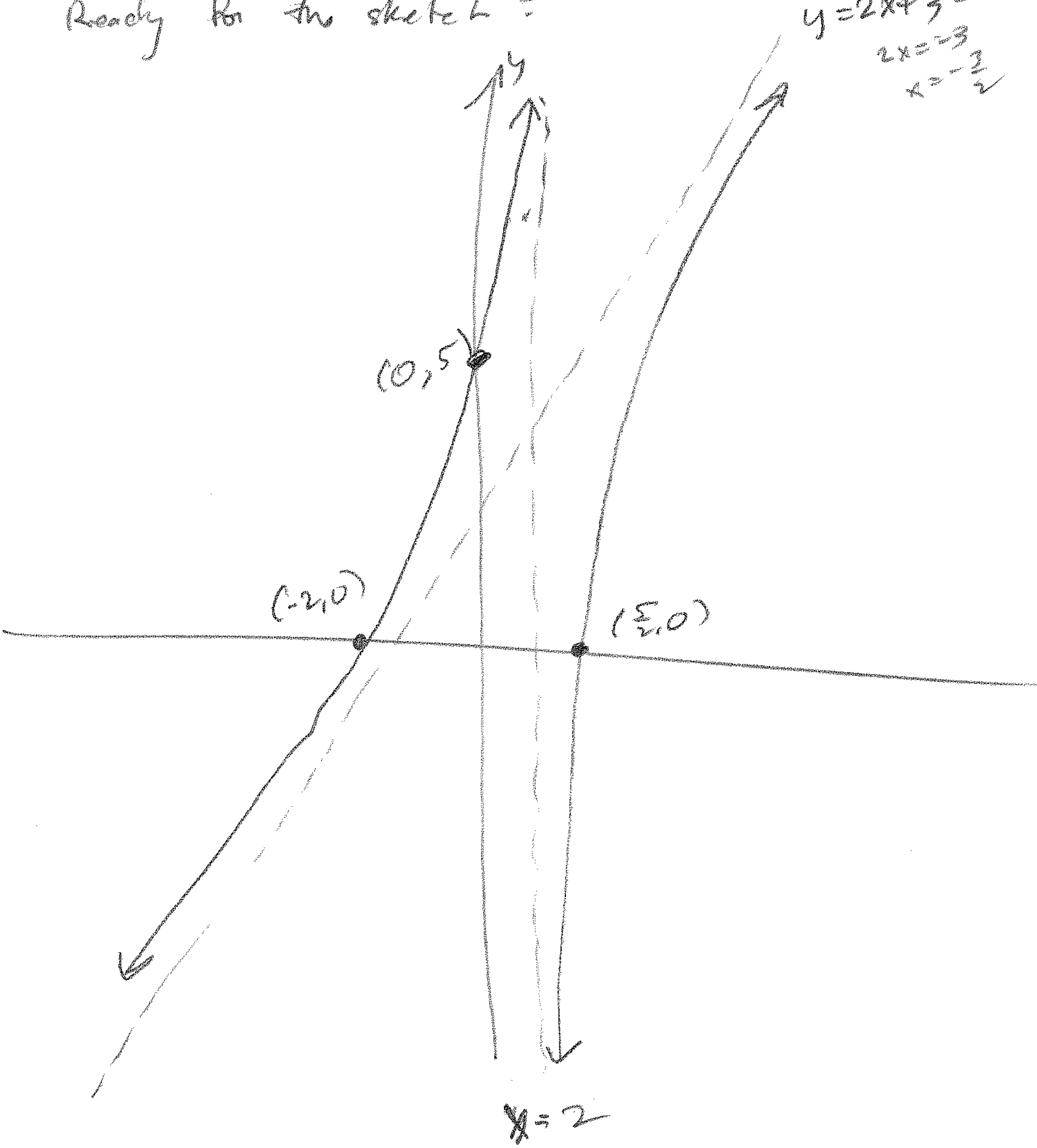
$$= 4\left[\frac{(x-2)^2 - (x^2-4x+6)}{(x-2)^3}\right] = 4\left[\frac{x^2-4x+4-x^2+4x-6}{(x-2)^3}\right]$$

$$= 4\left[\frac{-2}{(x-2)^3}\right] = \frac{-8}{(x-2)^3}$$



201 Test 2 Fall 2013 Take-home

Ready for the sketch :



201 §3.7 #s 3,5,7,9,15, 19, 35, 39

(3) 2 positive #s whose product is 100
& whose sum is a minimum

$xy = 100$

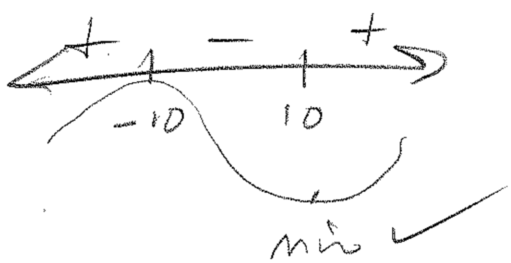
$P = x + y$ to be minimized

$y = \frac{100}{x} = 100x^{-1} \Longrightarrow$

$P = x + 100x^{-1} \Longrightarrow$

$P' = 1 - 100x^{-2} = 1 - \frac{100}{x^2} = \frac{x^2 - 100}{x^2} \overset{\text{SET}}{=} 0$

$\Longrightarrow x = \pm 10 \Longrightarrow \boxed{x = 10} \Longrightarrow \boxed{y = \frac{100}{10} = 10}$

