201 §4.2 #s 17, 19, 21-25, 27, 28, 33a, 37, 41

#s 17-20 Express the limit as a definite integral on the given interval.

(17) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1-x_k^2}{4+x_k^2}\Delta x$ on $[2,6]$

$$\boxed{= \int_2^6 \frac{1-x^2}{4+x^2}\,dx}$$

(19) $\lim_{n\to\infty} \sum_{k=1}^{n} \left[5(x_k^*)^3 - 4x_k^*\right]\Delta x$ on $[2,7]$

$$\boxed{= \int_2^7 (5x^3-4x)\,dx}$$

#s 21-25 Use limit defn of the integral to evaluate the integral.

(21) $\int_2^5 (4-2x)\,dx$ $\qquad \frac{b-a}{n} = \frac{5-2}{n} = \frac{3}{n} = \Delta x$

$x_k = 2 + \frac{3}{n}k$

$$\sum_{k=1}^{n}\left(4-2\left(2+\frac{3k}{n}\right)\right)\cdot\frac{3}{n} = \frac{3}{n}\left[\cancel{\sum 4 + \frac{3}{n}\sum k}\right]$$

$$\cancel{= \frac{3}{n}\left[4n + \frac{3}{n}\cdot\frac{n^2+n}{2}\right] = 12 + \frac{9}{n^2}\cdot\frac{n^2+n}{2} \xrightarrow{n\to\infty}}$$

$$\cancel{12 + \frac{9}{2} = \frac{24+9}{2}} = \boxed{\frac{33}{2}}$$

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21 $= \frac{3}{n}\left[\sum 4 - \sum\left(4 + \frac{6k}{n}\right)\right]$

$= \frac{3}{n}\left[\sum 4 - \sum 4 - \sum \frac{6k}{n}\right]$

$= \frac{3}{n}\left[-\frac{6}{n}\sum k\right]$

$= -\frac{18}{n^2}\left[\frac{n^2+n}{2}\right] = -\frac{18n^2}{2n^2} - \frac{n}{2n^2} \xrightarrow{n\to\infty}$

$\boxed{-9}$ (Check w/ FTC: $\int_2^5 (4-2x)\,dx$

$= \left[4x - x^2\right]_2^5 = 4(5) - 5^2 - [4(2) - 2^2]$

$= 20 - 25 - 8 + 4 = 24 - 33 = \boxed{-9}$ ✓

23 $\int_{-2}^{0} (x^2+x)\,dx$ $\Delta x = \frac{0-(-2)}{n} = \frac{2}{n}$

$x_k = a + \Delta x\, k = -2 + \frac{2}{n}k = \cancel{\frac{-2n+2}{n}k}$ → Nope

~~$\Delta x \sum f(x_k) = \frac{2}{n}\sum_{k=1}^{n}\left[\left(\frac{-2n+2}{n}k\right)^2 + \left(\frac{-2n+2}{n}\right)k\right]$~~

~~$= \frac{2}{n}\left[\sum \frac{4n^2-8n+4}{n^2}k^2 + \sum \frac{-2n+2}{n}k\right]$~~

~~$= \frac{2}{n^3}(4n^2-8n+4)\frac{n^3+\dots}{3} + \frac{2}{n^2}(-2n+2)\frac{n^2+n}{2}$~~

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23 $x_k = -2 + \frac{2}{n}k$

$$\sum f(x_k)\Delta x = \Delta x \sum f(x_k) = \frac{2}{n}\left[\sum_{k=1}^{n}(x_k^2 + x_k)\right]$$

$$= \frac{2}{n}\left[\sum\left(\left(-2+\frac{2k}{n}\right)^2 + \left(-2+\frac{2k}{n}\right)\right)\right]$$

$$= \frac{2}{n}\left[\sum\left(4 - \frac{8k}{n} + \frac{4k^2}{n^2}\right) + \left(-2 + \frac{2k}{n}\right)\right]$$

$$= \frac{2}{n}\left[\sum\left(2 - \frac{6k}{n} + \frac{4}{n^2}k^2\right)\right] = \frac{2}{n}\left[2n - \frac{6}{n}\cdot\frac{n^2+\text{mm}}{2} + \frac{4}{n^2}\cdot\frac{n^3+\text{mm}}{3}\right]$$

