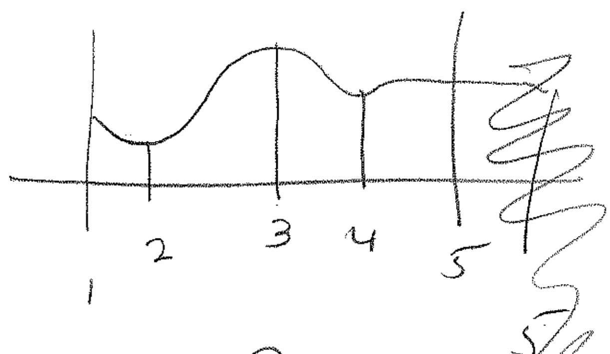
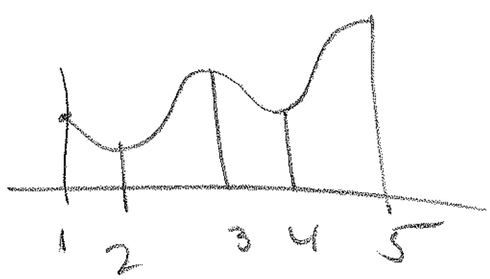
201 §3.1 #s 5, 6, 7, 9, 15 - 31, 35, 41, 47, 53

(5) No abs. min   Local min: (1, 3)
Abs max: (4, 5)   (2, 2), (5, 3)
Local max:
(4, 5), (6, 4)

(7) cnts on [1, 5],
Abs min @ 2, Abs max @ 3, Local min
at 4



(9) Abs max @ 5, abs min @ 2
loc max @ 3, local min @ 2, 4



201 §3.1 #s 15-31, 35, 41, 47, 53

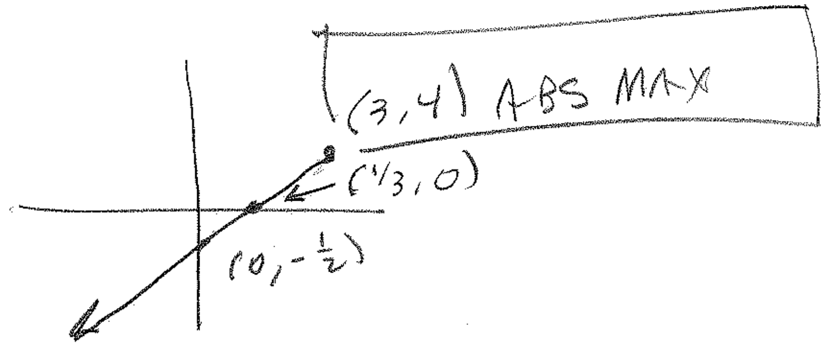
#s 15 - 28 sketch & use it to find

Abs & local extremes

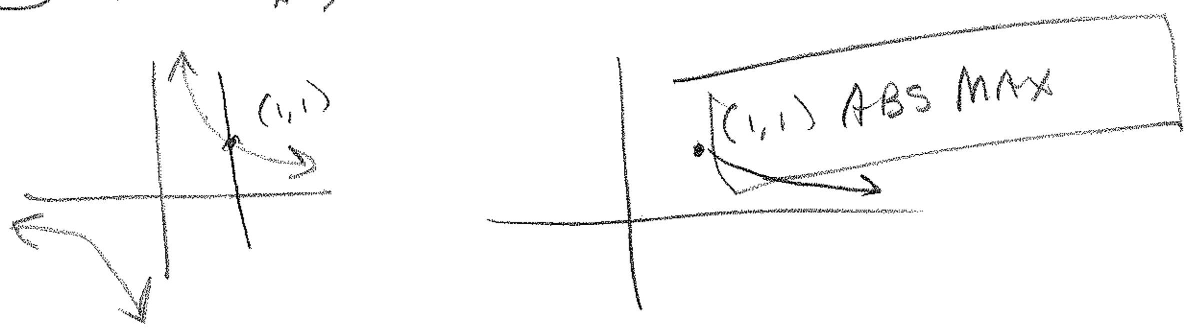
15 $f(x) = \frac{1}{2}(3x - 1)$ $x \le 3$