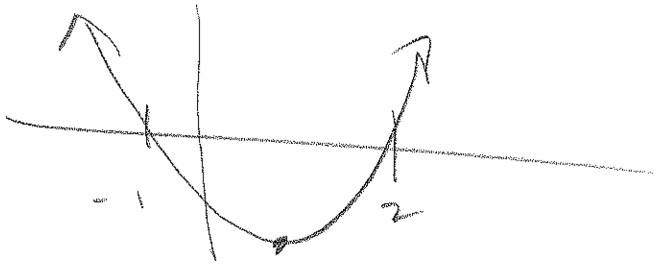


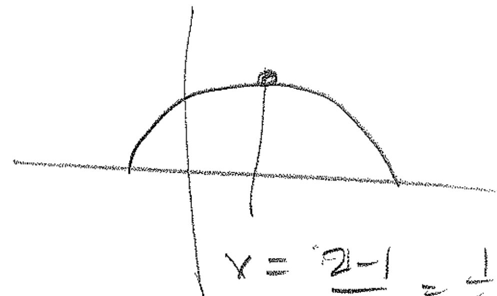
(5) Max Vertical Distance between $y = x + 2$
and $y = x^2$ $\forall$ $-1 \le x \le 2$.

201 §3.7 #s 5, 7, 9, 15, 19, 38, 39

5 $|x^2 - (x+2)| = |x^2 - x - 2|$ MAXIMIZE IT on $[-1, 2]$

$= |x-2||x+1|$





$x = \frac{1}{2}$ should do it.

$x = \frac{2-1}{2} = \frac{1}{2}$

$y' = 2x - 1$ wait. Since $|x^2-x-2| = -(x^2-x-2)$ on $[-1, 2]$, use

$y = -x^2 + x + 2 \Longrightarrow$

$y' = -2x + 1$ SET 0 $\Longrightarrow \boxed{x = \frac{1}{2}}$

Max Distance is

$-(\frac{1}{2})^2 + \frac{1}{2} + 2 = -\frac{1}{4} + \frac{2}{4} + \frac{8}{4} = \boxed{\frac{9}{4} = y}$

201 §3.7 #s 7, 9, 15, 19, 38, 39

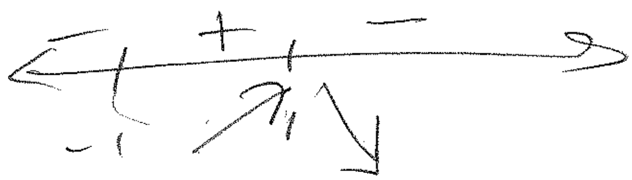
(9) Yield $= Y = \dfrac{kN}{1+N^2}$, where $k =$ konstant, and $N =$ nitrogen in soil ("appropriate")

Maximize the yield.

$$Y' = \frac{k(N^2+1) - kN(2N)}{(N^2+1)^2}$$

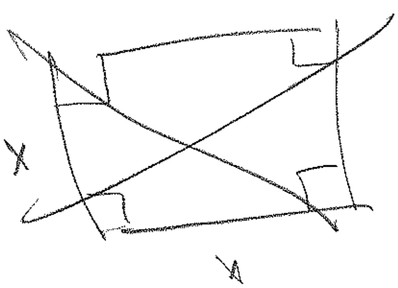
$$= \frac{kN^2 + k - 2kN^2}{(\quad)^2} = \frac{-kN^2 + k}{(\quad)^2}$$

$$= \frac{-k(N^2-1)}{(\quad)^2} \Rightarrow N = \pm 1 \Rightarrow \boxed{N = 1}$$



Yes. MAX ✓

(15) 1200 cm^2 of material for square box w/ open top. Maximize the vol.



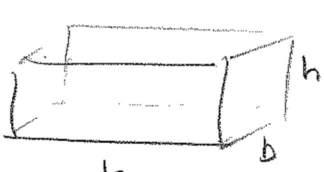
All we know is surface area

$= b^2 + 4bh = 1200 \Rightarrow 4bh = 1200 - b^2$, $h = \dfrac{1200 - b^2}{4b}$

Volume $= V = b^2 h$

$= b^2\left[\dfrac{1200 - b^2}{4b}\right]$

$= \dfrac{1200b - b^3}{4}$



$\Rightarrow \dfrac{dV}{db} = 300 - \dfrac{3b^2}{4} \overset{\text{SET}}{=} 0 \Rightarrow$

$\dfrac{3b^2}{4} = 300 \Rightarrow b^2 = 400 \Rightarrow \boxed{b = 20\text{cm}} \Rightarrow \boxed{h = 10 \text{ cm}}$

$\boxed{\text{Volume} = 4000\text{cm}^3}$

201 §3.7 #s 19, 35, 39

(19) Find pt on $y = 2x+3$ that's closest to the origin.

$D = \sqrt{(x-0)^2 + (2x+3-0)^2}$ we minimize D^2:

$y = x^2 + 4x^2 + 12x + 9 = 5x^2 + 12x + 9 \longrightarrow$

$y' = 10x + 12 \overset{\text{SET}}{=} 0 \longrightarrow 10x = -12$

$x = -\frac{12}{10} = -\frac{6}{5}$

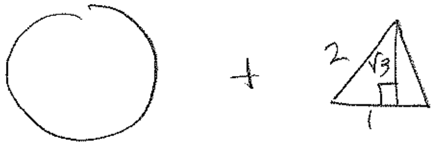
$\Longrightarrow y = 2\left(\frac{6}{5}\right) + 3$

$= -\frac{12}{5} + \frac{15}{5} = -\frac{3}{5} \leadsto \boxed{\left(\frac{6}{5}, -\frac{3}{5}\right)}$

(35) 10 m of wire. Cut it and make a circle and an equilateral triangle.

(a) Maximize Area enclosed

(b) Minimize Area enclosed.