$$= \frac{2}{n}\cdot 2n - \frac{12}{n^2}\cdot\frac{n^2+\text{mm}}{2} + \frac{8}{n^3}\cdot\frac{n^3+\text{mm}}{3}$$

$$= 4 - 6 - \frac{12\,\text{mm}}{2n^2} + \frac{8}{3} + \frac{8\,\text{mm}}{3n^3} \xrightarrow{n\to\infty}$$

$$\xrightarrow{n\to\infty} 4 - 6 + \frac{8}{3} = -2 + \frac{8}{3} = \frac{-6+8}{3} = \boxed{\frac{2}{3}}$$

201 §4.2 #s 25, 27, 28, 33a, 37, 41

(25) $\int_0^1 (x^3 - 3x^2)\,dx$

$\Delta x = \frac{1}{n} \quad x_k = \frac{k}{n}$

$\sum f(x_k)\Delta x = \Delta x \sum f(x_k) = \frac{1}{n}\sum\left(\left(\frac{k}{n}\right)^3 - 3\left(\frac{k}{n}\right)^2\right)$

$= \frac{1}{n}\left[\frac{1}{n^3}\sum k^3 - \frac{3}{n^2}\sum k^2\right] \xrightarrow{n \to \infty} \frac{1}{n^4}\cdot\frac{n^4}{4} - \frac{3}{n^3}\cdot\frac{n^3}{3}$

$= \frac{1}{n^4}\sum k^3 - \frac{3}{n^3}\sum k^2$

$= \frac{1}{4} - 1 = \boxed{-\frac{3}{4}}$

(28) Prove $\int_a^b x^2\,dx = \frac{b^3 - a^3}{3}$

$\Delta x = \frac{b-a}{n}, \quad x_k = a + \frac{b-a}{n}k \longrightarrow$

$\Delta x \sum f(x_k) = \frac{b-a}{n}\sum\left(a + \frac{b-a}{n}k\right)^2$

$= \frac{b-a}{n}\sum\left(a^2 + 2a\frac{b-a}{n}k + \left(\frac{b-a}{n}\right)^2 k^2\right)$

$= \frac{b-a}{n}\left[\sum a^2 + 2a\frac{b-a}{n}\sum k + \frac{(b-a)^2}{n^2}\sum k^2\right]$

$= \frac{b-a}{n}\left[a^2 n + 2a\frac{b-a}{n}\cdot\frac{n^2+ \cdots}{2} + \frac{(b-a)^2}{n^2}\cdot\frac{n^3+\cdots}{3}\right]$

$+ 2a\frac{(b-a)^2}{n^2}\cdot\frac{n^2+\cdots}{2} + \frac{(b-a)^3}{n^3}\cdot\frac{n^3+\cdots}{3}$

$\xrightarrow{n\to\infty} \frac{b-a}{n}\cdot a^2 n + 2a\frac{(b-a)^2}{n^2}$

201 §4.2 #s ~~27~~, 28, 33 a, 37, 41

28

$$\xrightarrow{n\to\infty} (b-a)a^2 + a(b-a)^2 + \frac{(b-a)^3}{3}$$

$$= (b-a)\left[a^2 + a(b-a) + \frac{(b-a)^2}{3}\right]$$

$$= (b-a)\left[a^2 + ab - a^2 + \tfrac{1}{3}(b^2 - 2ab + a^2)\right]$$

$$= (b-a)\left[ab + \tfrac{1}{3}b^2 - \tfrac{2}{3}ab + \tfrac{1}{3}a^2\right]$$

$$= (b-a)\left[\tfrac{1}{3}ab + \tfrac{1}{3}b^2 + \tfrac{1}{3}a^2\right]$$

$$(x-y)(x^2+xy+y^2) = x^3 - y^3$$

$$= \tfrac{1}{3}(b-a)\left[b^2 + ab + a^2\right]$$

$$= \tfrac{1}{3}\left[b^3 - a^3\right]$$

~~28~~ 27 Prove $\int_a^b x\,dx = \frac{b^2-a^2}{2}$

$$\frac{b-a}{n}\sum_{k=1}^{n}\left(a + \frac{b-a}{n}k\right) = \frac{b-a}{n}\sum_{k=1}^{n} a + \frac{b-a}{n}\sum \frac{b-a}{n}k$$

$$= \frac{b-a}{n}\cdot an + \frac{b-a}{n}\cdot\frac{b-a}{n}\cdot\frac{n^2+n}{2}$$

