201 §3.7 #s 3, 5, 7, 9, 15, 19, 35, 39

(3) 2 positive #s whose product is 100 & whose sum is a minimum

$xy = 100$

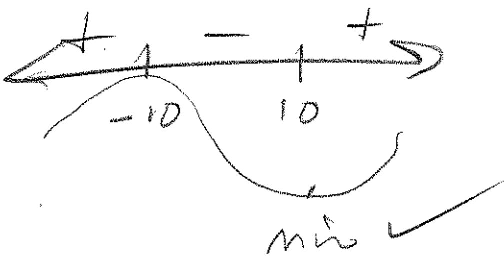
$P = x + y$ to be minimized

$y = \frac{100}{x} = 100x^{-1} \Longrightarrow$

$P = x + 100x^{-1} \Longrightarrow$

$P' = 1 - 100x^{-2} = 1 - \frac{100}{x^2} = \frac{x^2 - 100}{x^2} \overset{\text{SET}}{=} 0$

$\Longrightarrow x = \pm 10 \Longrightarrow \boxed{x = 10} \Longrightarrow \boxed{y = \frac{100}{10} = 10}$

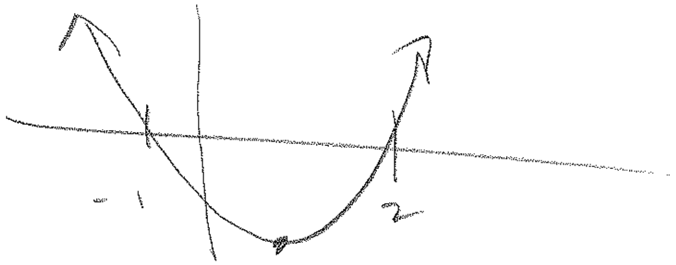


(5) Max Vertical Distance between $y = x + 2$ and $y = x^2$ $\forall$ $-1 \le x \le 2$.

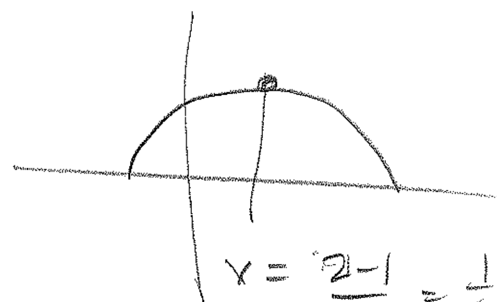
201 §3.7 #s 5, 7, 9, 15, 19, 38, 39

5 $|x^2 - (x+2)| = |x^2 - x - 2|$ MAXIMIZE IT. on $[-1, 2]$

$= |x-2||x+1|$



$x = \frac{1}{2}$ should do it.



$x = \frac{2-1}{2} = \frac{1}{2}$

$y' = 2x - 1$ wait. Since $|x^2 - x - 2| =$

$-(x^2 - x - 2)$ on $[-1, 2]$, use

$y = -x^2 + x + 2 \Longrightarrow$

$y' = -2x + 1$ SET 0 $\Longrightarrow \boxed{x = \frac{1}{2}}$

Max Distance is

$-(\frac{1}{2})^2 + \frac{1}{2} + 2 = -\frac{1}{4} + \frac{2}{4} + \frac{8}{4} = \boxed{\frac{9}{4} = \text{4}}$

201 §3.7 #s 7, 9, 15, 19, 38, 39

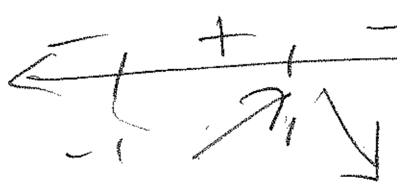
(9) Yield $= Y = \dfrac{kN}{1+N^2}$, where k = konstant, and N = nitrogen in soil ("appropriate")

Maximize the yield.

$$Y' = \frac{k(N^2+1) - kN(2N)}{(N^2+1)^2}$$

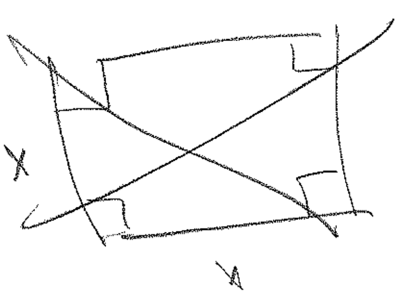
$$= \frac{kN^2 + k - 2kN^2}{(\quad)^2} = \frac{-kN^2 + k}{(\quad)^2}$$

$$= \frac{-k(N^2-1)}{(\quad)^2} \Rightarrow N = \pm 1 \Rightarrow \boxed{N = 1}$$



Yes. MAX ✓

(15) 1200 cm^2 of material for square box w/ open top. Maximize the vol.



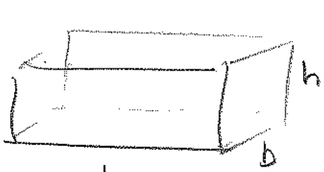
All we know is surface area

$= b^2 + 4bh = 1200 \Rightarrow 4bh = 1200 - b^2$, $h = \dfrac{1200 - b^2}{4b}$

Volume $= V = b^2 h$

$$= b^2\left[\frac{1200 - b^2}{4b}\right]$$

$$= \frac{1200b - b^3}{4}$$



$$\Rightarrow \frac{dV}{db} = 300 - \frac{3b^2}{4} \overset{\text{SET}}{=} 0 \Rightarrow$$

$$\frac{3b^2}{4} = 300 \Rightarrow b^2 = 400 \Rightarrow \boxed{b = 20\text{cm}} \Rightarrow \boxed{h = 10 \text{ cm}}$$

$\boxed{\text{Volume} = 4000 cm^3}$

201 §3.7 #s 19, 35, 39

(19) Find pt on $y=2x+3$ that's closest to the origin.

