201 §3.3 II #s 29-39

#s 29-40 Graph it! Show all key features

29) $x^3 - 12x + 2$ $\quad \mathcal{D} = \mathbb{R} = \mathbb{R}$

$f'(x) = 3x^2 - 12 \overset{\text{SET}}{=} 0$ $\qquad f''(x) = 6x \overset{\text{SET}}{=} 0$

$x^2 = 4$ $\qquad x = 0$

$x = \pm 2$

f': $+ \quad -2 \quad - \quad 2 \quad +$

$\nearrow \; -2 \; \searrow \; 2 \; \nearrow$

f'': $- \quad 0 \quad +$

$\cap \; 0 \; \cup$

combine: $-2 \quad 0 \quad 2$

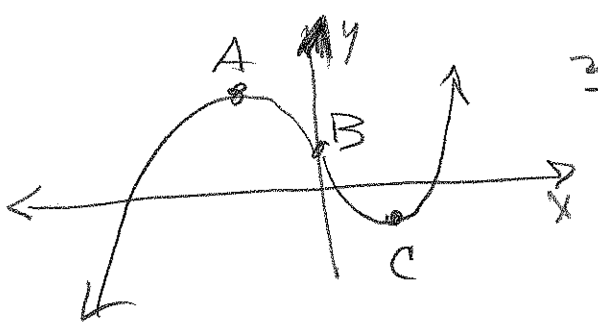
-2	1	0	-12	2
		-2	4	16
	1	-2	-8	18

A

$\boxed{(-2, 18) \text{ MAX}}$

B

$f(0) = 2 \rightarrow \boxed{(0, 2) \text{ I.P.}}$



2	1	0	-12	2
		2	4	-16
	1	2	-8	-14 = C

$\boxed{(2, -14) \text{ MIN}}$

201 §3.3 #s 31–39

(31) $f(x) = -x^4 + 2x^2 + 2$ $\boxed{\text{EVEN!}}$

$f'(x) = -4x^3 + 4x \overset{\text{SET}}{=} 0$

$-4x(x^2 - 1) = 0$

$x = 0, \pm 1$

$f''(x) = -12x^2 + 4 \overset{\text{SET}}{=} 0$

$12x^2 = 4$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{3}}{3}$

f': + | −1 | − | 0 | + | 1 | −

↗ −1 ↘ 0 ↗ 1 ↘

f'': − | $-\frac{\sqrt{3}}{3}$ | + | $\frac{\sqrt{3}}{3}$ | −

∩ $-\frac{\sqrt{3}}{3}$ ∪ $\frac{\sqrt{3}}{3}$ ∩

combine

−1, $-\frac{\sqrt{3}}{3}$, 0, $\frac{\sqrt{3}}{3}$, 1

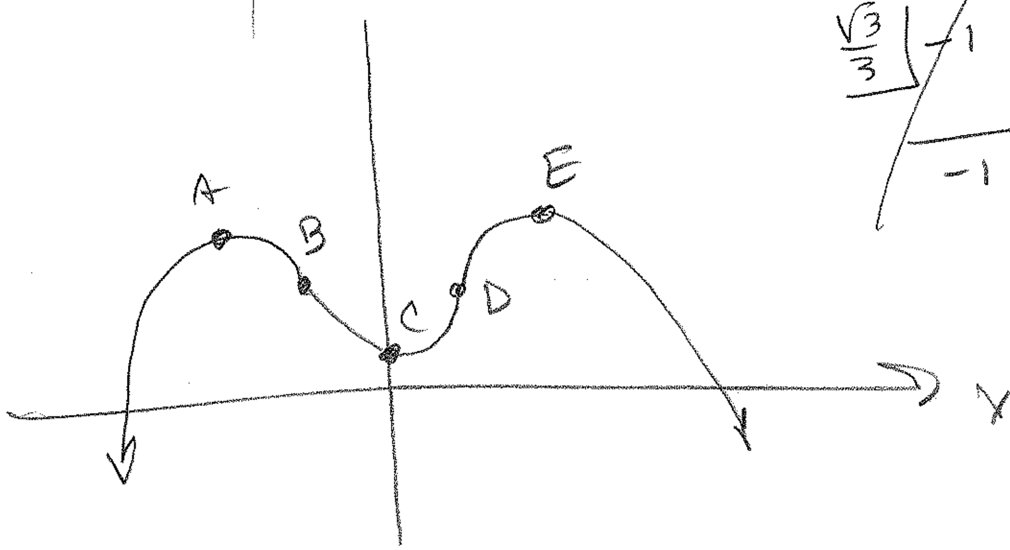
$f(0) = 2$

~~−1~~	−1	0	2	2
		1	−1	−1
	−1	1	1	1

~~(−1, 1) MAX~~

→ ~~(1, 1) MAX~~

~~$\frac{\sqrt{3}}{3}$~~	−1	0	2	2
		$-\frac{\sqrt{3}}{3}$	$-\frac{1}{3}$	$\frac{5\sqrt{3}}{9}$
