201 §4.4 #s 5, 11, 19, 27, 33, 39-42 ALL, 46, 48, 49, 55, 57

#s 5-16 Find gen'l antiderivative

5 $\int (x^2 + x^{-2})\,dx = \boxed{\frac{1}{3}x^3 - x^{-1} + C}$

11 $\int \frac{x^3 - 2\sqrt{x}}{x}\,dx = \int (x^2 - 2x^{-\frac{1}{2}})\,dx$

$= \boxed{\frac{1}{3}x^3 - 4x^{\frac{1}{2}} + C}$

#s 19-42 Evaluate the Integral

19 $\int_{-2}^{3} (x^2 - 3)\,dx = \left[\frac{1}{3}x^3 - 3x\right]_{-2}^{3}$

$= \frac{1}{3}(3)^3 - 3(3) - \left[\frac{1}{3}(-2)^3 - 3(-2)\right]$

$= 9 - 9 - \left[-\frac{8}{3} + 6\right] = \frac{8}{3} - \frac{18}{3} = \boxed{-\frac{10}{3}}$

27 $\int_1^4 \left(\frac{6u + 4}{\sqrt{u}}\right) du = \int_1^4 (6u^{\frac{1}{2}} + 4u^{-\frac{1}{2}})\,du$

$= \left[6\left(\frac{2}{3}\right)u^{\frac{3}{2}} + 4(2)u^{\frac{1}{2}}\right]_1^4$

$= 4(4)^{\frac{3}{2}} + 8(4)^{\frac{1}{2}} - \left[4(1)^{\frac{3}{2}} + 8(1)^{\frac{1}{2}}\right]$

$= 4(8) + 8(2) - (4 + 8) = 32 + 16 - 12 = \boxed{36}$

$= \boxed{}$

201 §4.4 #5 33, 39-42 ALL, 46, 48, 49, 55, 57

33 $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\cos^2\theta + 1}{\cos^2\theta}\, d\theta$

$$= \int_0^{\frac{\pi}{4}} (1 + \sec^2\theta)\, d\theta = \Big(\theta + \tan\theta\Big)\Big]_0^{\frac{\pi}{4}}$$

$$= \frac{\pi}{4} + \tan\left(\frac{\pi}{4}\right) - (0 - \tan 0)$$

$$= \boxed{\frac{\pi}{4} + \frac{1}{\sqrt{2}}} =$$

39 $\displaystyle\int_2^5 |x-3|\, dx$

$$|x-3| = \begin{cases} x-3 & \text{if } x \geq 3 \\ -x+3 & \text{if } x < 3 \end{cases}$$

$$= \int_2^3 (-x+3)\, dx + \int_3^5 (x-3)\, dx$$

$$= \left[-\frac{x^2}{2} + 3x\right]_2^3 + \left[\frac{x^2}{2} - 3x\right]_3^5$$

$$= -\frac{3^2}{2} + 3(3) - \left(-\frac{2^2}{2} + 3(2)\right) + \frac{5^2}{2} - 3(5) - \left(\frac{3^2}{2} - 3(3)\right)$$

$$= -\frac{9}{2} + 9 - (-2+6) + \frac{25}{2} - 15 - \frac{9}{2} + 9$$

$$= -9 + 9 - 4 + \frac{25}{2} - 15 + 9 = -10 + \frac{25}{2} = \frac{-20+25}{2} = \boxed{\frac{5}{2}}$$