$f(3) = \frac{1}{2}(9-1) = \frac{1}{2}(8) = 4$

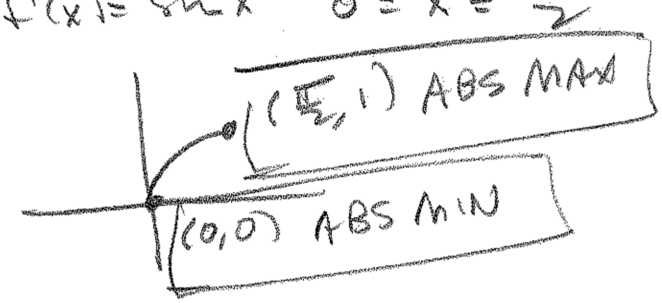
$f(0) = -\frac{1}{2}$



17 $f(x) = \frac{1}{x}$, $x \ge 1$

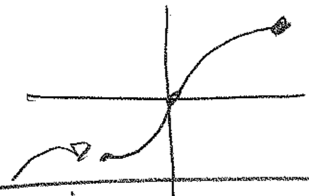


19 $f(x) = \sin x$ $0 \le x \le \frac{\pi}{2}$



201 §3.1 #s 21-31, 35, 41, 47, 53

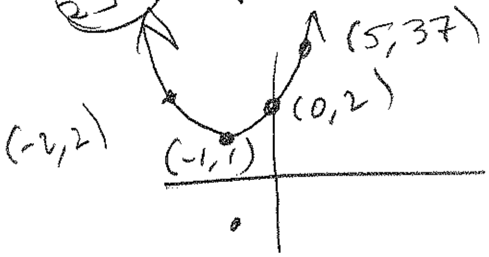
21 $f(x) = \sin x$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

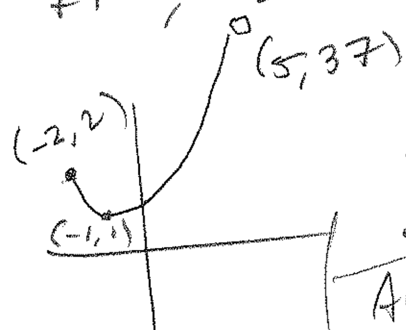


$\boxed{(\frac{\pi}{2}, 1) \text{ ABS MAX}}$

$\boxed{(-\frac{\pi}{2}, -1) \text{ ABS MIN}}$

23 $f(x) = (x+1)^2 + 1$; $-2 \le x < 5$



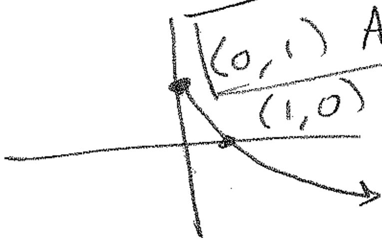


No ABS MAX

ABS MIN: (-1, 1)
& LOCAL MIN

25 $f(x) = -\sqrt{x} + 1$

$\boxed{(0, 1) \text{ ABS MAX}}$

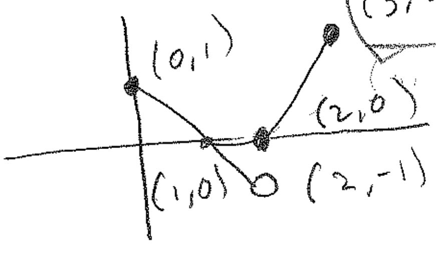


27 $f(x) = \begin{cases} 1-x & \text{if } 0 \le x < 2 \\ 2x-4 & \text{if } 2 \le x \le 3 \end{cases}$

$1-2=-1$
$2(2)-4=0$
$2(3)-4=2$

$\boxed{(3, 2) \text{ ABS MAX}}$

No local / ABS min



201 §3.1 #s 29, 31, 35, 41, 47, 43

#s 29-42 Find critical ~~#s~~

29 $f(x) = -\frac{1}{2}x^2 + \frac{1}{3}x + 4$

$f'(x) = -x + \frac{1}{3} \overset{\text{SET}}{=} 0 \longrightarrow \boxed{x = \frac{1}{3}}$

31 $f(x) = 2x^3 - 3x^2 - 36x$

$f'(x) = 6x^2 - 6x - 36 \overset{\text{SET}}{=} 0$

$\Longrightarrow x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$\Longrightarrow \boxed{x = -2, 3}$