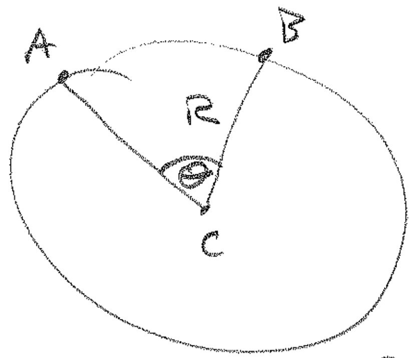
$x + y = 10$

x = circumference of circle

Then Heck, we did this in class.

201 §3.7 #39

(39)



Maximize capacity of drinking cup made by cutting a sector out of the paper disk.

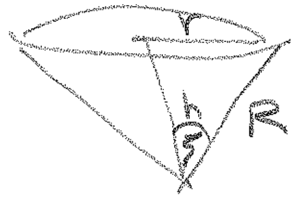
Circumference @ the top is $2\pi R - \pi\theta$

Radius @ the top is $r = \dfrac{2\pi R - \pi\theta}{2\pi} = R - \dfrac{\theta}{2}$

$V = \frac{1}{3}\pi r^2 h$

$\frac{h}{r} = \tan\xi$

$h = r\tan\xi$ ugh



$r^2 + r^2 = R^2 \longrightarrow$

$h^2 = R^2 - r^2 \longrightarrow$

$h = \sqrt{R^2 - r^2}$

FORGET θ

$\cancel{① \; V = \frac{1}{3}\pi \frac{\sqrt{2}}{\sqrt{3}}R \cdot \frac{1}{\sqrt{3}}R = \frac{\sqrt{2}}{9}R}$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2\sqrt{R^2 - r^2}$

$\Rightarrow \dfrac{dV}{dr} = \frac{2}{3}\pi r\sqrt{R^2 - r^2} + \left(\frac{1}{3}\pi r^2\right)\left(\frac{1}{2}(R^2 - r^2)^{-\frac{1}{2}}(-2r)\right)$

$= \dfrac{2\pi r\sqrt{R^2 - r^2}}{3} - \dfrac{\pi r^3}{3\sqrt{R^2 - r^2}}$

$= \dfrac{2\pi r(R^2 - r^2) - \pi r^3}{3\sqrt{R^2 - r^2}} = \dfrac{\pi r\left[2R^2 - 2r^2 - r^2\right]}{3\sqrt{R^2 - r^2}}$

SET $= 0 \Rightarrow 2R^2 - 3r^2 = 0 \longrightarrow r^2 = \dfrac{2R^2}{3} \Rightarrow \boxed{r = \dfrac{\sqrt{2}}{\sqrt{3}}R}$

$\Rightarrow h = \sqrt{R^2 - \frac{2}{3}R^2} = \sqrt{\frac{1}{3}R^2} = \boxed{\dfrac{\sqrt{3}}{3}R = h}$

201 §3.7 #39

39 Contd

$h = \frac{\sqrt{3}}{3} R$

$r = \frac{\sqrt{2}}{\sqrt{3}} R$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{\sqrt{2}}{\sqrt{3}}R\right)^2\left(\frac{\sqrt{3}}{3}R\right)$$