201 §4.4 #s 40–42 All, 46, 48, 49, 55, 57

(40) $$\int_0^2 |2x-1|\,dx = \int_0^{\frac{1}{2}} (-2x+1)\,dx + \int_{\frac{1}{2}}^{2} (2x-1)\,dx$$

$$= \Big[-x^2+x\Big]_0^{\frac{1}{2}} + \Big[x^2-x\Big]_{\frac{1}{2}}^{2}$$

$2x-1 \geq 0$

$2x \geq 1$

$x \geq \frac{1}{2}$

$$= -\left(\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{2}\right) - \left(-0^2+0\right) + \left(2^2-2-\left(\left(\tfrac{1}{2}\right)^2-\tfrac{1}{2}\right)\right)$$

$$= -\tfrac{1}{4} + \tfrac{1}{2} + \left(4-2-\left(\tfrac{1}{4}-\tfrac{1}{2}\right)\right)$$

$$= \tfrac{1}{4} + \left(2-\left(-\tfrac{1}{4}\right)\right) = \tfrac{1}{4} + 2 + \tfrac{1}{4} = \boxed{2.5 = \tfrac{5}{2}}$$

(41) $$\int_{-1}^{2} (x - 2|x|)\,dx$$

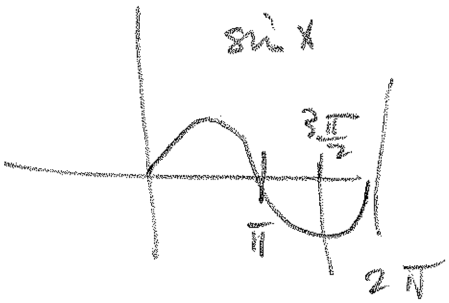
$$= \int_{-1}^{0} (x-2(-x))\,dx + \int_0^2 (x-2x)\,dx$$

$$= \int_{-1}^{0} 3x\,dx + \int_0^2 -x\,dx = \tfrac{3}{2}x^2\Big]_{-1}^{0} - \tfrac{x^2}{2}\Big]_0^2$$

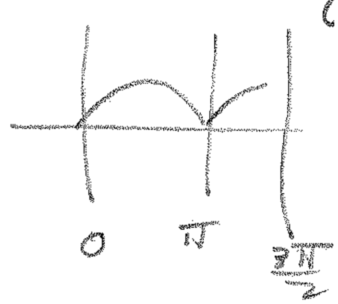
$$= \tfrac{3}{2}(0^2) - \tfrac{3}{2}(-1)^2 - \left(\tfrac{2^2}{2} - \tfrac{0^2}{2}\right) = -\tfrac{3}{2} - 2 = -\tfrac{7}{2}$$

201 §4.4 #s 42, 46, 48, 49, 55, 57

(42) $\displaystyle\int_0^{\frac{3\pi}{2}} |\sin x|\, dx = \int_0^{\pi} \sin x\, dx - \int_{\pi}^{\frac{3\pi}{2}} \sin x\, dx$



$|\sin x| = \begin{cases} \sin x & 0 \le x \le \pi \\ -\sin x & \pi < x \le \frac{3\pi}{2} \end{cases}$

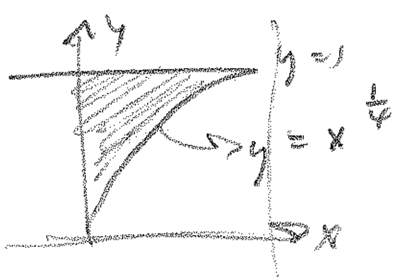


$= \Big[-\cos x\Big]_0^{\pi} + \Big[\cos x\Big]_{\pi}^{\frac{3\pi}{2}} = -\cos(\pi) - (-\cos(0)) + \cos\left(\frac{3\pi}{2}\right) - \cos(\pi)$

$= -(-1) + 1 + 0 - (-1) = \boxed{3}$

(46) We find the area in $\underline{2}$ ways:

