201 §4.5 #s 1, 2, 7, 11, 15, 21, 25, 35, 37, 41, 42, 46, 51

~~#s~~ 1-6 Evaluate using the given substitution

(1) $\int \sin(\pi x)\,dx$, $u = \pi x \Rightarrow$

$du = \pi\,dx \Rightarrow$

$dx = \frac{du}{\pi}$

$= \int \sin u \frac{du}{\pi} = \frac{1}{\pi}\int \sin u\,du = \boxed{-\frac{1}{\pi}\cos(\pi x) + C}$

(2) $\int x^3 (x^4+2)^5\,dx$, $u = x^4 + 2$

$du = 4x^3\,dx$

$dx = \frac{du}{4x^3}$

$= \int x^3 (u)^5 \frac{du}{4x^3} = \frac{1}{4}\int u^5\,du = \frac{1}{4}\cdot\frac{1}{6}u^6 + C$

$= \boxed{\frac{1}{24}(x^4+2)^6 + C}$

(7) $\int x \sin(x^2)\,dx$ $u = x^2$

$du = 2x\,dx$

$= \frac{1}{2}\int \sin(x^2)(2x\,dx) = \frac{1}{2}\int \sin u\,du$

$= -\frac{1}{2}\cos u + C = \boxed{-\frac{1}{2}\cos(x^2) + C}$

201 §4.5 #s 11, 15, 21, 25, 35, 37, 41, 42, 46, 51

(11) $\int (x+1)\sqrt{x^2+2x}\, dx$

M1

$u = x^2+2x$

$du = (2x+2)\,dx = 2(x+1)\,dx$

$dx = \frac{du}{2(x+1)}$

$= \int (x+1)\sqrt{u}\,\frac{du}{2(x+1)}$

$= \frac{1}{2}\int u^{\frac{1}{2}}du = \frac{1}{2}\cdot\frac{2}{3}u^{\frac{3}{2}}+C$

$= \boxed{\frac{1}{3}(x^2+2x)^{\frac{3}{2}}+C}$

straight from this to $\int\sqrt{x^2+2x}\,(x+1)\,dx$

Then see you're missing a factor of 2 and make it so:

$\frac{1}{2}\int\sqrt{x^2+2x}\,(2(x+1)\,dx)$

$= \frac{1}{2}\int u^{\frac{1}{2}}du$, etc.

(15) $\int \frac{bx^2+a}{\sqrt{3ax+bx^3}}\,dx$

$u = bx^3+3ax$

$du = (3bx^2+3a)\,dx$

$= 3(bx^2+a)\,dx$

$= \frac{1}{3}\int (bx^3+3ax)^{-\frac{1}{2}}(3(bx^2+a)\,dx)$

$= \boxed{\frac{1}{3}\cdot 2(bx^3+3ax)^{\frac{1}{2}}+C}$

201 §4.5 #s 21, 25, 35, 37, 41, 42, 46, 51

(21) $\int \frac{\cos x}{\sin^2 x}\,dx$

$\boxed{M1}$ $u = \sin x$

$du = \cos x\,dx$

$\Longrightarrow$

$\int u^{-2}\,du$

$= -u^{-1} + C$

$= -\frac{1}{\sin x} + C$

$= -\csc x + C$

$\boxed{M2}$ $= \int \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}\,dx$

$= \int \cot x \csc x\,dx$

$= -\csc x + C$

$\left(\frac{d}{dx}[\csc x] = -\csc x \cot x\right.$

(25) $\int \sqrt{\cot x}\,\csc^2 x\,dx$

$\begin{pmatrix} u = \cot x \\ du = -\csc^2 x\,dx \end{pmatrix}$

$= -\int (\cot x)^{\frac{1}{2}}(-\csc^2 x\,dx)$

$= \boxed{-\frac{2}{3}(\cot x)^{\frac{3}{2}} + C}$

(35) $\int_0^1 \cos\left(\frac{\pi}{2}t\right)dt = \frac{2}{\pi}\int_{t=0}^{t=1} \cos\left(\frac{\pi}{2}t\right)\left(\frac{\pi}{2}dt\right)$

$\left(u = \frac{\pi}{2}t \rightarrow du = \frac{\pi}{2}dt\right)$

$t = 0 \Longrightarrow u = 0$

$t = 1 \Longrightarrow u = \frac{\pi}{2}$

$= \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \cos u\,du$

$= \frac{2}{\pi}\left[\sin u\right]_0^{\frac{\pi}{2}} = \frac{2}{\pi}\left[\sin\left(\frac{\pi}{2}\right) - \sin(0)\right]$