35 $g(y) = \dfrac{y-1}{y^2 - y + 1}$

$y - 1 = 0$

$\boxed{y = 1}$

x-int

$y^2 - y + 1 = 0$

$y^2 - y + \left(\frac{1}{2}\right)^2 - \frac{1}{4} + 1$

$= \left(y - \frac{1}{2}\right)^2 + \frac{3}{4} \overset{\text{SET}}{=} 0$

$\longrightarrow$ Never, for $y \in \mathbb{R}$

$g'(y) = \dfrac{1(y^2 - y + 1) - (y-1)(2y-1)}{(y^2 - y + 1)^2} = \dfrac{y^2 - y + 1 - (2y^2 - 3y + 1)}{(\;)^2}$

$= \dfrac{y^2 - y + 1 - 2y^2 + 3y - 1}{(\;)^2} = \dfrac{-y^2 + 2y}{(\;)^2} \overset{\text{SET}}{=} 0$

$\Longrightarrow y^2 - 2y = y(y-2) = 0 \longrightarrow \boxed{y = 0, 2 \text{ critical}}$ #s.

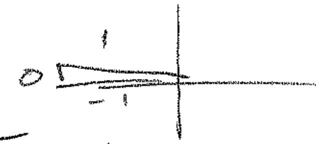
201 §3.1 #s 41, 47, 53

41 $f(\theta) = 2\cos\theta + \sin^2\theta$

$\Rightarrow f'(\theta) = -2\sin\theta - 2\sin\theta\cos\theta$

$= -2\sin\theta(1+\cos\theta)$

$\cos\theta = -1$



$\theta = \pm\pi, \pm 3\pi, \ldots$

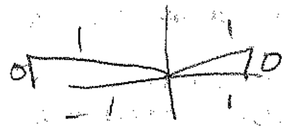
$\theta = (2n+1)\pi, n \in \mathbb{Z}$

$\sin\theta = 0$

$\theta = \pm\pi, \pm 2\pi, \ldots$

$n\pi, n \in \mathbb{Z}$

$\boxed{\text{C.V.s} : \{ n\pi \mid n \in \mathbb{Z} \}}$



#s 45–56 Find abs max, min on [a, b]

47 $f(x) = 2x^3 - 3x^2 - 12x + 1$ on $[-2, 3]$

$f'(x) = 6x^2 - 6x - 12 \overset{\text{SET}}{=} 0$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$

$\Rightarrow \boxed{x = -1, 2}$

$$\begin{array}{r|rrrr} -1 & 2 & -3 & -12 & 1 \\ & & -2 & 5 & 7 \\ \hline & 2 & -5 & -7 & \boxed{8 = f(-1)} \end{array}$$

ABS MAX

$$\begin{array}{r|rrrr} -2 & 2 & -3 & -12 & 1 \\ & & -4 & 14 & -4 \\ \hline & 2 & -7 & 2 & \boxed{-3 = f(-2)} \end{array}$$

$$\begin{array}{r|rrrr} 2 & 2 & -3 & -12 & 1 \\ & & 4 & 2 & -20 \\ \hline & 2 & 1 & -10 & \boxed{-19 = f(2)} \end{array}$$

ABS MIN

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -12 & 1 \\ & & 6 & 9 & -9 \\ \hline & 2 & 3 & -3 & \boxed{-8 = f(3)} \end{array}$$

201 § 3.1 #53

53 $f(t) = t\sqrt{4-t^2}$ $[-1, 2]$

$= t(4-t^2)^{\frac{1}{2}}$

$\Rightarrow f'(t) = \sqrt{4-t^2} + t\left(\frac{1}{2}(4-t^2)^{-\frac{1}{2}}\right)(-2t)$

$= \sqrt{4-t^2} - \frac{t^2}{\sqrt{4-t^2}} = \frac{4-t^2-t^2}{\sqrt{4-t^2}} = \frac{4-2t^2}{\sqrt{4-t^2}}$

$4-2t^2 = 0$ $\sqrt{4-t^2} = 0$

$2t^2 = 4$ $4-t^2 = 0$

$t^2 = 2$ $t = \pm 2$

$t = \pm\sqrt{2} \notin$ $t = +2 \in [-1, 2]$

$t = +\sqrt{2} \in [-1, 2]$

$f(-1) = -1\sqrt{4-(-1)^2} = -1\sqrt{3} = -\sqrt{3}$ $\boxed{\overset{\text{MIN}}{(-1, -\sqrt{3})}}$

$f(\sqrt{2}) = \sqrt{2}\sqrt{4-\sqrt{2}^2} = \sqrt{2}\sqrt{4-2} = \sqrt{2}\sqrt{2} = 2$ $\boxed{\underset{\text{MAX}}{(\sqrt{2}, 2)}}$

$f(2) = 2\sqrt{4-2^2} = 2\sqrt{0} = 0$ $(2, 0)$

201 §3.2 #s 1-5, 9-11, 15, 17, 19, 25, 27

#s 1-4 Rolle's & Verify Hypotheses & find c(s).