$$= \frac{1}{3}\pi\left(\frac{2}{3}R^2\right)\left(\frac{\sqrt{3}}{3}\right)R$$

$$= \boxed{\frac{2\sqrt{3}\,\pi}{27}R^3 = \text{Max Vol}}$$

201 §3.5 #s 4, 11, 26, 34

4

$y = x^4 - 8x^2 + 8 \overset{\text{SET}}{=} 0 \implies$

$u^2 - 8u + 8 = 0$

$u^2 - 8u + 4^2 = -8 + 16 = 8$

$(u-4)^2 = 8$

$u - 4 = \pm 2\sqrt{2}$

$u = 4 \pm 2\sqrt{2} = x^2$

$\mathcal{D} = \mathbb{R}$

<u>EVEN</u>

$x = \pm\sqrt{4 \pm 2\sqrt{2}}$ all zeros of f,

$\approx \pm 2.6131259\overline{3},$

± 1.0823922

$y' = 4x^3 - 16x$

$= 4x(x^2 - 4) = 4x(x-2)(x+2) \overset{\text{SET}}{=} 0 \implies$

$x \in \{0, \pm 2\}$

$y'' = 12x^2 - 16 = 4(3x^2 - 4) \overset{\text{SET}}{=} 0 \implies$

$x \in \{\pm \frac{2\sqrt{3}}{3}\}$

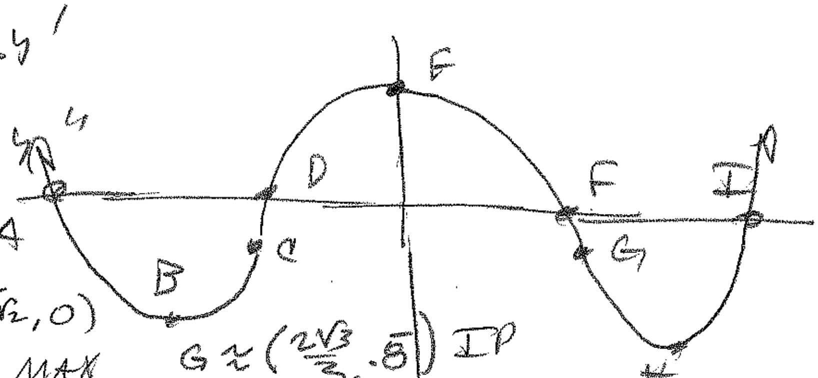
$3x^2 = 4$

$x^2 = \frac{4}{3}$

$x = \pm\frac{2}{\sqrt{3}} = \pm\frac{2\sqrt{3}}{3} \approx \pm 1.1547$

y': $-$ | -2 (MIN) | $+$ | 0 (MAX) | $-$ | 2 (MIN) | $+$

y'': $+$ | $-\frac{2\sqrt{3}}{3}$ | $-$ | $\frac{2\sqrt{3}}{3}$ | $+$



$A \approx (-\sqrt{4+2\sqrt{2}}, 0)$

$B \approx (-2, -8)$ MIN

$C \approx (-\frac{2\sqrt{3}}{3}, -.8\overline{8})$

$D = (-\sqrt{4-2\sqrt{2}}, 0)$

$E = (0, 8)$ MAX

$F = (\sqrt{4-2\sqrt{2}}, 0)$

$G \approx (\frac{2\sqrt{3}}{3}, .\overline{8})$ IP

$H = (2, -8)$ MIN

$I = (\sqrt{4+2\sqrt{2}}, 0)$

201 § 3.5 #s 11, 26, 34

(11) $y = \dfrac{-x^2+x}{x^2-3x+2} = -\dfrac{x^2-x}{x^2-3x+2} = -\dfrac{x(x-1)}{(x-2)(x-1)}$

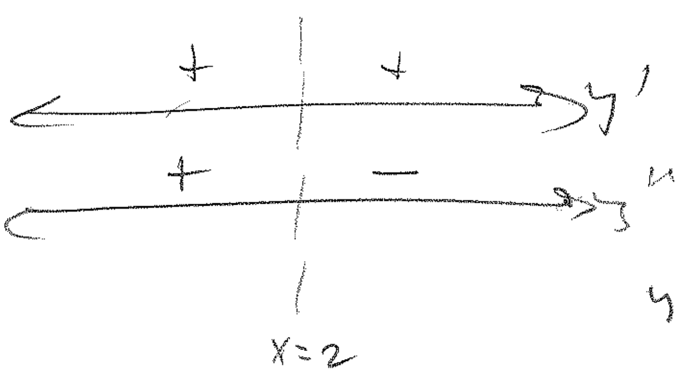
$\mathcal{D} = \mathbb{R} \setminus \{1,2\}$, $\boxed{\begin{array}{l} x=2 \text{ is V.A.} \\ (1,-1) \text{ is HOLE} \\ \text{H.A}: y=-1 \end{array}}$

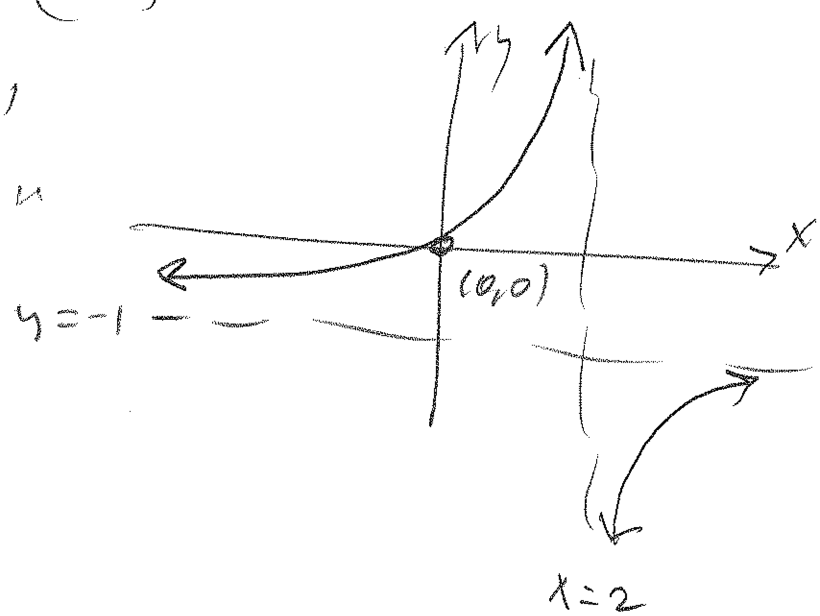
Now, except for the hole, $y = -\dfrac{x}{x-2}$

$$y' = -\left[\frac{1(x-2) - x(1)}{(x-2)^2}\right]$$

$$= -\left[\frac{x-2-x}{(x-2)^2}\right] = \frac{2}{(x-2)^2} = 2(x-2)^{-2}$$

$$y'' = -4(x-2)^{-3} = -\frac{4}{(x-2)^3}$$





201 §3.5 #s 26, 34

$\mathcal{D} = [-\sqrt{2}, \sqrt{2}]$

(26) $y = x\sqrt{2-x^2} \overset{SET}{=} 0 \Rightarrow x = 0, \pm\sqrt{2}$

$y' = \sqrt{2-x^2} + x\left(\frac{1}{\cancel{2}}(2-x^2)^{-\frac{1}{2}}(-\cancel{2}x)\right)$

$= \frac{2-x^2}{\sqrt{2-x^2}} - \frac{x^2}{\sqrt{2-x^2}} = \frac{2-2x^2}{\sqrt{2-x^2}} = -\frac{2(x^2-1)}{\sqrt{2-x^2}}$

$\overset{SET}{=} 0 \Rightarrow x = \pm 1, ~~\pm\sqrt{2}~~ \quad \overset{SET}{=} \nexists \Rightarrow x = \pm\sqrt{2}$

