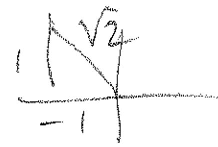
201 §4.1 #4 Doh! I just did #32 / oh well continuez vous

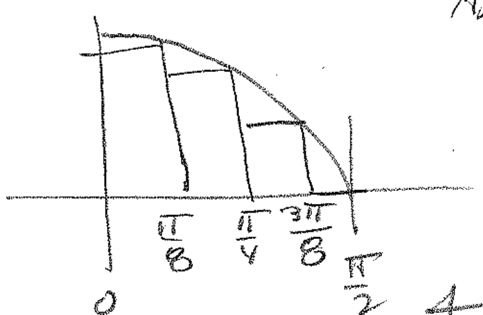
(a) Estimate area under $f(x) = \cos x$, from $x=0$ to $x=\frac{\pi}{2}$ using $4=n$ rectangles and right end points. Sketch. Is this an over- or under-estimate?

$$\frac{b-a}{n} = \frac{\frac{\pi}{2}-0}{4} = \frac{\pi}{8} = \Delta x$$



k	x_k	$f(x_k)$
0	0	$\cos(0)=1$
1	$\frac{\pi}{8}$	$\cos(\frac{\pi}{8}) = \sqrt{\frac{1+\cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}}$
2	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$
3	$\frac{3\pi}{8}$	$\cos(\frac{3\pi}{8}) = \sqrt{\frac{1+\cos\frac{3\pi}{4}}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}}$
4	$\frac{\pi}{2}$	$\cos\frac{\pi}{2} = 0$

$$\text{Area} \approx \sum_{k=1}^{4} f(x_k)\Delta x = \Delta x \sum_{k=1}^{4} f(x_k)$$



$$= \frac{\pi}{8}\left[\frac{\sqrt{2+\sqrt{2}}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2-\sqrt{2}}}{2} + 0\right]$$

$$= \frac{\pi}{8}\left[\frac{\sqrt{2+\sqrt{2}} + \sqrt{2} + \sqrt{2-\sqrt{2}}}{2}\right]$$

$$= \frac{\pi}{16}\left[\sqrt{2+\sqrt{2}} + \sqrt{2} + \sqrt{2-\sqrt{2}}\right] \approx \boxed{.790766201}$$

(b) This is an underestimate, due to right endpts & $\cos x$ is decreasing on the interval $(0, \frac{\pi}{2}]$.

201 § 4.1 #4 For reals

(4) (a) Same as #3a, but for $f(x)=\sqrt{x}$ on $[0,4]$

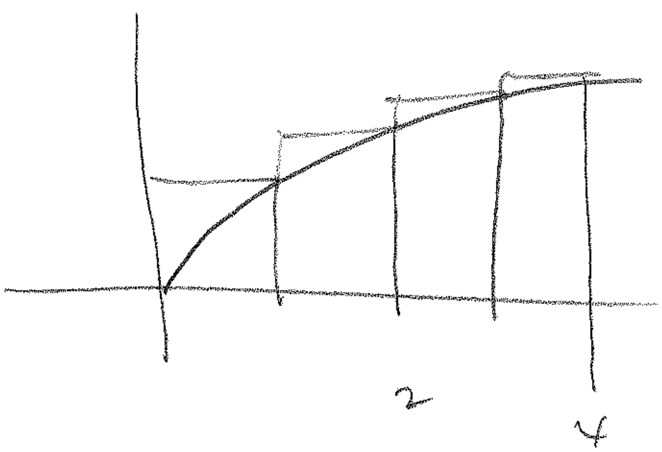
$\Delta x = \frac{b-a}{n} = \frac{4}{4} = 1 = \Delta x$, $x_k = 0 + k\Delta x = k$

k	x_k	$f(x_k)$
0	0	0
1	1	1
2	2	$\sqrt{2}$
3	3	$\sqrt{3}$
4	4	2

$R_4 = \text{Area} \approx \sum_{k=1}^{4} f(x_k)\Delta x = \Delta x \sum_{k=1}^{4} f(x_k)$

$= 1\left[1 + \sqrt{2} + \sqrt{3} + 2\right]$

$= 3 + \sqrt{2} + \sqrt{3} \approx \boxed{6.14626437}$

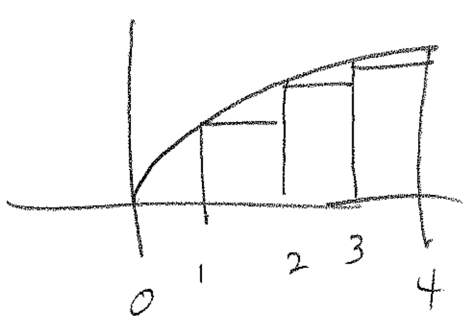


It's an overestimate right endpts & increasing.

(b) $L_4 = \sum_{k=0}^{3} f(x_k)\Delta x = \Delta x \sum_{k=0}^{3} f(x_k) = 0 + 1 + \sqrt{2} + \sqrt{3}$

$= 1 + \sqrt{2} + \sqrt{3}$

$\approx \boxed{4.14626437}$



This is an underestimate Left endpoints & increasing function