(1) $f(x) = 3x^2 - 12x + 5$ on $[1, 3]$

f is polynomial $\Longrightarrow$ cnt & difbl $\forall x \in \mathbb{R}$ ✓

$f(1) = 3 - 12 + 5 = -4$ ✓

$f(3) = 27 - 36 + 5 = -4$ ✓

$f'(x) = 6x - 12 \overset{SET}{=} 0 \Longrightarrow$

$\boxed{x = 2 = c}$

(3) $f(x) = \sqrt{x} - \frac{1}{3}x$ on $[0, 9]$

f is $\sqrt{x} - \frac{1}{3}x$ is cnts on $[0, \infty)$ ✓

and difbl on $(0, \infty)$ ✓

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{3} = \frac{1}{2\sqrt{x}} - \frac{1}{3} \overset{SET}{=} 0 \Longrightarrow$

$\frac{3 - 2\sqrt{x}}{6\sqrt{x}} = 0 \Longrightarrow 2\sqrt{x} = 3 \Longrightarrow$

$\sqrt{x} = \frac{3}{2} \Longrightarrow$

$\boxed{x = \frac{9}{4} = c}$

201 §3.2 #s 5, 9–11, 13, ...

(5) $f(x) = 1 - x^{2/3}$. Then $f(1) = f(-1) = 0$, BUT

$$f'(x) = -\frac{2}{3}x^{-\frac{5}{3}} = -\frac{2}{3x^{5/3}} \neq 0 \quad \forall x.$$

This doesn't contradict Rolle's, b/c $f(x)$ isn't dif'b'l @ $x = 0 \in (-1, 1)$

#s 9–12 Verify MVT Hypo. Find c(s).

(9) $f(x) = 2x^2 - 3x + 1$; $[0, 2]$

f is polynomial $\longrightarrow$ cont's & dif'b'l $\forall x \in \mathbb{R}$.

$$f'(x) = 4x - 3 \underset{=}{\text{SET}} \frac{f(2) - f(0)}{2 - 0}$$

$$4x - 3 = \frac{8 - 3(2) + 1 - [1]}{2}$$

$$4x - 3 = \frac{2}{2} = 1$$

$$4x = 4$$

$$\boxed{x = 1 = c}$$

201 § 3.2 #s 11, 15, 17, 19, 25, 27 ...

11 $f(x) = \sqrt[3]{x}$, $[0,1]$

$= x^{\frac{1}{3}}$

cnt^s on $\mathbb{R}$ ✓ f co

difb^l on $(-\infty, 0) \cup (0, \infty)$ ✓

$f'(x) = \frac{1}{3} x^{-\frac{2}{3}} \overset{\text{SET}}{=} \frac{f(1) - f(0)}{1-0}$

$\frac{1}{3x^{2/3}} = \frac{1-0}{1} = 1$

$1 = 3x^{2/3}$

$x^{2/3} = \frac{1}{3}$

$x = \left(\frac{1}{3}\right)^{3/2} = \frac{1}{3} \cdot \left(\frac{1}{3}\right)^{\frac{1}{2}} = \frac{1}{3} \cdot \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{9} = c}$

15 $f(x) = (x-3)^{-2}$. Show $\not\exists$ $c \in (1,4) \ni$

$f(4) - f(1) = f'(c)(4-1)$ (i.e, $\frac{f(4) - f(1)}{4-1} = f'(c)$)

$f'(x) = -2(x-3)^{-3} = -\frac{2}{(x-3)^3}$

$f(4) - f(1) = (4-3)^{-2} - (1-3)^{-2} = 1^{-2} - (-2)^{-2}$

$= 1 - \frac{1}{4} = \frac{3}{4} \overset{\text{SET}}{=} -\frac{2}{(x-3)^3}(4-1) = -\frac{2 \cdot 3}{(x-3)^3}$

$\frac{3}{4} = -\frac{6}{(x-3)^3}$

$(x-3)^3 = -8$

$x - 3 = \sqrt[3]{-8} = -2$

$x = 3 - 2 = \boxed{1 = x = c}$

But $c \notin (1,4)$
No contradiction, b/c
f(x) not cnt^s @ $x = 3 \in [1,4]$

201 §3.2 #s 17, 19, 25, 27

(17) Show that $2x + \cos x = 0$ has exactly one root.