	−1	$-\frac{\sqrt{3}}{3}$	$\frac{5}{3}$	$2+\frac{5\sqrt{3}}{9}$



A = (−1, 3) MAX D = (1, 3) MAX

B = $\left(-\frac{\sqrt{3}}{3}, \frac{23}{9}\right)$ IP E = $\left(\frac{\sqrt{3}}{3}, \frac{23}{9}\right)$ IP

C = (0, 2)

201 §3.3 II #s 31-39

(31) Re-do f(x)'s

$$\begin{array}{r|rrrrr} -1 & -1 & 0 & 2 & 0 & 2 \\ & & 1 & -1 & -1 & 1 \\ \hline & -1 & 1 & 1 & -1 & 3 \end{array} \qquad (-1, 3) \Rightarrow (1, 3)$$

$$\begin{array}{r|rrrrr} \frac{\sqrt{3}}{3} & -1 & 0 & 2 & 0 & 2 \\ & & -\frac{\sqrt{3}}{3} & -\frac{1}{3} & \frac{5\sqrt{3}}{9} & \frac{5}{9} \\ \hline & -1 & -\frac{\sqrt{3}}{3} & \frac{5}{3} & \frac{5\sqrt{3}}{9} & \frac{23}{9} \end{array} \qquad \left(\frac{\sqrt{3}}{3}, \frac{23}{9}\right)$$

$$\Rightarrow \left(-\frac{\sqrt{3}}{3}, \frac{23}{9}\right)$$

201 §3.3 ~~II~~ #s 33–39

(33) $h(x) = (x+1)^5 - 5x - 2$

$h'(x) = 5(x+1)^4 - 5 \overset{\text{SET}}{=} 0$

$(x+1)^4 = 1$

$x+1 = \pm 1$

$x = -1 \pm 1 \to 0, -2$

$h''(x) = 2(x+1)^3 \overset{\text{SET}}{=} 0$

$x = -1$

$h(0) = 1 - 2 = -1$ $\boxed{(0,-1)}$

$h(-2) = (-1) + 10 - 2 = 7$ $\boxed{(-2,7)}$

$h(-1) = 5 - 2 = 3$ $\boxed{(-1,3)}$

f': $+$ $\nearrow$ -2 $-$ $\searrow$ 0 $+$ $\nearrow$

f'': $-$ $\frown$ -1 $+$ $\smile$

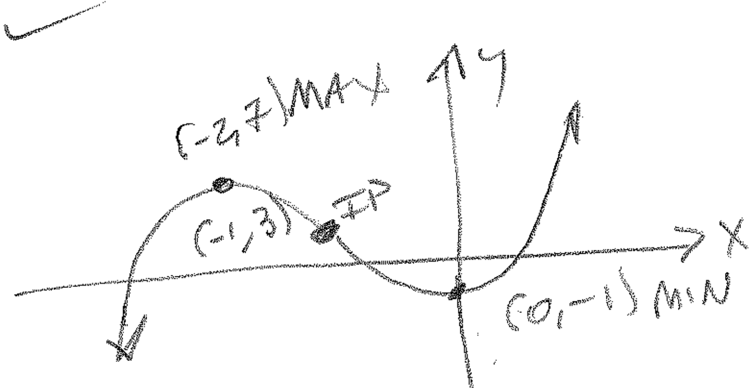
Combine: -2, -1, 0

Check: $(x+1)^4 - 1$

$= ((x+1)^2 - 1)((x+1)^2 + 1)$

$= (x+1-1)(x+1+1)((x+1)^2+1)$

$x = 0, -2$ ✓



201 § 3.3 II #s 35–39

(35)

