201 §3.4 #s 9–35, 45, 47, 49, 51, 53, 55

#s 9–30 Find lim or $\nexists$ lim.

(9) $\lim_{x\to\infty} \frac{3x-2}{2x+1} = \boxed{\frac{3}{2}}$

(11) $\lim_{x\to-\infty} \frac{x-2}{x^2+1} = \boxed{0}$

(13) $\lim_{t\to\infty} \frac{\sqrt{t}+t^2}{2t-t^2} = \boxed{-1}$ $\left(\frac{t^2}{-t^2} = -1 \xrightarrow{t\to\infty} -1 \right)$

(15) $\lim_{t\to\infty} \frac{(2x^2+1)^2}{(x-1)^2(x^2+x)} = \frac{4x^4+\cdots}{x^4+\cdots} \xrightarrow{x\to\infty} \boxed{4}$

(17) $\frac{\sqrt{9x^6-x}}{x^3+1} = \frac{3x^3\sqrt{1-\frac{1}{9x^5}}}{x^3\left(1+\frac{1}{x^3}\right)} \xrightarrow{x\to\infty} \boxed{3}$

(19) $\sqrt{9x^2+x} - 3x =$

$\left(\sqrt{9x^2+x} - 3x\right)\left(\frac{\sqrt{9x^2+x}+3x}{\sqrt{9x^2+x}+3x}\right) = \frac{9x^2+x-9x^2}{\sqrt{9x^2+x}+3x}$

$= \frac{x}{3x\sqrt{1+\frac{1}{9x}}+3x} \xrightarrow{x\to\infty} \boxed{\frac{1}{3}}$

(21)

201 §3.4 #s 21-35, 45, 47, 49, 51, 53, 55

(21) $\sqrt{x^2+ax} - \sqrt{x^2+bx}$

$$= \left(\sqrt{x^2+ax} - \sqrt{x^2+bx}\right)\left(\frac{\sqrt{x^2+ax} + \sqrt{x^2+bx}}{\sqrt{x^2+ax} + \sqrt{x^2+bx}}\right)$$

$$= \frac{x^2+ax-(x^2+bx)}{\sqrt{x^2+ax} + \sqrt{x^2+bx}} = \frac{ax-bx}{|x|\left(\sqrt{1+\frac{a}{x}} + \sqrt{1+\frac{b}{x}}\right)}$$

$$= \frac{x(a-b)}{x\left(\sqrt{1+\frac{a}{x}} + \sqrt{1+\frac{b}{x}}\right)} \xrightarrow{x \to \infty} \boxed{a-b}$$

(23) $$\frac{x^4-3x^2+x}{x^3-x+2} \xrightarrow{x\to\infty} \infty, \text{ i.e., } \nexists.$$

(25) $$(x^4+x^5) \xrightarrow{x\to -\infty} -\infty, \text{ i.e., } \nexists.$$

(27) $$(x-\sqrt{x}) \xrightarrow{x\to\infty} \infty, \text{ i.e., } \nexists.$$

(29) $$\left(x \sin\left(\frac{1}{x}\right)\right) \xrightarrow{x\to\infty} ?$$

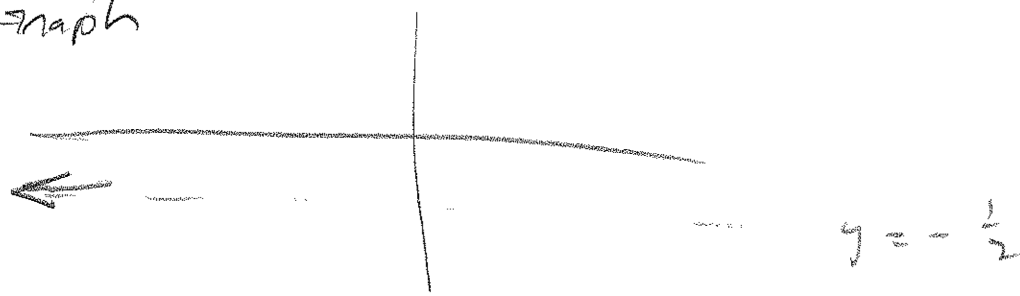
$$\frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = \frac{\sin u}{u} \xrightarrow{u\to 0} \boxed{1}$$

