201 §3.3 I #s 3, 5, 8, 9, 11, 13, 15, 17.

③ f has a formula

(a) f is inc/dec where $f'>0$ / $f'<0$, respectively.

(b) f is concave up where $f''>0$

" " concave down " $f''<0$

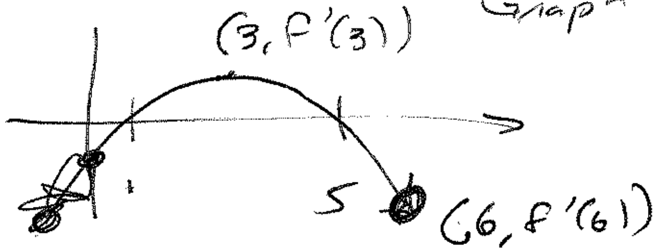
(c) IPs found by $f''=0$ or $f''\nexists$

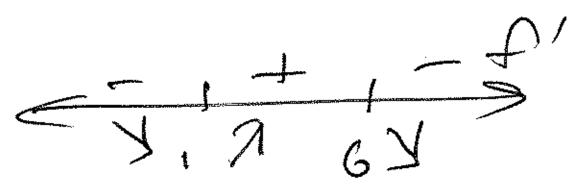
⑤ f' is shown

(a) where is f inc? dec?

(b) where are the local extremes?

Graph of f'





This says

(a) f inc. on [1, 5]

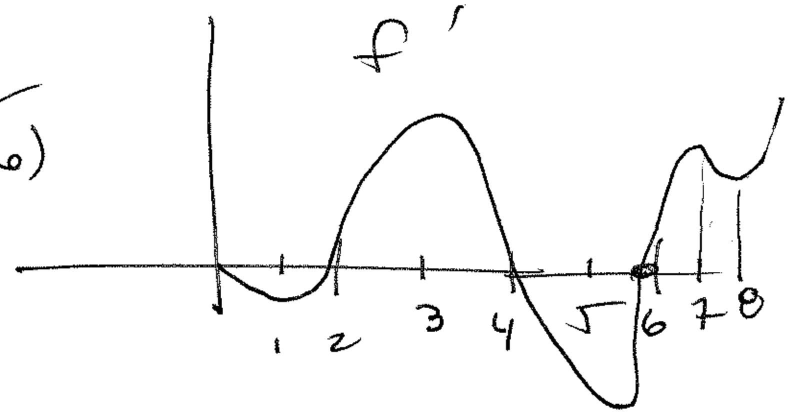
f dec. on [0, 1] ∪ [5, 6]

(b) Local max @ (6, f(6))

" min @ (1, f(1))

201 § 3.3 I #s 8, 9, 11, 13, 15, 17

(8) f' is shown



(a) ~~f inc on [2,4] ∪ [5,∞) b/c f' ≥ 0~~

(b)

(a) Textbook is inconsistent with its defn of increasing / decreasing. Pg 19, we'd've said what I started to say, but NOW, they don't count where the overlap is, and just look @ sign of f' (+/−?)

(a) f increasing on $(2,4) \cup (6,\infty)$ b/c $f' > 0$

~~(b)~~ f decreasing on $(0,2) \cup (4,6)$ b/c $f' < 0$

(b) Local max @ $x = 4$ ⇝ $(4, f(4))$, $f' = 0$

1st deriv. test $\underset{\nearrow \; 4 \; \searrow}{+ \mid -}$ f'
$=0$

Local min @ $x = 2$. $f' = 0$ ⇝ $(2, f(2))$

1st deriv test $\underset{\searrow \; 2 \; \nearrow}{- \mid +}$ f'
$=0$

Also @ $x = 6$ $f' = 0$

1st deriv. test $\underset{\searrow \; 6 \; \nearrow}{- \mid +}$
$=0$

201 §3.3 I #s 8, 9, 11, 13, 15, 17

(8) (c) f is concave up where f' is increasing. That's on $(1,3) \cup (5,7) \cup (8,\infty)$:)

concave DOWN on $(0,1) \cup (3,5) \cup (7,8)$ b/c that's where f' is decreasing :(

(d) IP's of f are @ $x = 1, 3, 5, 7, 8$, b/c that's where concavity changes (local extremes of f'!

$$\begin{array}{r|rrrr} -3 & 2 & 3 & -36 & 0 \\ & & -6 & 9 & 81 \\ \hline & 2 & -3 & -27 & 81 \end{array}$$

~~(9)~~ #s 9–14.

