

201 § 3.2 # 5

Roll 15 & Verify Hypotheses & find $c(s)$.

1 $f(x) = 3x^2 - 12x + 5$ on $[1, 3]$

f is polynomial \rightarrow cont^s & diff^l $\forall x \in \mathbb{R}$ ✓

$$f(1) = 3 - 12 + 5 = -4 \quad \checkmark$$

$$f(3) = 27 - 36 + 5 = -4 \quad \checkmark$$

$$f'(x) = 6x - 12 \stackrel{\text{set}}{=} 0 \Rightarrow$$

$$x = 2 = c$$

2 $f(x) = \sqrt{x} - \frac{1}{3}x$ on $[0, 9]$

f is $\sqrt{x} - \frac{1}{3}x$ is cont^s on $[0, \infty)$ ✓
and diff^l on $(0, \infty)$ ✓

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{3} = \frac{1}{2\sqrt{x}} - \frac{1}{3} \stackrel{\text{set}}{=} 0 \Rightarrow$$

$$\frac{3 - 2\sqrt{x}}{6\sqrt{x}} = 0 \Rightarrow 2\sqrt{x} = 3 \Rightarrow$$

$$\sqrt{x} = \frac{3}{2} \Rightarrow$$

$$x = \frac{9}{4} = c$$

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3 $f(x) = 1 - x^{2/3}$. Then $f(1) = f(-1) = 0$, BUT

$$f'(x) = -\frac{2}{3}x^{-5/3} = -\frac{2}{3x^{5/3}} \neq 0 \quad \forall x.$$

This doesn't contradict Rolle's, b/c
 $f(x)$ isn't diff^{ble} @ $x=0 \in (-1, 1)$

Verify MVT Hypo. Find $c(s)$.

4 $f(x) = 2x^2 - 3x + 1$; $[0, 2]$

f is polynomial \rightarrow cont^s & diff^{ble} $\forall x \in \mathbb{R}$.

$$f'(x) = 4x - 3 \quad \text{SET } \frac{f(2) - f(0)}{2 - 0}$$

$$4x - 3 = \frac{8 - 3(2) + 1 - [1]}{2}$$

$$4x - 3 = \frac{2}{2} = 1$$

$$4x = 4$$

$$x = 1 = c$$

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$$f(x) = \sqrt[3]{x}, [0, 1]$$

$$= x^{1/3}$$

cont^s on \mathbb{R}

f' c

dif^{bl} on $(-\infty, 0) \cup (0, \infty)$

$$f'(x) = \frac{1}{3} x^{-2/3} \quad \text{SET } \frac{f(1) - f(0)}{1 - 0}$$

$$\frac{1}{3x^{2/3}} = \frac{1-0}{1} = 1$$

$$1 = 3x^{2/3}$$

$$x^{2/3} = \frac{1}{3}$$

$$x = \left(\frac{1}{3}\right)^{3/2} = \frac{1}{3} \cdot \left(\frac{1}{3}\right)^{1/2} = \frac{1}{3} \cdot \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{9} = c}$$

6

$f(x) = (x-3)^{-2}$. Show $\nexists c \in (1, 4) \exists$

$$f(4) - f(1) = f'(c)(4-1) \quad \text{i.e., } \frac{f(4) - f(1)}{4-1} = f'(c)$$

$$f'(x) = -2(x-3)^{-3} = -\frac{2}{(x-3)^3}$$

$$f(4) - f(1) = (4-3)^{-2} - (1-3)^{-2} = 1^{-2} - (-2)^{-2}$$

$$= 1 - \frac{1}{4} = \frac{3}{4} \quad \text{SET } -\frac{2}{(x-3)^3} (4-1) = -\frac{2 \cdot 3}{(x-3)^3}$$

$$\frac{3}{4} = -\frac{6}{(x-3)^3}$$

$$(x-3)^3 = -8$$

$$x-3 = \sqrt[3]{-8} = -2$$

$x = 3 - 2 = 1 = x = c$
 But $c \notin (1, 4)$
 No contradiction, b/c
 $f(x)$ not cont^s @ $x=3 \in [1, 4]$

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71 Show that $2x + \cos x = 0$ has exactly one root.

$2(10) + \cos(10) > 0$
 $2(-10) + \cos(10) < 0 \implies$ At least one. (IVT)

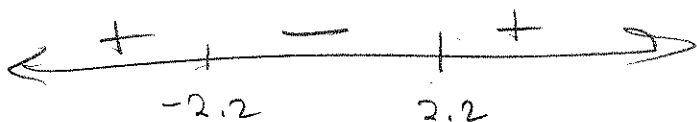
$f'(x) = 2 - \sin x > 0 \forall x \implies$ At most one
 $f(x) = 2x + \cos x$ is strictly increasing $\forall x$.

8 $x^3 - 15x + c = 0$ has at MOST one root.
in $[-2, 2]$

$f'(x) = 3x^2 - 15 \stackrel{\text{SET}}{=} 0$

$x^2 = 5$

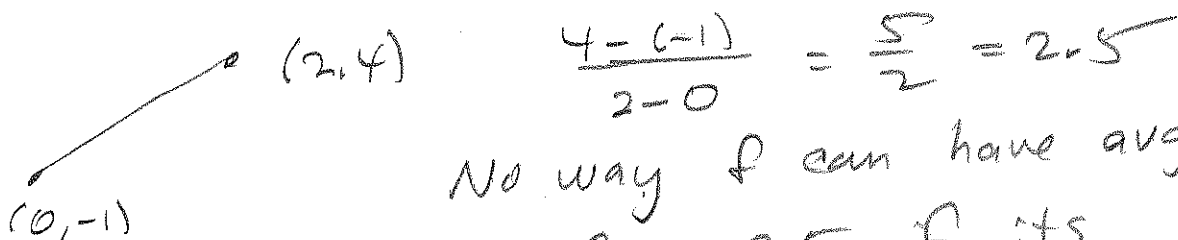
$x = \pm\sqrt{5} \approx \pm 2.236067977$



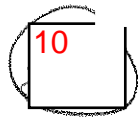
$f(x) = x^3 - 15x + c$ is strictly decreasing $\forall x \in [-2, 2]$

Can't cross x-axis twice between $\pm\sqrt{5}$.

9 Does $\exists f \ni f(0) = -1, f(2) = 4$ &
 $f'(x) \leq 2 \forall x$?



No way f can have avg slope of $m = 2.5$ if its slope is everywhere less than 2!
Violates MVT.



Show that $\sqrt{x+1} < 1 + \frac{1}{2}x$ if $x > 0$

@ $x=0$, we have $\sqrt{1} = 1$ on LHS
and 1 on RHS.

We show $f(x) = \sqrt{x+1} - (1 + \frac{1}{2}x) < 0$ if $x > 0$.

Note $f(0) = 0$.

$$f'(x) = \frac{1}{2}(x+1)^{-\frac{1}{2}} - \frac{1}{2}$$

$$= \frac{1}{2\sqrt{x+1}} - \frac{1}{2} < 0 \text{ if } x > 0$$

So it's decreasing, strictly.

Basically, $\sqrt{x+1}$ grows slower than $\frac{1}{2}x + 1$
CLOSE to $x=0$, it's a little harder to be
sure. They're equal at $x=0$, but as soon
as you move to the right, $\sqrt{x+1}$ is ducking
underneath.