$u = \frac{1}{x}$

201 §3.4 #s 31–35, 45, 47, 49, 51, 53, 55

(31) Estimate $\lim_{x\to-\infty}\left(\sqrt{x^2+x+1}+x\right)$ by

(a) Graph



(b) Numerically $(-1000, -.4996)$

$(-10^6, -.5)$ (Calculator)

(c) Analytically.

$$\sqrt{x^2+x+1}+x=\left(\sqrt{x^2+x+1}+x\right)\left(\frac{\sqrt{x^2+x+1}-x}{\sqrt{x^2+x+1}-x}\right)$$

$$=\frac{x^2+x+1-x^2}{\sqrt{x^2+x+1}-x}=\frac{x+1}{|x|\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}-x}$$

$$=\frac{x+1}{-x\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}-x}=\frac{x+1}{-x\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1\right)}$$

$$=\frac{x\left(1+\frac{1}{x}\right)}{-x\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1\right)}\xrightarrow{x\to\infty}\boxed{-\frac{1}{2}}$$

201 §3.4 #5 33, 38, 45, 47, 49, 5#, 53, 55

#s 33–38 Find H.A., V.A. Check w/ graph

33

$$y = \frac{2x+1}{x-2}$$

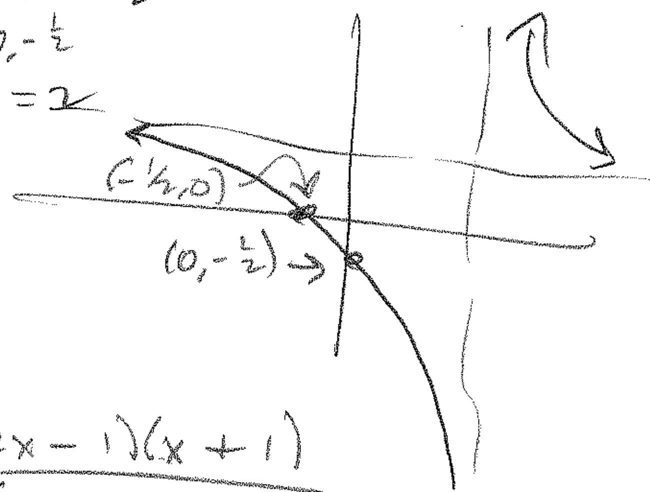
x-int: $x = -\frac{1}{2}$

y-int: $(0, -\frac{1}{2})$

$y = 2$