$F(x) = x(6-x)^{\frac{1}{2}}$

$F'(x) = (6-x)^{\frac{1}{2}} + x\left(\frac{1}{2}(6-x)^{-\frac{1}{2}}(-1)\right)$

$= \left(\sqrt{6-x}\right)\left(\frac{2\sqrt{6-x}}{2\sqrt{6-x}}\right) - \frac{x}{2\sqrt{6-x}}$

$= \frac{2(6-x)-x}{2\sqrt{6-x}} = \frac{12-2x-x}{2\sqrt{6-x}} = \frac{12-3x}{2\sqrt{6-x}}$

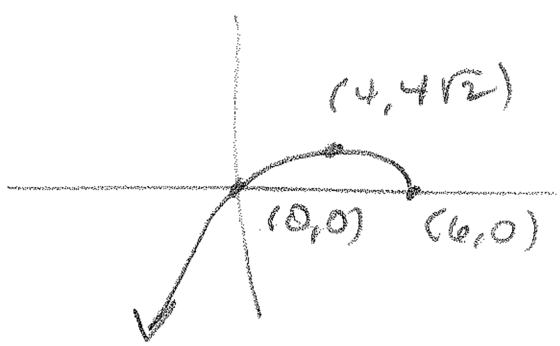
$\overset{\text{SET}}{=} 0 \Longrightarrow x = 4$

$F''(x) = \frac{-3\left(2\sqrt{6-x}\right) - (12-3x)(2)\left(\frac{1}{2}(6-x)^{-\frac{1}{2}}(-1)\right)}{4(6-x)}$

$= \cdots$ Wow!

$f'' < 0$ on its domain

Max of $4(2^{\frac{1}{2}}) = 4\sqrt{2}$ @ $x = 4$



201 §3.3 II #s 35–39

(35) $F(x) = x\sqrt{6-x} = x(6-x)^{\frac{1}{2}}$ $\mathcal{D} = \{x \mid 6-x \geq 0\}$ $= \{x \mid 6 \geq x\}$ $= (-\infty, 6]$

$F'(x) = (6-x)^{\frac{1}{2}} + x\left(\frac{1}{2}(6-x)^{-\frac{1}{2}}(-1)\right)$

$= (6-x)^{\frac{1}{2}} - \dfrac{x}{2(6-x)^{1/2}}$

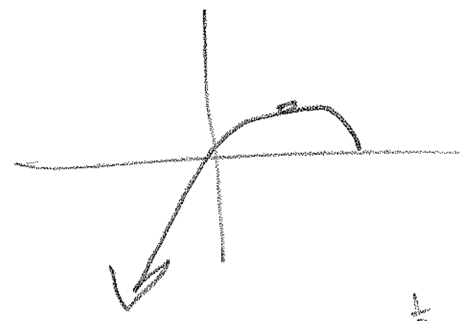
$= \dfrac{6-x-x}{2\sqrt{6-x}} = \dfrac{6-2x}{2\sqrt{6-x}}$

Goes vertical @ edge of its domain $(x=6)$

$6-2x=0$

$x=3$ $3\sqrt{6-3} = 3\sqrt{3}$

$(3, 3\sqrt{3})$ MAX



$\xleftarrow{\quad + \quad \underset{3}{|} \quad - \quad}$ F'

$\nearrow$ 3 $\searrow$

$F''(x) = \dfrac{-2(2\sqrt{6-x}) - (6-2x)\left[2\left(\frac{1}{2}(6-2x)^{-\frac{1}{2}}(-2)\right)\right]}{4(6-x)}$