CVS: $x = \pm 1$

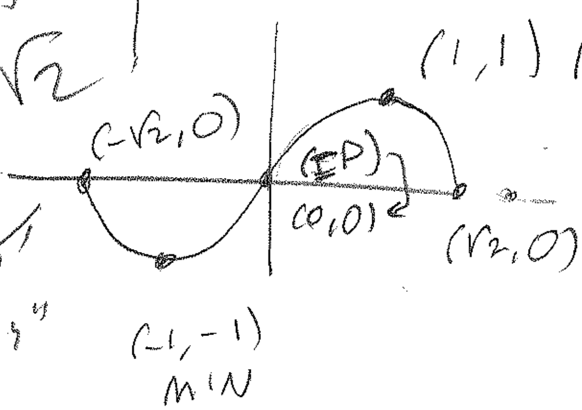
$y'' = -2\left[\frac{2x(2-x^2)^{\frac{1}{2}} - (x^2-1)\left(\frac{1}{2}(2-x^2)^{-\frac{1}{2}}(-2x)\right)}{2-x^2}\right]$

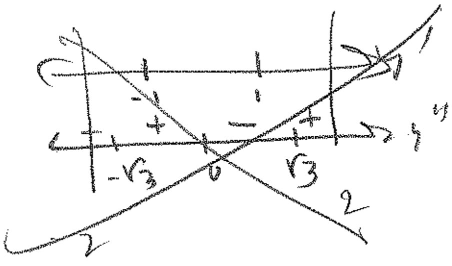
$= -2\left[\frac{\frac{2x(2-x^2)}{\sqrt{2-x^2}} + \frac{x(x^2-1)}{\sqrt{2-x^2}}}{2-x^2}\right]$

$= -2\left[\frac{x[4-2x^2+x^2-1]}{(2-x^2)^{3/2}}\right] = -2\left[\frac{x[-x^2+3]}{(2-x^2)^{3/2}}\right]$

$= \frac{2x[x^2-3]}{(2-x^2)^{3/2}} \overset{SET}{=} 0 \rightarrow x \in \{0, \pm\sqrt{3}\}$

$\sqrt{3} > \sqrt{2}$!





2 01 §3.5 #34

(34) $y = x + \cos x$ Hard to find zeros analytically

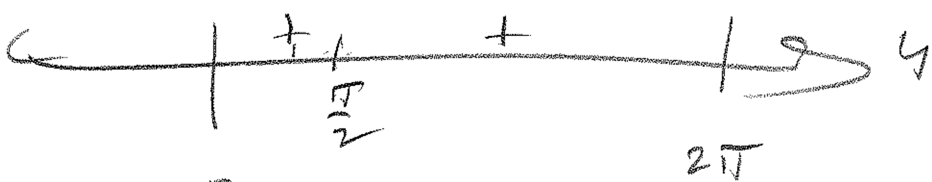
$y' = 1 - \sin x \overset{\text{SET}}{=} 0 \implies$

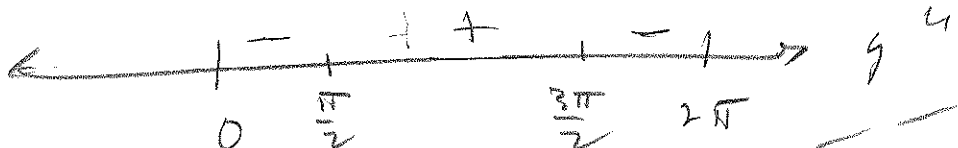
$x = \frac{\pi}{2} + 2n\pi, \; n \in \mathbb{Z}$

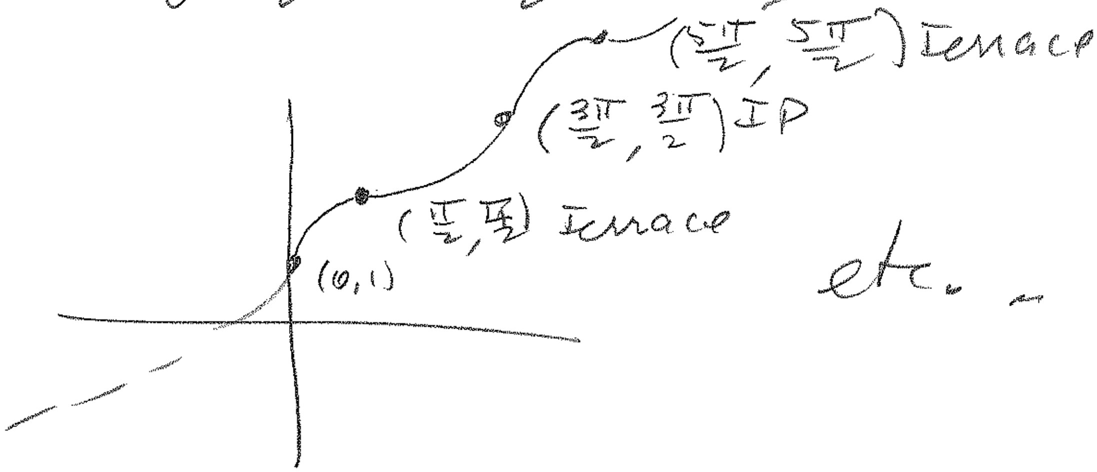
$y'' = -\cos x \overset{\text{SET}}{=} 0$

$x = \frac{(2n+1)\pi}{2}, \; n \in \mathbb{Z}.$

Note: $y' = 1 - \sin x \geq 0 \;\forall x$, so no extremes. Just terraces







etc. ..

201 §3.4 #s 9-35, 45, 47, 49, 51, 53, 55

#s 9-30 Find lim or ∄ lim.