V.A.: $x = 2$

H.A.: $y = 2$



$x = 2$

38

$$y = \frac{2x^2 + x - 1}{x^2 + x - 2} = \frac{(2x-1)(x+1)}{(x-1)(x+2)}$$

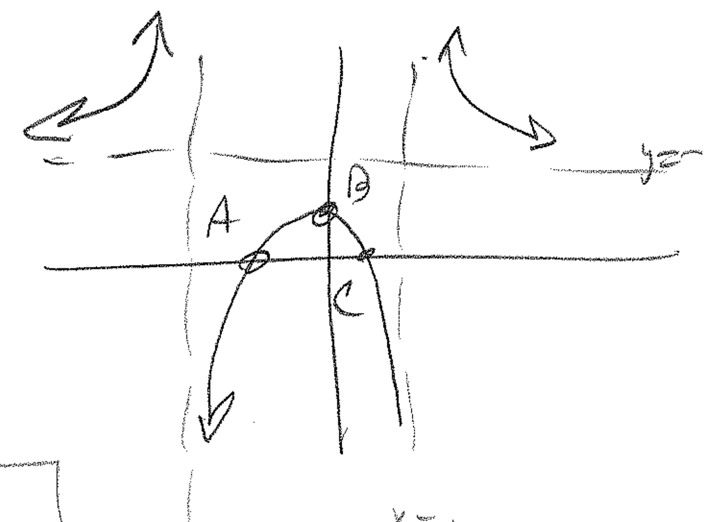
V.A.: $x = 1$, $x = -2$

H.A.: $y = 2$

x-int: $\boxed{(\frac{1}{2}, 0)}$ C $\boxed{(-1, 0)}$ A

y-int: $\boxed{(0, \frac{1}{2})}$ B

A



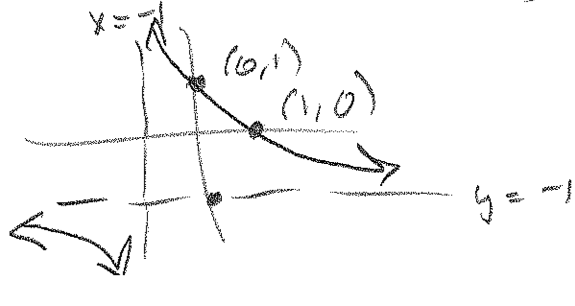
$x = 1$

$x = -2$

45 Use H.A. & Calc. to sketch $\boxed{HA: y = -1}$

$$y = \frac{1-x}{1+x} \qquad y' = \frac{-1(x+1) - (1-x)}{(1+x)^2} = \frac{-x-1-1+x}{(x+1)^2}$$

$$= -\frac{2}{(x+1)^2} \Longrightarrow y'' = -2\left(-2(x+1)^{-3}\right) = \frac{4}{(x+1)^3}$$



201 §2.4 #s 47, 49, 51, 53, 55

(47) $y = \frac{x}{x^2+1}$ No V.A. (0,0),
H.A.: $y=0$

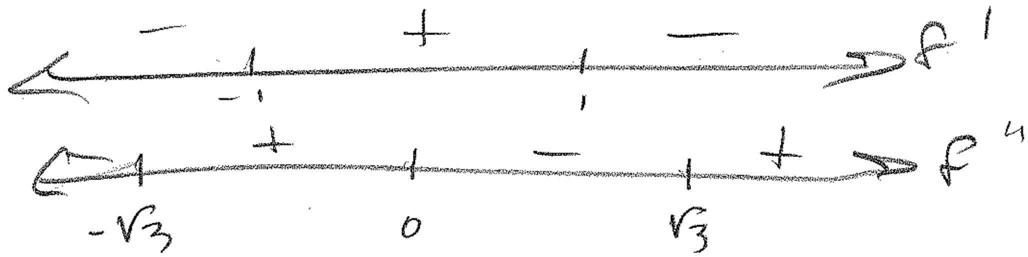
$$y' = \frac{1(x^2+1) - x(2x)}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = -\frac{x^2-1}{(x^2+1)^2}$$

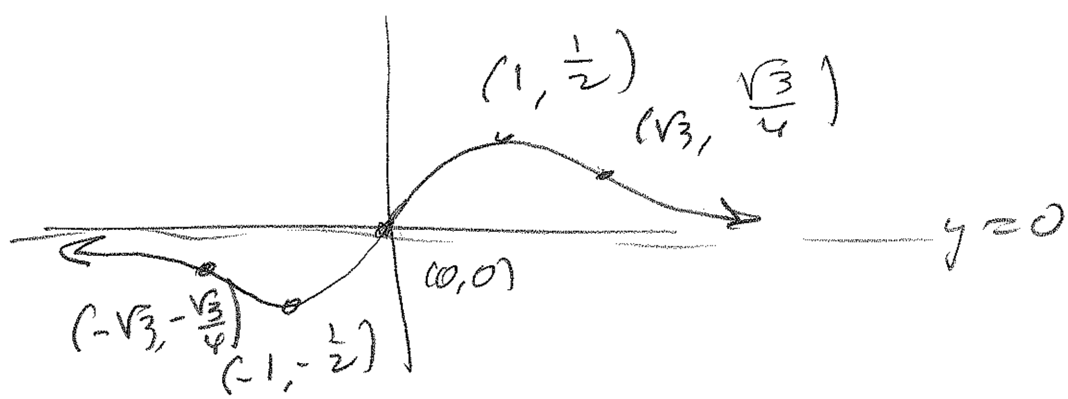
$$y'' = -\left[\frac{2x(x^2+1)^2 - (x^2-1)(2(x^2+1)(2x))}{(x^2+1)^4}\right]$$