$2(10) + \cos(10) > 0$
$2(-10) + \cos(10) < 0$ $\Longrightarrow$ At <u>least</u> one. (IVT)

$f'(x) = 2 - \sin x > 0 \ \forall x \Longrightarrow$ At <u>Most</u> one

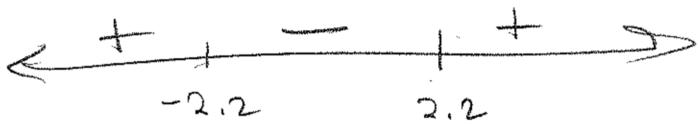
$f(x) = 2x + \cos x$ is strictly increasing $\forall x$.

(19) $x^3 - 15x + c = 0$ has at MOST one root. in $[-2,2]$

$f'(x) = 3x^2 - 15 \overset{\text{set}}{=} 0$

$x^2 = 5$

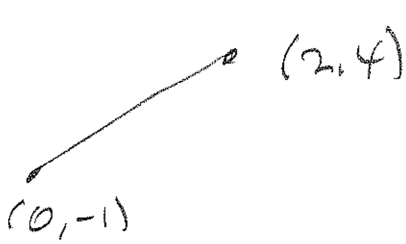
$x = \pm\sqrt{5} \approx \pm 2.236067977$



$f(x) = x^3 - 15x + c$ is <u>strictly</u> <u>decreasing</u> $\forall x \in [-2,2]$

Can't cross x-axis twice between $\pm\sqrt{5}$.

(25) Does $\exists f \ni f(0) = -1$, $f(2) = 4$ & $f'(x) \leq 2 \ \forall x$?



$\dfrac{4-(-1)}{2-0} = \dfrac{5}{2} = 2.5$

No way f can have avg slope of $m = 2.5$ if its slope is everywhere less than 2!

Violates MVT.

201 §3.2 #27

27) Show that $\sqrt{x+1} < 1 + \frac{1}{2}x$ if $x > 0$

@ $x=0$, we have $\sqrt{1} = 1$ on LHS

and 1 on RHS.

We show $f(x) = \sqrt{x+1} - \left(1 + \frac{1}{2}x\right) < 0$ if $x > 0$.

Note $f(0) = 0$.

$$f'(x) = \frac{1}{2}(x+1)^{-\frac{1}{2}} - \frac{1}{2}$$

$$= \frac{1}{2\sqrt{x+1}} + \frac{1}{2} < 0 \text{ if } x > 0$$

So it's decreasing, strictly.

Basically, $\sqrt{x+1}$ grows slower than $\frac{1}{2}x + 1$

CLOSE to $x=0$, it's a little harder to ~~be~~ sure. They're equal at $x=0$, but as soon as you move to the right, $\sqrt{x+1}$ is ducking underneath.

201 §3.3 #s 3, 5, 8, 9, 11, 13, 15, 17.

(3) f has a formula

(a) f is inc/dec where $f' > 0$ / $f' < 0$, respectively.

(b) f is concave up where $f'' > 0$

.. .. concave down .. $f'' < 0$

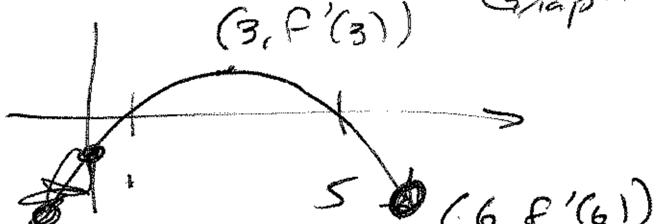
(c) IPs found by $f'' = 0$ or $f'' \not\exists$

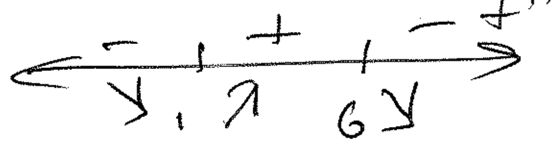
(5) f' is shown

(a) where is f inc? dec?

