201 §5.5 #s 5, 6, 9, 13, 15, 20

#s 1-8 Find f_{AVG}

(5) $f(t) = t^2(1+t^3)^4$ on $[0, 2]$

$$\frac{1}{b-a}\int_a^b f(t)dt = \frac{1}{2}\int_0^2 (t^3+1)^4(t^2 dt) = \frac{1}{6}\int_0^2 (t^3+1)^4(3t^2 dt)$$

$u = t^3+1 \Rightarrow du = 3t^2 dt$

$$= \frac{1}{6}\left.\frac{(t^3+1)^5}{5}\right]_0^2 = \frac{1}{30}(2^3+1)^5 - \frac{1}{30}(1)^5$$

$$= \frac{1}{30}\left[9^5 - 1\right] = \boxed{\frac{29524}{15} = 1968.2\overline{6}}$$

(6) $f(\theta) = \sec^2\left(\frac{\theta}{2}\right)$ on $\left[0, \frac{\pi}{2}\right]$

$$f_{AVG} = \frac{1}{b-a}\int_a^b f(t)dt = \frac{1}{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \sec^2\left(\frac{\theta}{2}\right)d\theta$$

$u = \frac{\theta}{2}$ $du = \frac{1}{2}d\theta$ $d\theta = 2du$

$$2\cdot\frac{2}{\pi}\int_0^{\frac{\pi}{2}} \sec^2\left(\frac{\theta}{2}\right)\left(\frac{1}{2}d\theta\right) = \frac{4}{\pi}\left[\tan\left(\frac{\theta}{2}\right)\right]_0^{\frac{\pi}{2}}$$

$$= \frac{4}{\pi}\left[\tan\frac{\pi}{4} - \tan(0)\right] = \boxed{\frac{4}{\pi}}$$

201 § 8.5 #s 9, 13, 15, 20

(9) (a) Find f_{AVG}

(b) Find $c \ni f(c) = f_{AVG}$

(c) sketch f & a rectangle whose area = area under the graph of f.

(a) $f(x) = (x-3)^2$ on $[2,5]$

$$f_{AVG} = \frac{1}{3}\int_2^5 (x-3)^2\,dx = \frac{1}{3}\left[\frac{(x-3)^3}{3}\right]_2^5$$

$$= \frac{1}{3}\left[\frac{2^3}{3} - \frac{(-1)^3}{3}\right] = \frac{1}{9}\left[8+1\right] = \boxed{1 = f_{AVG}}$$

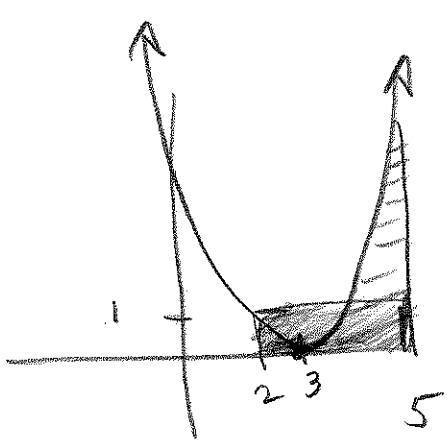
(b) $(x-3)^2 \overset{SET}{=} 1$

$x - 3 = \pm\sqrt{1} = \pm 1$

$x = 3 \pm 1$ $\boxed{4 = c}$

$2 \notin (2,5)$ (Interior of interval)

(c)



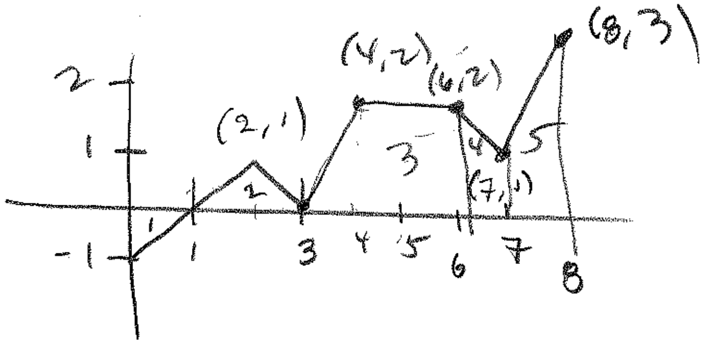
201 8.5 #s 13, 15, 20

(13) f is cont & $\int_1^3 f(x)\,dx = 8$, Show that f takes on the value 4 at least once on the interval.