$$= -\left[\frac{2x(x^2+1)[x^2+1-2(x^2-1)]}{(x^2+1)^4}\right]$$

$$= -\left[\frac{2x[-2x^2+2+x^2+1]}{(x^2+1)^3}\right]$$

$$= -\left[\frac{2x[-x^2+3]}{(x^2+1)^3}\right] = \frac{2x(x^2-3)}{(x^2+1)^3}$$



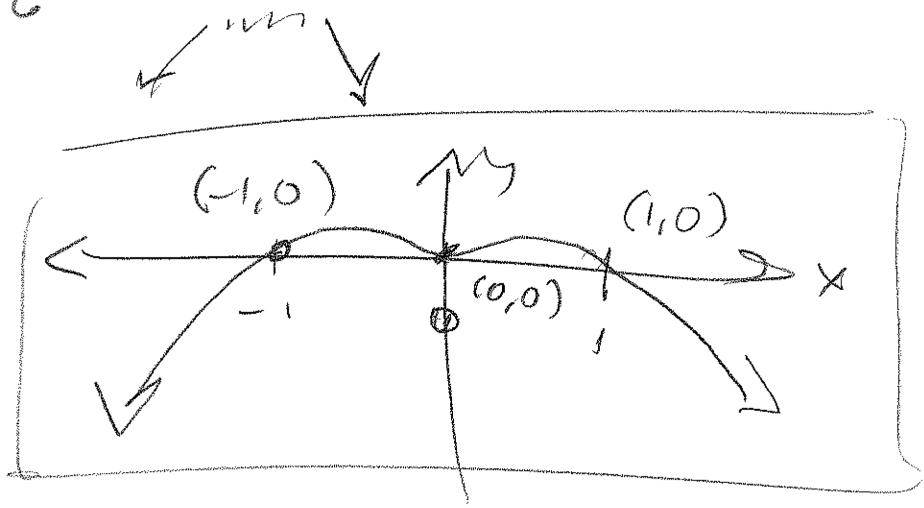


201 §3.4 #s 49, 51, 53, 55

~~#~~s 48–52 E.B. & x-ints & y-ints for rough sketch.