$D = \sqrt{(x-0)^2 + (2x+3-0)^2}$ we minimize D^2:

$y = x^2 + 4x^2 + 12x + 9 = 5x^2 + 12x + 9 \longrightarrow$

$y' = 10x + 12 \overset{\text{SET}}{=} 0 \longrightarrow 10x = -12$

$x = -\frac{12}{10} = -\frac{6}{5}$

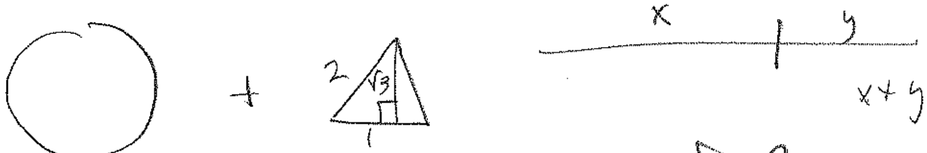
$\Longrightarrow y = 2\left(\frac{6}{5}\right) + 3$

$= \frac{-12}{5} + \frac{15}{5} = \frac{-3}{5} \rightsquigarrow \boxed{\left(\frac{6}{5}, \frac{-3}{5}\right)}$

(35) 10 m of wire. Cut it and make a circle and an equilateral triangle.

(a) Maximize Area enclosed

(b) Minimize Area enclosed.



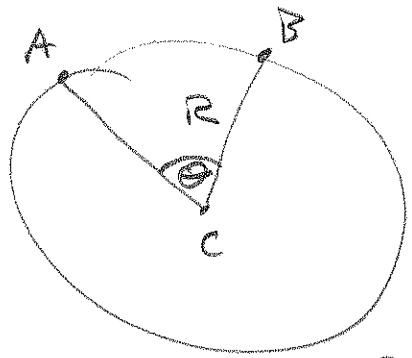
$x + y = 10$

x = circumference of circle

Then Heck, we did this in class.

201 §3.7 #39

(39)



Maximize capacity of drinking cup made by cutting a sector out of the paper disk.

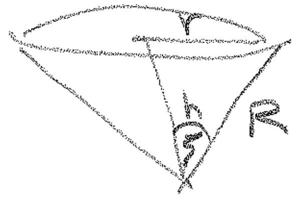
Circumference @ the top is $2\pi R - \pi\theta$

Radius @ the top is $r = \dfrac{2\pi R - \pi\theta}{2\pi} = R - \dfrac{\theta}{2}$

$V = \frac{1}{3}\pi r^2 h$

$\frac{h}{r} = \tan\xi$

$h = r\tan\xi$ ugh



$h^2 + r^2 = R^2 \longrightarrow$

$h^2 = R^2 - r^2 \longrightarrow$

$h = \sqrt{R^2 - r^2}$

FORGET θ

$\cancel{\textcircled{1}\ V = \frac{1}{3}\pi \frac{\sqrt{2}}{\sqrt{3}}R \cdot \frac{1}{\sqrt{3}}R = \frac{\sqrt{2}}{9}R}$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2 \sqrt{R^2 - r^2}$

$\Rightarrow \dfrac{dV}{dr} = \frac{2}{3}\pi r\sqrt{R^2 - r^2} + \left(\frac{1}{3}\pi r^2\right)\left(\frac{1}{2}(R^2 - r^2)^{-\frac{1}{2}}(-2r)\right)$

$= \dfrac{2\pi r\sqrt{R^2 - r^2}}{3} - \dfrac{\pi r^3}{3\sqrt{R^2 - r^2}}$

$= \dfrac{2\pi r(R^2 - r^2) - \pi r^3}{3\sqrt{R^2 - r^2}} = \dfrac{\pi r\left[2R^2 - 2r^2 - r^2\right]}{3\sqrt{R^2 - r^2}}$

SET $= 0 \Rightarrow 2R^2 - 3r^2 = 0 \longrightarrow r^2 = \dfrac{2R^2}{3} \Rightarrow \boxed{r = \dfrac{\sqrt{2}}{\sqrt{3}}R}$

$\Longrightarrow h = \sqrt{R^2 - \frac{2}{3}R^2} = \sqrt{\frac{1}{3}R^2} \boxed{= \frac{\sqrt{3}}{3}R = h}$

201 §3.7 #39

39 Contd

$h = \frac{\sqrt{3}}{3} R$

$r = \frac{\sqrt{2}}{\sqrt{3}} R$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{\sqrt{2}}{\sqrt{3}} R\right)^2\left(\frac{\sqrt{3}}{3} R\right)$$

$$= \frac{1}{3}\pi\left(\frac{2}{3} R^2\right)\left(\frac{\sqrt{3}}{3}\right) R$$

$$= \boxed{\frac{2\sqrt{3}\,\pi}{27} R^3 = \text{Max Vol}}$$