(9) $\lim_{x\to\infty} \frac{3x-2}{2x+1} = \boxed{\frac{3}{2}}$

(11) $\lim_{x\to-\infty} \frac{x-2}{x^2+1} = \boxed{0}$

(13) $\lim_{t\to\infty} \frac{\sqrt{t}+t^2}{2t-t^2} = \boxed{-1}$ $\left(\frac{t^2}{-t^2} = -1 \xrightarrow{t\to\infty} -1 \right)$

(15) $\lim_{t\to\infty} \frac{(2x^2+1)^2}{(x-1)^2(x^2+x)} = \frac{4x^4+\cdots}{x^4+\cdots} \xrightarrow{x\to\infty} \boxed{4}$

(17) $\frac{\sqrt{9x^6-x}}{x^3+1} = \frac{3x^3\sqrt{1-\frac{1}{9x^5}}}{x^3\left(1+\frac{1}{x^3}\right)} \xrightarrow{x\to\infty} \boxed{3}$

(19) $\sqrt{9x^2+x} - 3x =$

$\left(\sqrt{9x^2+x} - 3x\right)\left(\frac{\sqrt{9x^2+x}+3x}{\sqrt{9x^2+x}+3x}\right) = \frac{9x^2+x-9x^2}{\sqrt{9x^2+x}+3x}$

$= \frac{x}{3x\sqrt{1+\frac{1}{9x}}+3x} \xrightarrow{x\to\infty} \boxed{\frac{1}{3}}$

(21)

201 §3.4 #s 21-35, 45, 47, 49, 51, 53, 55

(21) $\sqrt{x^2+ax} - \sqrt{x^2+bx}$

$$= \left(\sqrt{x^2+ax} - \sqrt{x^2+bx}\right)\left(\frac{\sqrt{x^2+ax} + \sqrt{x^2+bx}}{\sqrt{x^2+ax} + \sqrt{x^2+bx}}\right)$$

$$= \frac{x^2+ax-(x^2+bx)}{\sqrt{x^2+ax} + \sqrt{x^2+bx}} = \frac{ax-bx}{|x|\left(\sqrt{1+\frac{a}{x}} + \sqrt{1+\frac{b}{x}}\right)}$$

$$= \frac{x(a-b)}{x\left(\sqrt{1+\frac{a}{x}} + \sqrt{1+\frac{b}{x}}\right)} \xrightarrow{x \to \infty} \boxed{a-b}$$

(23) $\frac{x^4-3x^2+x}{x^3-x+2} \xrightarrow{x \to \infty} \infty$, i.e., $\nexists$.

(25) $(x^4+x^5) \xrightarrow{x \to -\infty} -\infty$, i.e., $\nexists$.

(27) $(x-\sqrt{x}) \xrightarrow{x \to \infty} \infty$, i.e., $\nexists$.

(29) $\left(x \sin\left(\frac{1}{x}\right)\right) \xrightarrow{x \to \infty} ?$

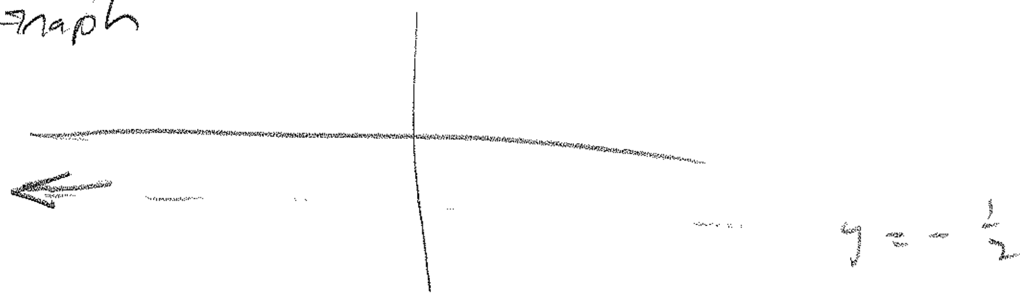
$$\frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = \frac{\sin u}{u} \xrightarrow{u \to 0} \boxed{1}$$

$u = \frac{1}{x}$

201 §3.4 #s 31-35, 45, 47, 49, 51, 53, 55

31 Estimate $\lim_{x \to -\infty}\left(\sqrt{x^2+x+1} + x\right)$ by

(a) Graph



(b) Numerically $(-1000, -.4996)$
$(-10^6, -.5)$ (Calculator)

(c) Analytically.

$$\sqrt{x^2+x+1} + x = \left(\sqrt{x^2+x+1} + x\right)\left(\frac{\sqrt{x^2+x+1} - x}{\sqrt{x^2+x+1} - x}\right)$$

$$= \frac{x^2+x+1-x^2}{\sqrt{x^2+x+1} - x} = \frac{x+1}{|x|\sqrt{1+\frac{1}{x}+\frac{1}{x^2}} - x}$$

$$= \frac{x+1}{-x\sqrt{1+\frac{1}{x}+\frac{1}{x^2}} - x} = \frac{x+1}{-x\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}} + 1\right)}$$

$$= \frac{x\left(1+\frac{1}{x}\right)}{-x\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}} + 1\right)} \xrightarrow{x \to \infty} \boxed{-\frac{1}{2}}$$