$$f_{AVG} = \frac{1}{3-1}\int_1^3 f(x)\,dx = \frac{8}{2} = 4$$

$\Longrightarrow \exists\ c \in (1,3) \ni f(c) = 4$, by MVT.

(15) Find AVG Val. of f on [0, 8]



$$\int_0^8 f(x)dx = -\tfrac{1}{2}(1)(1) + \tfrac{1}{2}(2)(1) + \tfrac{1}{2}(3+2)(2) + \tfrac{1}{2}(2+1)(1) + \tfrac{1}{2}(3+1)(1)$$

$$= -\tfrac{1}{2} + 1 + 5 + \tfrac{3}{2} + 2 = 1 + 5 + 1 + 2 = 9$$

$$f_{AVG} = \frac{1}{b-a}\int_a^b f(x)dx = \left(\frac{1}{8-0}\right)(9) = \boxed{\frac{9}{8}}$$

201 §8.5 #20

We discussed this in class.
It's unusual to express velocity as a function of distance, and it takes an unusual step to rewrite $v(t)$ as $v(t(s))$ (TIME AS FUNC. OF DISTANCE)

but somewhat useful, I suppose!

$$s = \frac{1}{2}gt^2 = s$$

$$t^2 = \frac{2s}{g}$$

$$t = \pm\sqrt{\frac{2s}{g}} \longrightarrow \text{Take positive.} \rightarrow v(t(s))$$

$$v(t) = gt = \boxed{g\sqrt{\frac{2s}{g}} = \sqrt{2sg} = (2sg)^{\frac{1}{2}}} \longrightarrow$$

$$\frac{dv}{ds} = \frac{1}{2}(2sg)^{-\frac{1}{2}}(2g) = \frac{g}{\sqrt{2sg}}$$ Pointless. We're

after avg val. on $[0,T]$:

Integrate wrt t:

$$\frac{1}{T-0}\int_0^T gt\,dt = \frac{1}{T}\cdot\frac{1}{2}\left[gt^2\right]_0^T = \frac{1}{T}\cdot\frac{1}{2}gT^2 = \frac{1}{2}gT = \frac{1}{2}V(T) = \boxed{\frac{1}{2}V_T}$$

Now, wrt s: $s(T) = \frac{1}{2}gT^2$

$$\frac{1}{\frac{1}{2}gT^2 - 0}\int_0^{\frac{1}{2}gT^2}(2sg)^{\frac{1}{2}}ds = \frac{2\sqrt{2g}}{gT^2}\int_0^{\frac{1}{2}gT^2} s^{\frac{1}{2}}ds$$

$$= \frac{2\sqrt{2}}{\sqrt{g}\,T^2}\cdot\frac{2}{3}s^{\frac{3}{2}}\Big]_0^{\frac{1}{2}gT^2} = \frac{4\sqrt{2}}{3T^2\sqrt{g}}\left[\left(\frac{1}{2}gT^2\right)^{\frac{3}{2}}\right]$$

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$$= \frac{4\sqrt{2}}{3T^2\sqrt{g}} \cdot \left(\frac{1}{2}\right)^{\frac{3}{2}} g^{\frac{3}{2}} T^3 = \frac{4\sqrt{2}}{3T^2\sqrt{g}} \left(\frac{1}{2\sqrt{2}}\right) g\sqrt{g}\, T^3$$

$$= \frac{2gT}{3} = \frac{2}{3} gT = \boxed{\frac{2}{3} v_T}$$

Average value with respect to WHERE you are is different than average value with respect to WHEN.

Time increases linearly

Distance is increasing at quadratic pace.