(b) where are the local extremes?

Graph of f'





This says

(a) f inc. on [1, 5]

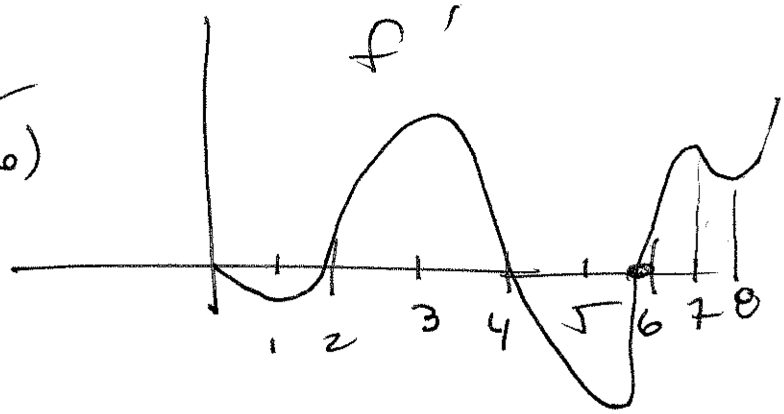
f dec. on [0, 1] ∪ [5, 6]

(b) Local max @ (6, f(6))

.. min @ (1, f(1))

201 §3.3 I #s 8, 9, 11, 13, 15, 17

(8) f' is shown



(a) ~~f inc on [2,4] ∪ [5, ∞) b/c f' ≥ 0~~

(b)

(a) Textbook is inconsistent with its defn of increasing / decreasing. Pg 19, we'd've said what I started to say, but NOW, they don't count where the overlap is, and just look @ sign of f' (+/-?)

(a) f increasing on $(2,4) \cup (6,\infty)$ b/c $f' > 0$

~~(b)~~ f decreasing on $(0,2) \cup (4,6)$ b/c $f' < 0$

(b) Local max @ $x = 4$ ⇝ $(4, f(4))$, $f' = 0$

1st deriv. test $\underset{\nearrow\ 4\ \searrow}{+\ |\ -}$ f' $\quad 4 = 0$

Local min @ $x = 2$. $f' = 0$ ⇝ $(2, f(2))$

1st deriv test $\underset{\searrow\ 2\ \nearrow}{\longleftarrow -\ |\ + \longrightarrow}$ f' $\quad 2 = 0$

Also @ $x = 6$ $f' = 0$

1st deriv. test $\underset{\searrow\ 6\ \nearrow}{\longleftarrow -\ |\ + \longrightarrow}$ $\quad 6 = 0$

201 §3.3 I #s 8, 9, 11, 13, 15, 17

(8) (c) f is concave up where f' is increasing. That's on $(1,3) \cup (5,7) \cup (8,\infty)$

concave DOWN on $(0,1) \cup (3,5) \cup (7,8)$ b/c

that's where f' is decreasing

(d) IP's of f are @ $x = 1, 3, 5, 7, 8$, b/c that's where concavity changes (local extremes of f'!

$$\begin{array}{r|rrrr} -3 & 2 & 3 & -36 & 0 \\ & & -6 & 9 & 81 \\ \hline & 2 & -3 & -27 & 81 \end{array}$$

~~(9)~~ #s 9–14.

(a) f is inc/dec where?

(b) Local extrema

(c) Concavity & IPs.

$$\begin{array}{r|rrrr} 2 & 2 & 3 & -36 & 0 \\ & & 4 & 14 & -44 \\ \hline & 2 & 7 & -22 & -44 \end{array}$$

(9) $f(x) = 2x^3 + 3x^2 - 36x$

(a) $f'(x) = 6x^2 + 6x - 36 = 6(x^2 + x - 6)$

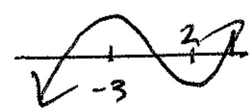
$= 6(x+3)(x-2) \overset{\text{SET}}{=} 0 \Rightarrow x \in \{2, -3\}$

$$\begin{array}{ccccc} + & & - & & + \\ \nearrow & -3 & \searrow & 2 & \nearrow \\ & =0 & & =0 & \\ & \text{MAX} & & \text{MIN} & \end{array} \quad f'$$