201 §3.4 #s 33, 38, 45, 47, 49, 51, 53, 55

#s 33–38 Find H.A., V.A. Check w/ graph

33

$y = \frac{2x+1}{x-2}$

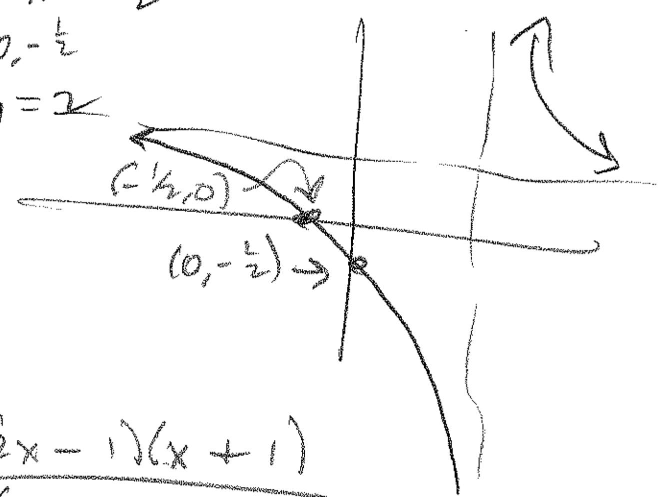
x-int: $x = -\frac{1}{2}$

y-int: $(0, -\frac{1}{2})$

$y = 2$

V.A.: $x = 2$

H.A.: $y = 2$



38

$y = \frac{2x^2+x-1}{x^2+x-2} = \frac{(2x-1)(x+1)}{(x-1)(x+2)}$

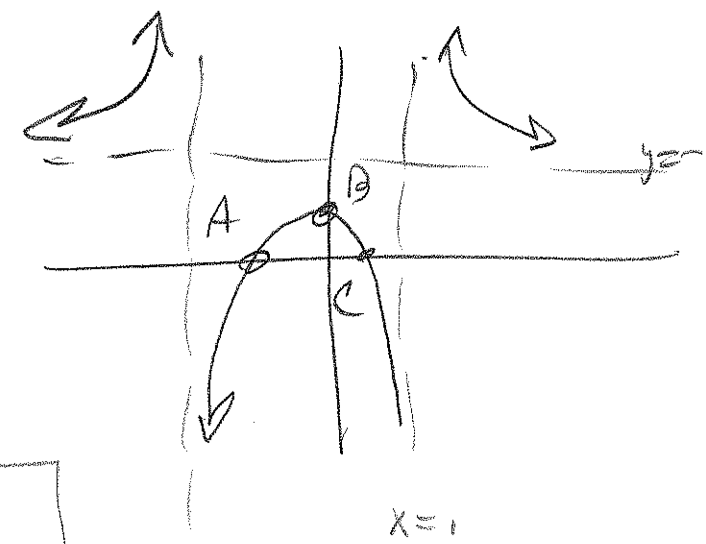
V.A.: $x = 1, x = -2$

H.A.: $y = 2$

x-int: $\boxed{(\frac{1}{2}, 0)}$ C $\boxed{(-1, 0)}$ A

y-int: $\boxed{(0, \frac{1}{2})}$ B

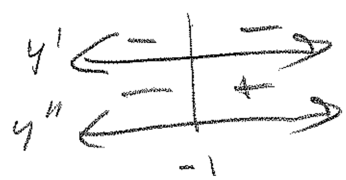
A

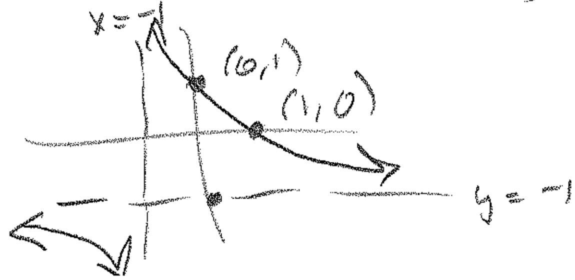


45 Use H.A. & Calc. to sketch $\boxed{HA: y = -1}$

$y = \frac{1-x}{1+x}$ $y' = \frac{-1(x+1) - (1-x)}{(1+x)^2} = \frac{-x-1-1+x}{(x+1)^2}$

$= -\frac{2}{(x+1)^2} \Longrightarrow y'' = -2\left(-2(x+1)^{-3}\right) = \frac{4}{(x+1)^3}$





201 §2.4 #s 47, 49, 51, 53, 55

(47) $y = \dfrac{x}{x^2+1}$ No V.A. $(0,0)$,

H.A.: $y=0$

$$y' = \frac{1(x^2+1) - x(2x)}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = -\frac{x^2-1}{(x^2+1)^2}$$

$$y'' = -\left[\frac{2x(x^2+1)^2 - (x^2-1)(2(x^2+1)(2x))}{(x^2+1)^4}\right]$$

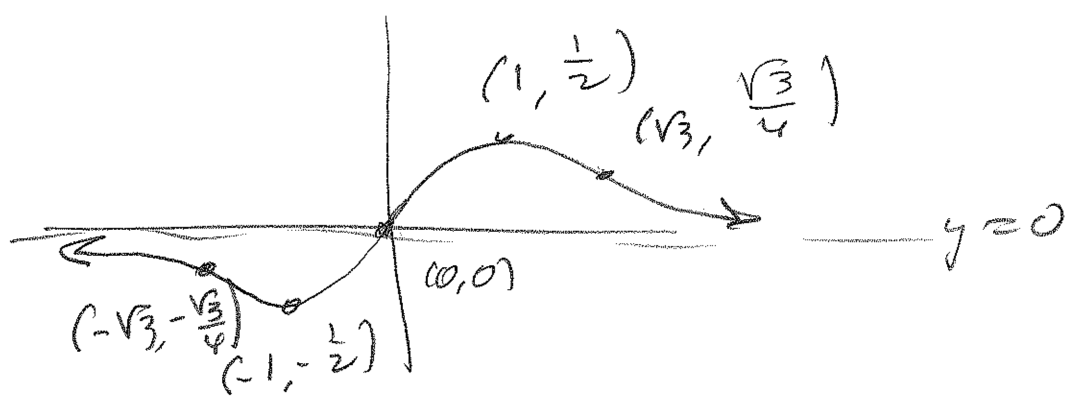
$$= -\left[\frac{2x(x^2+1)[x^2+1-2(x^2-1)]}{(x^2+1)^4}\right]$$