(a) f is inc/dec where?

(b) Local extrema

(c) Concavity & IPs.

$$\begin{array}{r|rrrr} 2 & 2 & 3 & -36 & 0 \\ & & 4 & 14 & -44 \\ \hline & 2 & 7 & -22 & -44 \end{array}$$

(9) $f(x) = 2x^3 + 3x^2 - 36x$

(a) $f'(x) = 6x^2 + 6x - 36 = 6(x^2 + x - 6)$

$= 6(x+3)(x-2) \overset{\text{SET}}{=} 0 \Rightarrow x \in \{2, -3\}$

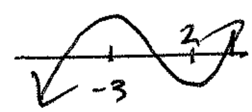
f': + on $(-\infty,-3)$ ↗, − on $(-3,2)$ ↘, + on $(2,\infty)$ ↗

-3: $=0$ MAX; 2: $=0$ MIN

$$\boxed{\begin{array}{l} \text{INC: } (-\infty,-3) \\ \quad \cup (2,\infty) \\ \text{DEC: } (-3,2) \end{array}}$$

(b) Min of $y = -44$ @ $x = 2$

Max of $y = 81$ @ $x = -3$



201 §3.3 I #s 9, 11, 13, 15, 17

9 (c) $f''(x) = 12x + 6 \overset{SET}{=} 0 \Longrightarrow 2x + 1 = 0$

$\Longrightarrow x = -\frac{1}{2}$

$-$ | $+$ f''

$-\frac{1}{2}$

concave down: $(-\infty, -\frac{1}{2})$

" up: $(-\frac{1}{2}, \infty)$

IP: $x = -\frac{1}{2}, y = \frac{37}{2} \leadsto (-\frac{1}{2}, \frac{37}{2})$

$-\frac{1}{2}$	2	3	-36	0
		-1	-1	$\frac{37}{2}$
	2	2	-37	$\frac{37}{2}$

---

will graph this in class, 10/7

---

11 $f(x) = x^4 - 2x^2 + 3$

(a) $f'(x) = 4x^3 - 4x \overset{SET}{=} 0 \Longrightarrow 4x(x^2 - 1) = 0 \Longrightarrow$

$4x(x-1)(x+1) = 0$

↘ ↗ ↘ ↗

$-$ | $+$ | $-$ | $+$

-1 0 1

~~(c)~~

Inc: $(-1, 0) \cup (1, \infty)$

Dec: $(-\infty, -1) \cup (0, 1)$

(b) max @ $x = 0. \rightarrow \cancel{(0,0)}$ $(0, 3)$

min @ $x = -1, 1 \rightarrow \cancel{(1,0)}$ $(-1, 2)$

201 §3.3 I #s 11, 13, 15, 17

(11) (c) $f''(x) = 12x^2 - 4 = 4(3x^2 - 1) \overset{\text{SET}}{=} 0 \Rightarrow$

$3x^2 = 1$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}}$

$= \pm\frac{\sqrt{3}}{3}$

$+ \quad - \quad +$ f'

$\cup \quad -\frac{\sqrt{3}}{3} \quad \cap \quad \frac{\sqrt{3}}{3} \quad \cup$

c. up: $\left(-\infty, -\frac{\sqrt{3}}{3}\right) \cup \left(\frac{\sqrt{3}}{3}, \infty\right)$

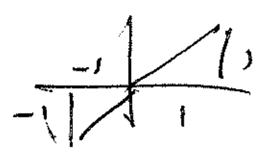
c. down: $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$

(13) $f(x) = \sin x + \cos x, \quad 0 \le x \le 2\pi$

(a) $f'(x) = \cos x - \sin x \overset{\text{SET}}{=} 0 \Rightarrow$

$\cos x = \sin x$ OR

$\cot x = 1 \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$



$+ \quad - \quad +$

$\frac{\pi}{4} \quad \frac{5\pi}{4}$

Inc: $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right)$

Dec: $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

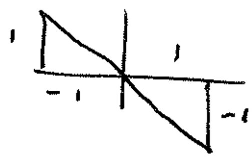
(b) ~~Abs~~ Max @ $x = \frac{\pi}{4} \to \left(\frac{\pi}{4}, \sqrt{2}\right)$