(49) $y = x^4 - x^6$

E.B



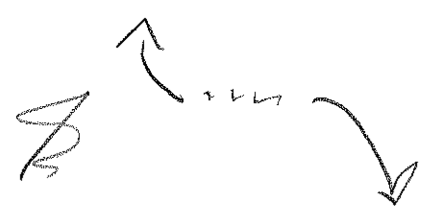
$-x^4(x^2-1) = 0$

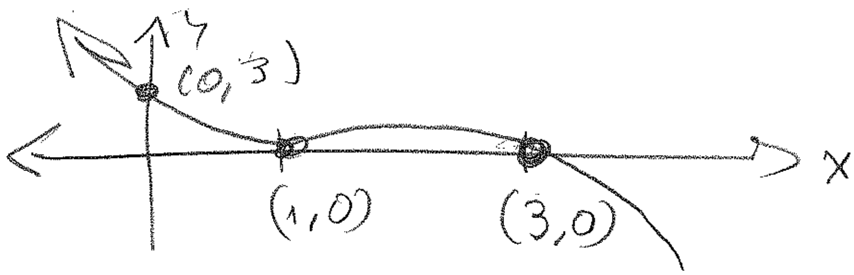
$x = 0, \pm 1$

(51) $y = (3-x)(x+1)^2(1-x)^4$

$= -(x-3)(x^2+1)^2(x-1)^4$

E.B.: $-x \cdot x^4 \cdot x^4 = -x^5$





201 § 3.4 #s 53, 58

53

$f'(2)=0$, $f(2)=-1$, $f(0)=0$

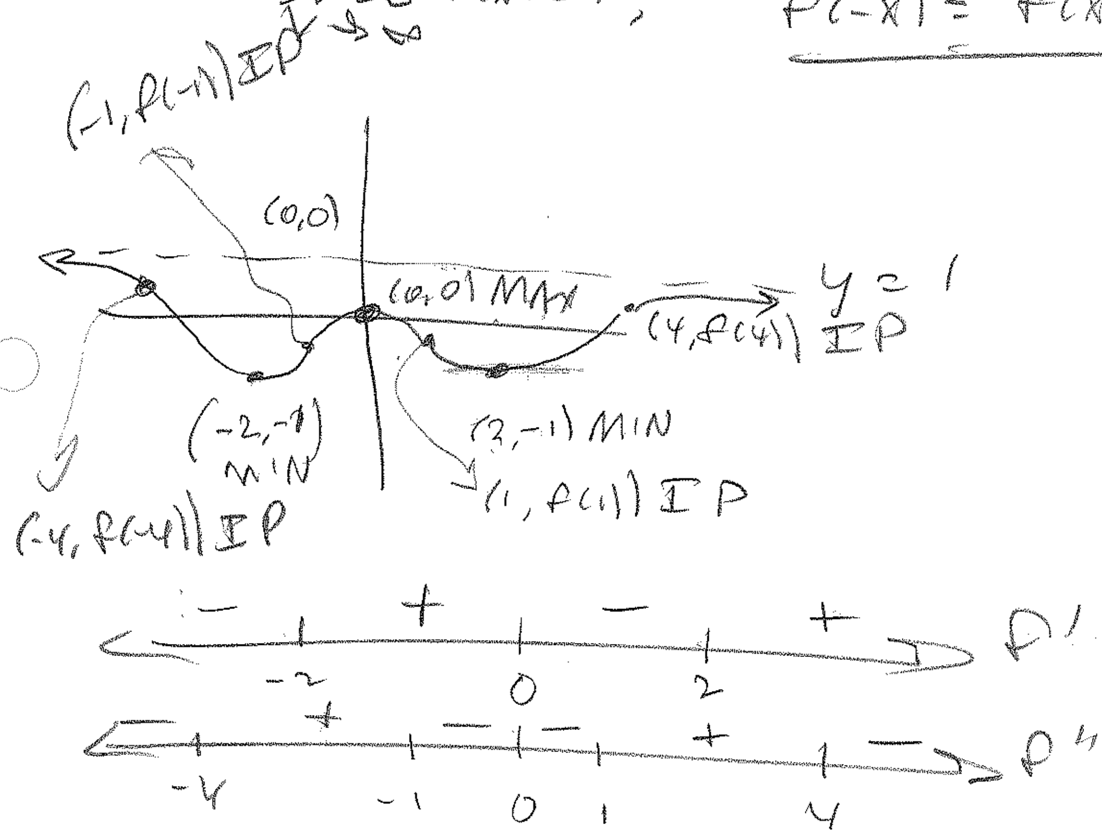
$f'(x)<0$ if $0<x<2$

$f'(x)>0$ if $x>2$

$f''(x)<0$ if $0\le x<1$ OR $x>4$

$f''(x)>0$ if $1<x<4$

$\lim_{x\to\infty} f(x)=1$, $f(-x)=f(x)\ \forall x$ EVEN



201 §3.4 #58

58 $f(1) = f'(1) = 0$ $\lim_{x \to 2^+} f(x) = +\infty$

$\lim_{x \to 0} f(x) = -\infty$ $\lim_{x \to 2^-} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = \infty$ $f''(x) > 0 \ \forall x > 2,$

$\lim_{x \to \infty} f(x) = 0$ $f''(x) < 0 \ \forall x < 0$ and $\forall \ 0 < x < 2$