$\boxed{\begin{array}{l} \text{INC: } (-\infty,-3) \\ \quad \cup (2,\infty) \\ \text{DEC: } (-3,2) \end{array}}$

(b) Min of $y = -44$ @ $x = 2$

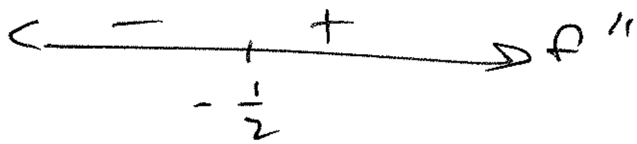
Max of $y = 81$ @ $x = -3$



201 §3.3 I #s 9, 11, 13, 15, 17

(9) (c) $f''(x) = 12x + 6 \overset{\text{SET}}{=} 0 \implies 2x + 1 = 0$

$\longrightarrow x = -\frac{1}{2}$



concave down: $(-\infty, -\frac{1}{2})$

" up: $(-\frac{1}{2}, \infty)$

IP: $x = -\frac{1}{2}, y = \frac{37}{2} \rightsquigarrow (-\frac{1}{2}, \frac{37}{2})$

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 3 & -36 & 0 \\ & & -1 & -1 & \frac{37}{2} \\ \hline & 2 & 2 & -37 & \frac{37}{2} \end{array}$$

---

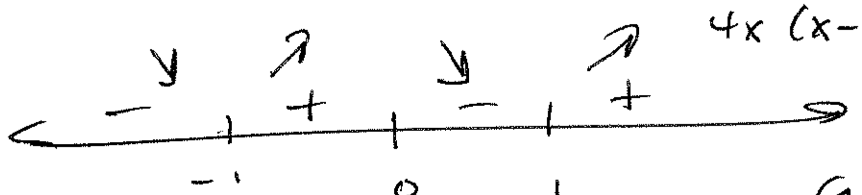
will graph this in class, 10/7

---

(11) $f(x) = x^4 - 2x^2 + 3$

(a) $f'(x) = 4x^3 - 4x \overset{\text{SET}}{=} 0 \implies 4x(x^2 - 1) = 0 \implies$

$4x(x-1)(x+1) = 0$



~~(c)~~

Inc: $(-1, 0) \cup (1, \infty)$

Dec: $(-\infty, -1) \cup (0, 1)$

(b) max @ $x = 0$. $\to$ ~~(0,0)~~ $(0, 3)$

min @ $x = -1, 1 \to$ ~~(1,0)~~ $(-1, 2)$

201 § 3.3 I #s 11, 13, 15, 17

(11) (c) $f''(x) = 12x^2 - 4 = 4(3x^2 - 1) \overset{\text{SET}}{=} 0 \Rightarrow$

$3x^2 = 1$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}}$

$= \pm \frac{\sqrt{3}}{3}$

$+ \quad - \quad +$

$\cup \quad -\frac{\sqrt{3}}{3} \quad \cap \quad \frac{\sqrt{3}}{3} \quad \cup \quad f'$

c. up: $(-\infty, -\frac{\sqrt{3}}{3}) \cup (\frac{\sqrt{3}}{3}, \infty)$

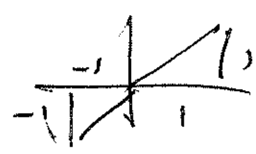
c. down: $(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3})$

(13) $f(x) = \sin x + \cos x, \quad 0 \le x \le 2\pi$

(a) $f'(x) = \cos x - \sin x \overset{\text{SET}}{=} 0 \Rightarrow$

$\cos x = \sin x$ OR

$\cot x = 1 \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$



$+ \quad - \quad +$

$\frac{\pi}{4} \quad \frac{5\pi}{4}$

Inc: $(0, \frac{\pi}{4}) \cup (\frac{5\pi}{4}, 2\pi)$

Dec: $(\frac{\pi}{4}, \frac{5\pi}{4})$

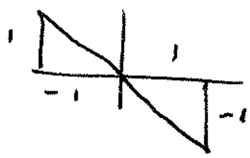
(b) Max @ $x = \frac{\pi}{4} \rightsquigarrow (\frac{\pi}{4}, \sqrt{2})$