Min @ $x = \frac{5\pi}{4} \to \left(\frac{5\pi}{4}, -\sqrt{2}\right)$

201 §3.3 I

13 (c) $f''(x) = -\sin x - \cos x \overset{SET}{=} 0 \Rightarrow$

$\sin x + \cos x = 0 \Rightarrow x \in \{\frac{3\pi}{4}, \frac{7\pi}{4}\}$



$-$ | $+$ | $-$

$\frac{3\pi}{4}$ $\frac{7\pi}{4}$

c. up : $(\frac{3\pi}{4}, \frac{7\pi}{4})$

c. down : $(0, \frac{3\pi}{4}) \cup (\frac{7\pi}{4}, 2\pi)$

IPs: $\frac{3\pi}{4} \rightsquigarrow (\frac{3\pi}{4}, 0)$

$\frac{7\pi}{4} \rightarrow (\frac{7\pi}{4}, 0)$

#s 15–17 Find local max / min w/ 1st & 2nd Deriv. Tests

15 $f(x) = 1 + 3x^2 - 2x^3$

$= -2x^3 + 3x^2 + 1$

$f'(x) = -6x^2 + 6x = -6x(x-1)$

Max @ $x = 1$

Min @ $x = 0$

1st: $-$ | $+$ | $-$

$\searrow$ 0 $\nearrow$ 1 $\searrow$

Min Max

$f(0) = 1$

$f(1) = 2$

$\boxed{\begin{array}{l}(0,1) \text{ MIN} \\ (1,2) \text{ MAX}\end{array}}$

2nd $f''(x) = -12x + 6$

$f''(0) = 6$ ☺ MIN $f''(1) = -6$ ☹ MAX.

1st works 4 Me!

201 §3.1 I #17

17) $f(x) = \sqrt{x} - \sqrt[4]{x}$

$= x^{\frac{1}{2}} - x^{\frac{1}{4}}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{4}x^{-\frac{3}{4}}$

$= \frac{1}{2x^{\frac{1}{2}}} - \frac{1}{4x^{3/4}}$

$= \frac{1}{2x^{\frac{1}{2}}} \cdot \frac{2x^{\frac{1}{4}}}{2x^{\frac{1}{4}}} - \frac{1}{4x^{\frac{3}{4}}}$

$= \frac{2x^{\frac{1}{4}} - 1}{4x^{3/4}}$

$2x^{\frac{1}{4}} = 1$

$x^{\frac{1}{4}} = \frac{1}{2}$

$x = \frac{1}{2^4} = \frac{1}{16}$

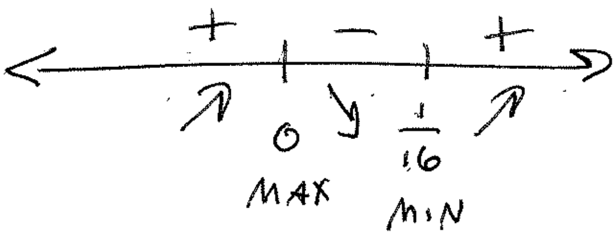
$4x^{\frac{3}{4}} = 0$

$x = 0$

$f(\frac{1}{16}) = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$ MIN

$(\frac{1}{16}, -\frac{1}{4})$

$f(0) = 0 \rightsquigarrow (0,0)$ MAX



There's some debate on if $(0,0)$ is local max.

Certainly $x = \frac{1}{16}$ gives local min.

$f''(x) = -\frac{1}{4}x^{-\frac{3}{2}} - \frac{1}{4}\left(-\frac{3}{4}\right)x^{-\frac{7}{4}}$

1st Deriv was Easier, here, too

$f''(0) \nexists$

$f''(\frac{1}{16}) = -\frac{1}{4} \cdot \frac{1}{(\frac{1}{4})^3} + \frac{3}{16} \cdot \frac{1}{(\frac{1}{2})^7}$

$= -\frac{1}{4} \cdot 4^3 + \frac{3}{16} \cdot 2^7 = -4^2 + \frac{3}{16} \cdot 2^4 \cdot 2^3 = -4^2 + 3 \cdot 2^3$

$= -16 + 24 = 8$ ☺ MIN