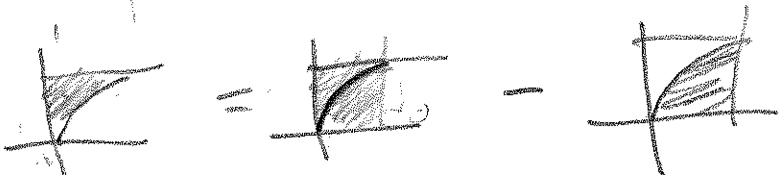
$x = f(y)$ & $y = g(x)$



$\boxed{M1}$ (Book) $y = x^{\frac{1}{4}} \Longrightarrow x = y^4$

$\text{Area} = \displaystyle\int_0^1 y^4\, dy = \frac{1}{5} y^5\Big]_0^1 = \frac{1}{5}$

$\boxed{M2}$



$= \displaystyle\int_0^1 1\, dx - \int_0^1 x^{\frac{1}{4}}\, dx = \Big[x\Big]_0^1 - \Big[\frac{4}{5} x^{\frac{5}{4}}\Big]_0^1$

$= 1 - \frac{4}{5} = \boxed{\frac{1}{5}}$

201 § 4.4 #s 48, 49, 55, 57

48 Current = $\frac{d}{dt}$[Charge]

$$I(t) = Q'(t)$$

Then $\int_a^b I(t)\,dt = Q(b) - Q(a)$ = net change in charge.

49 Oil leaks @ $r(t)$ gallons/minute. What's $\int_0^{120} r(t)\,dt$ represent?

It represents the # of gallons that leak in 2 hours' time.

#s 55-6 Velocity func. (in $\frac{m}{s}$) is given for a particle moving in a straight line.
Find (a) Displacement
(b) Distance

55 $v(t) = 3t - 5$, $0 \le t \le 3$

(a) $\int_0^3 (3t-5)\,dt = \left[\frac{3}{2}t^2 - 5t\right]_0^3 = \frac{3}{2}(9) - 5(3) = \frac{27}{2} - \frac{30}{2} = \boxed{-\frac{3}{2}\,m}$

(b) $\int_0^3 |3t-5|\,dt = \int_0^{\frac{5}{3}} (3t-5)\,dt - \int_{\frac{5}{3}}^{3} (3t-5)\,dt$ $= \boxed{\frac{41}{6}} = 6.8\overline{3}$

201 §4.4 #57

#s 57-8 $a(t)$ = acceleration (m/s^2) & $v(0)$ = initial velocity are given. Find (a) $v(t)$ & (b) distance travelled in the time given

(57) (a) $a(t) = t+4$, $v(0)=5$, $0 \le t \le 10$

$$v(t) = \int a(t)\,dt = \int (t+4)\,dt = \tfrac{1}{2}t^2 + 4t + C$$

$$v(0) = 5 \implies C = 5 \longrightarrow \boxed{v(t) = \tfrac{1}{2}t^2 + 4t + 5}$$

(b) Distance Travelled $= \displaystyle\int_0^{10} |v(t)|\,dt$

$$= \int_0^{10} v(t)\,dt \overset{*}{=} \int_0^{10} \left(\tfrac{1}{2}t^2 + 4t + 5\right) dt$$

$$= \left[\tfrac{1}{6}t^3 + 2t^2 + 5t\right]_0^{10} = \tfrac{1}{6}(1000) + 2(100) + 50$$

$$= \frac{1000 + 1200 + 300}{6} = \frac{2500}{6} = \boxed{\frac{1250}{3}} \doteq 416.\overline{6}\ m$$

* Scratch $\tfrac{1}{2}t^2 + 4t + 5$

$$= \tfrac{1}{2}\left[t^2 + 8t\qquad\right] + 5$$

$$= \tfrac{1}{2}\left[t^2 + 8t + 4^2\right] + 5 - \tfrac{1}{2}(16)$$

$5 - 8 = -3$

$$= \tfrac{1}{2}\left[t+4\right]^2 - 3 \overset{\text{SET}}{=} 0$$

$$\tfrac{1}{2}\left[t+4\right]^2 = 3$$

$$\left[t+4\right]^2 = 6$$

$t + 4 = \pm\sqrt{6}$

$t = -4 \pm \sqrt{6}$ No positive roots