$$= -\left[\frac{2x[-2x^2+2+x^2+1]}{(x^2+1)^3}\right]$$

$$= -\left[\frac{2x[-x^2+3]}{(x^2+1)^3}\right] = \frac{2x(x^2-3)}{(x^2+1)^3}$$

f': $-$ | -1 | $+$ | 1 | $-$

f'': $-\sqrt{3}$ | $+$ | 0 | $-$ | $\sqrt{3}$ | $+$



201 §3.4 #s 49, 51, 53, 55

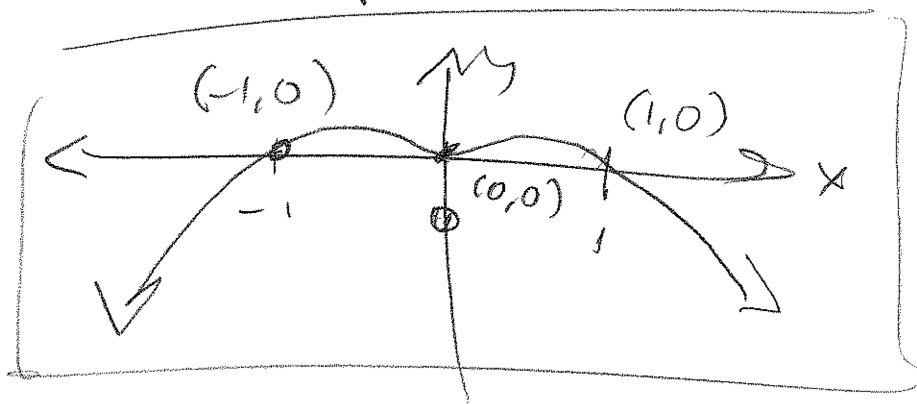
~~#~~s 48–52 E.B. & x-ints & y-ints for rough sketch.

49 $y = x^4 - x^6$

E.B

$-x^4(x^2-1) = 0$

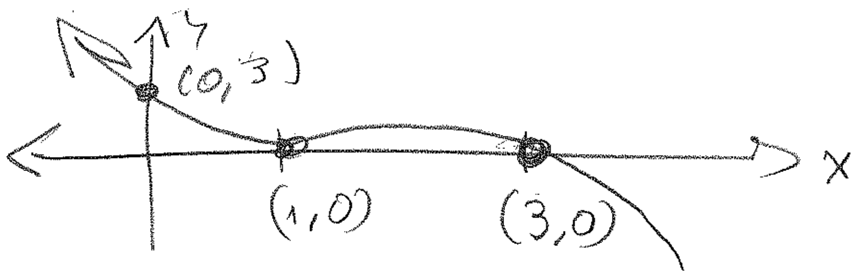
$x = 0, \pm 1$



51 $y = (3-x)(x+1)^2(1-x)^4$

$= -(x-3)(x^2+1)^2(x-1)^4$

E.B.: $-x \cdot x^4 \cdot x^4 = -x^5$



201 § 3.4 #s 53, 58

53 $f'(2)=0$, $f(2)=-1$, $f(0)=0$

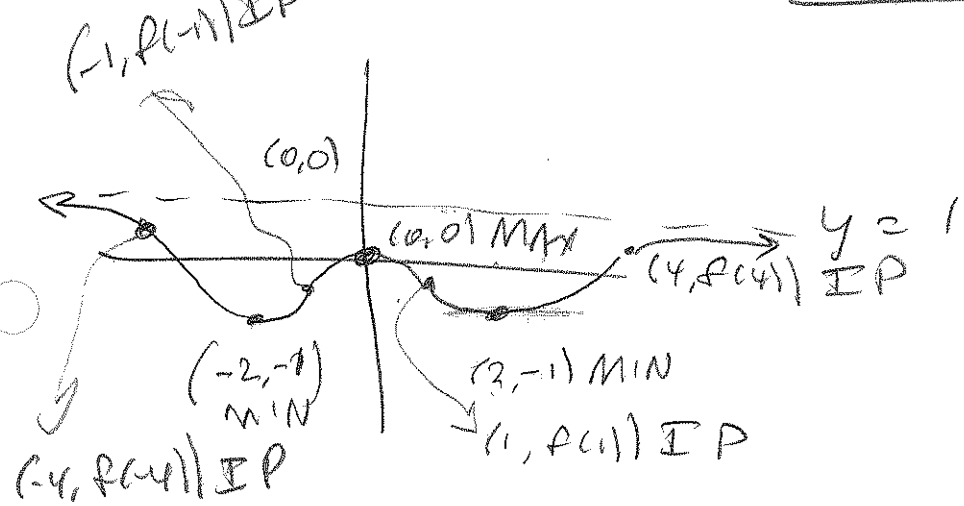
$f'(x)<0$ if $0<x<2$

$f'(x)>0$ if $x>2$

$f''(x)<0$ if $0\le x<1$ OR $x>4$

$f''(x)>0$ if $1<x<4$

$\lim_{x\to\infty} f(x)=1$, $f(-x)=f(x)\ \forall x$ EVEN



f': $-$ on $(-\infty,-2)$, $+$ on $(-2,0)$, $-$ on $(0,2)$, $+$ on $(2,\infty)$

f'': $-$ on $(-\infty,-4)$, $+$ on $(-4,-1)$, $-$ on $(-1,0)$, $-$ on $(0,1)$, $+$ on $(1,4)$, $-$ on $(4,\infty)$

201 §3.4 #58

58 $f(1)=f'(1)=0$ $\qquad \lim_{x\to 2^+} f(x)=+\infty$

$\lim_{x\to 0} f(x)=-\infty$ $\qquad \lim_{x\to 2^-} f(x)=-\infty$

$\lim_{x\to -\infty} f(x)=\infty$ $\qquad f''(x)>0 \ \forall\, x>2,$

$\lim_{x\to \infty} f(x)=0$ $\qquad f''(x)<0 \ \forall\, x<0$ and $\forall\, 0<x<2$