Min @ $x = \frac{5\pi}{4} \rightsquigarrow (\frac{5\pi}{4}, -\sqrt{2})$

201 §3.3 I

13 (c) $f''(x) = -\sin x - \cos x \overset{\text{SET}}{=} 0 \Rightarrow$

$\sin x + \cos x = 0 \Rightarrow x \in \{\frac{3\pi}{4}, \frac{7\pi}{4}\}$



$\longleftarrow$ − | + | − $\longrightarrow$

$\frac{3\pi}{4}$ $\frac{7\pi}{4}$

c. up : $(\frac{3\pi}{4}, \frac{7\pi}{4})$

c. down : $(0, \frac{3\pi}{4}) \cup (\frac{7\pi}{4}, 2\pi)$

IPs: $\frac{3\pi}{4} \rightsquigarrow (\frac{3\pi}{4}, 0)$

$\frac{7\pi}{4} \rightarrow (\frac{7\pi}{4}, 0)$

#s 15–17 Find local max / min w/ 1st & 2nd Deriv. Tests

15 $f(x) = 1 + 3x^2 - 2x^3$

$= -2x^3 + 3x^2 + 1$

$f'(x) = -6x^2 + 6x = -6x(x-1)$

1st $\longleftarrow$ − | + | − $\longrightarrow$

↘ 0 ↗ 1 ↘

Min Max

Max @ $x = 1$

Min @ $x = 0$

$f(0) = 1$

$f(1) = 2$

$\boxed{\begin{array}{l}(0,1) \text{ MIN} \\ (1,2) \text{ MAX}\end{array}}$

2nd $f''(x) = -12x + 6$

$f''(0) = 6$ ☺ MIN $f''(1) = -6$ ☹ MAX.

1st works 4 Me !

201 §3.1 I #17

17) $f(x) = \sqrt{x} - \sqrt[4]{x}$

$= x^{\frac{1}{2}} - x^{\frac{1}{4}}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{4}x^{-\frac{3}{4}}$

$= \dfrac{1}{2x^{\frac{1}{2}}} - \dfrac{1}{4x^{3/4}}$

$= \dfrac{1}{2x^{\frac{1}{2}}} \cdot \dfrac{2x^{\frac{1}{4}}}{2x^{\frac{1}{4}}} - \dfrac{1}{4x^{\frac{3}{4}}}$

$= \dfrac{2x^{\frac{1}{4}} - 1}{4x^{3/4}}$

$2x^{\frac{1}{4}} = 1$

$x^{\frac{1}{4}} = \frac{1}{2}$

$x = \frac{1}{2^4} = \frac{1}{16}$

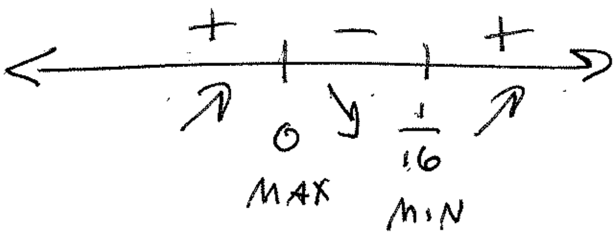
$4x^{\frac{3}{4}} = 0$

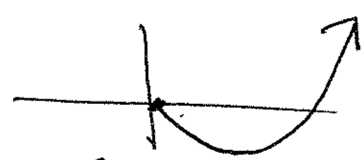
$x = 0$

$f\left(\frac{1}{16}\right) = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$ MIN

$\left(\frac{1}{16}, -\frac{1}{4}\right)$

$f(0) = 0 \rightsquigarrow (0,0)$ MAX





There's some debate on if (0,0) is local max.

Certainly $x = \frac{1}{16}$ gives local min.

$f''(x) = -\frac{1}{4}x^{-\frac{3}{2}} - \frac{1}{4}\left(-\frac{3}{4}\right)x^{-\frac{7}{4}}$

1st Deriv was Easier, here, too

$f''(0)\ \cancel{\exists}$

$f''\left(\frac{1}{16}\right) = -\frac{1}{4} \cdot \dfrac{1}{\left(\frac{1}{4}\right)^3} + \frac{3}{16} \cdot \dfrac{1}{\left(\frac{1}{2}\right)^7}$

$= -\frac{1}{4} \cdot 4^3 + \frac{3}{16} \cdot 2^7 = -4^2 + \frac{3}{16} \cdot 2^4 \cdot 2^3 = -4^2 + 3 \cdot 2^3$

$= -16 + 24 = 8$ ☺ MIN