$= \boxed{\frac{2}{\pi}}$

201 §4.5 #s 37, 41, 42, 46, 51

(37) $\int_0^3 \sqrt[3]{7x+1}\,dx = \frac{1}{7}\int_0^3 (7x+1)^{\frac{1}{3}}\,(7\,dx)$

$(u = 7x+1 \quad du = 7\,dx)$

$= \left[\frac{1}{7}\cdot\frac{3}{4}(7x+1)^{\frac{4}{3}}\right]_0^3 = \frac{3}{28}\left[(7(3)+1)^{\frac{4}{3}} - 1^{\frac{4}{3}}\right]$

$= \frac{3}{28}\left[22^{\frac{4}{3}} - 1\right] = \frac{3}{28}\left[22^1\cdot 22^{\frac{1}{3}} - 1\right] = \boxed{\frac{3}{28}\left(22\sqrt[3]{22} - 1\right)}$

$\approx \boxed{6.4976641}$

(41) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (x^3 + x^4 \tan x)\,dx = 0$

ODD function on symmetric interval.

~~Still, we have a bit of a problem @ x=0~~

No. tan x is cont's on $[-\frac{\pi}{4}, \frac{\pi}{4}]$

(42) $\int_0^{\frac{\pi}{2}} \cos x \sin(\sin x)\,dx$

$\left(\begin{array}{ll} u = \sin x & x = \frac{\pi}{2}:\ \sin\frac{\pi}{2} = 1 \\ du = \cos x\,dx & x = 0:\ \sin 0 = 0 \end{array}\right)$

$= \int_0^1 \sin u\,du = \sin(1) - \sin(0) = \boxed{\sin(1)}$

201 §4.5 #s 46, 51

46 $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x^4 \sin x \, dx = 0$

51 $\displaystyle\int_0^1 \frac{dx}{(\sqrt{x}+1)^4}$

$u = \sqrt{x} + 1 = x^{\frac{1}{2}} + 1$ $\quad x=0 \quad u=1$

$du = \frac{1}{2} x^{-\frac{1}{2}} dx \longrightarrow$ $\quad x=1 \quad u=16$

$dx = 2x^{\frac{1}{2}} du$

$= \displaystyle\int_1^{16} \frac{2x^{\frac{1}{2}} du}{u^4}$ $\qquad u = x^{\frac{1}{2}} + 1 \Longrightarrow$

$x^{\frac{1}{2}} = u - 1$

$= 2\displaystyle\int_1^{16} \frac{(u-1)\, du}{u^4} = 2\int_1^{16} (u^{-3} - u^{-4})\, du$

$= 2\left[-\frac{1}{2}u^{-2} - \left(-\frac{1}{3}\right)u^{-3}\right]_1^{16}$

$= 2\left[-\frac{1}{2u^2} + \frac{1}{3u^3}\right]_1^{16} = 2\left[-\frac{1}{2(256)} + \frac{1}{3(4096)} - \left(-\frac{1}{2} + \frac{1}{3}\right)\right]$

$= 2\left[-\frac{1}{512} + \frac{1}{12288} + \frac{1}{6}\right] \approx .3295898438$

Supposed to come out $= \frac{1}{6}$

Try again

201 §4.5 #51

51 $\int_0^1 \frac{dx}{(\sqrt{x}+1)^4}$

$u = \sqrt{x}+1 = x^{\frac{1}{2}}+1$

$du = \frac{1}{2}x^{-1/2}\,dx$

$2x^{\frac{1}{2}}du = 2x^{\frac{1}{2}} \cdot \frac{1}{2}x^{-\frac{1}{2}}dx = dx$

$x=0 \Rightarrow u=1$ $\longrightarrow$

$x=1 \Rightarrow u=(1+1)^4 = 2^4 = 16$

$\int_1^{16} \frac{2x^{\frac{1}{2}}du}{u^4}$ $\quad x^{\frac{1}{2}}+1 = u$ $\longrightarrow$

$x^{\frac{1}{2}} = u-1$

$\int_1^{16} \frac{2(u-1)\,du}{u^4} = 2\int_1^{16}(u^{-3}-u^{-4})\,du$

$= 2\left[\frac{u^{-2}}{-2} - \frac{u^{-3}}{-3}\right]_1^{16} = 2\left[-\frac{1}{2u^2}+\frac{1}{3u^3}\right]_1^{16}$

$= 2\left[-\frac{1}{2(16)^2}+\frac{1}{3(16)^3} - \left(-\frac{1}{2}+\frac{1}{3}\right)\right]$

$= -\frac{2}{2(16)^2} + \frac{2}{3(16)^3} + \frac{1}{6}$

$= \frac{1}{16^2}\left[-1+\frac{2}{3(16)}\right] + \frac{1}{6}$

Still not getting it. Dunno...