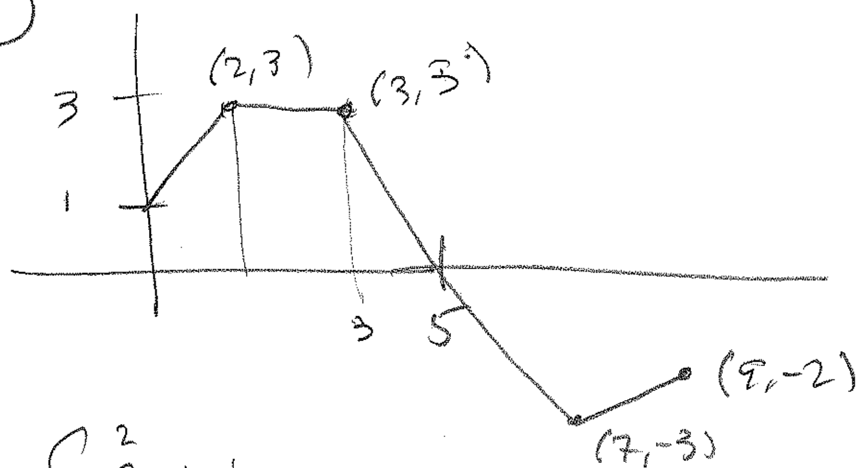
$$= (b-a)a + \frac{(b-a)^2}{2n^2}\cdot(n^2+n) \xrightarrow{n\to\infty}$$

$$ab - a^2 + \frac{b^2 - 2ab + a^2}{2} = \frac{2ab - 2a^2 + b^2 - 2ab + a^2}{2}$$

$$= \frac{b^2 - a^2}{2}$$ ✓

201 §4.2 #s 33a, 37, 41

33a

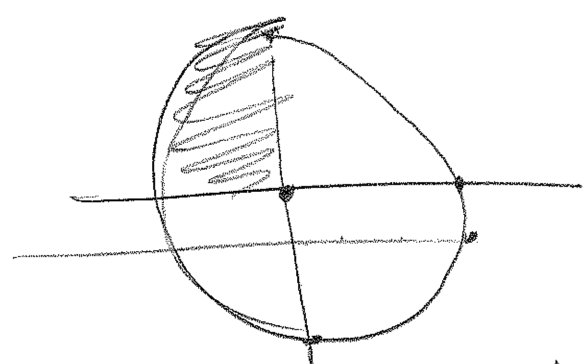



(a) $\int_0^2 f(x)\,dx$

is trapezoid: $b_1 = 1$, $b_2 = 3$, $h = 2$

$\frac{1}{2}(b_1+b_2)h = \frac{1}{2}(1+3)(2) = \frac{1}{2}(4)(2) = \boxed{4}$

37 $\int_{-3}^{0} \left(1 + \sqrt{9-x^2}\right) dx$



$y = 1 + \sqrt{9-x^2}$

$y - 1 = \sqrt{9-x^2}$

$(y-1)^2 = 9 - x^2$

$x^2 + (y-1)^2 = 9$

$r = 3$, $(h,k) = (0,1)$

$y = 1 + \sqrt{9-x^2}$ is top $\frac{1}{2}$.

$\int_{-3}^{0}$ is shaded area $= \frac{1}{4}$ circle!

$\frac{1}{4}(\pi r^2) = \frac{1}{4}(\pi(3^2))$

$= \boxed{\frac{9\pi}{4}}$

201 §4.2 #41

41 $\int_{\pi}^{\pi} \sin^2 x \cos^4 x \, dx \boxed{= 0}$