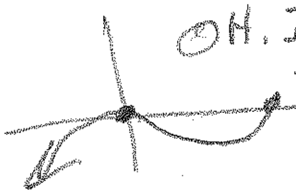
$= \dfrac{\left(-4\sqrt{6-x}\right)\left(\frac{\sqrt{6-x}}{\sqrt{6-x}}\right) - (6-2x)\left(\frac{-2}{\sqrt{6-x}}\right)}{4(6-x)}$

$= \dfrac{-4(6-x) + 12-4x}{4(6-x)^{3/2}} = \dfrac{-24+4x+12-4x}{4(6-x)^{3/2}} = \dfrac{-12}{4(6-x)^{3/2}}$

$f'' < 0$ on its domain. ✓

Missing something

OH, I mis-interpreted the sign pattern on f!



$F(0) = F(6) = 0$

Didn't find a min!

201 §3.3 II #s 37, 39

(37) $f(x) = x^{\frac{1}{3}}(x+4) = 0$ @ $x = 0, -4$

$\cancel{f'(x)} = x^{\frac{4}{3}} + 4x^{\frac{1}{3}}$

$\Longrightarrow f'(x) = \frac{4}{3}x^{\frac{1}{3}} + \frac{4}{3}x^{-\frac{2}{3}} = \frac{4}{3}x^{-\frac{2}{3}}(x+1)$

$\nexists$ @ $x = 0$

$= 0$ @ $x = -1$

No sign change f' @ $x = 0$

$x^{-\frac{2}{3}} = \frac{1}{x^{\frac{2}{3}}} = \frac{1}{(x^{\frac{1}{3}})^2}$

See the 2? It's even.

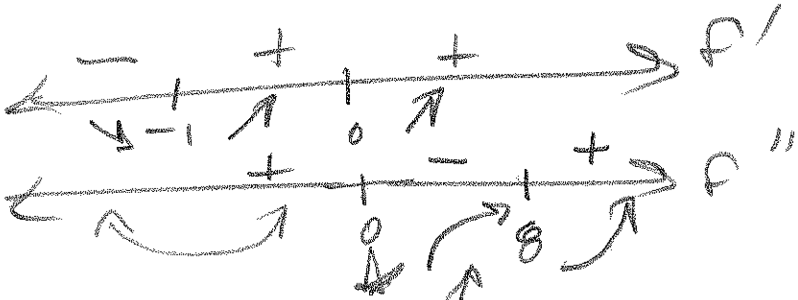
$f''(x) = \frac{4}{9}x^{-\frac{2}{3}} - \frac{8}{9}x^{-\frac{5}{3}}$

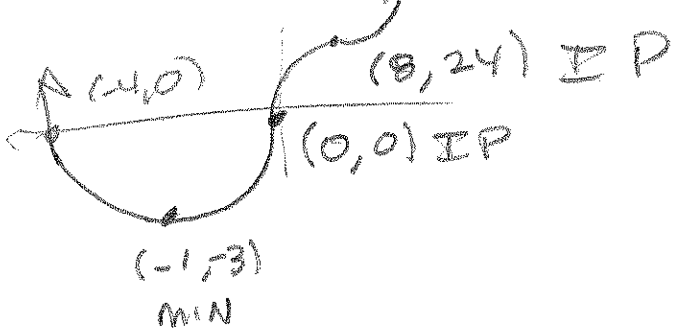
$= \frac{4}{9}x^{-\frac{5}{3}}(x^{\frac{1}{3}} - 2)$

$\nexists$ @ $x = 0$

$= 0$ @ $x^{\frac{1}{3}} = 2$

$x = 8$





201 § 3.3 IV #39

39) $f(\theta) = 2\cos\theta + \cos^2\theta \quad 0 \le \theta \le 2\pi$

$f(\theta) = 0 \longrightarrow$

$\cos\theta(\cos\theta + 2) = 0 \longrightarrow$

$\cos\theta = 0 \Longrightarrow$ $\qquad \cos\theta + 2 = 0$ Never

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ x-ints

$f'(\theta) = -2\sin\theta + 2\cos\theta(-\sin\theta)$

$= -2\sin\theta(1 + \cos\theta)$

$-2\sin\theta = 0 \qquad \cos\theta = -1$

$\theta = 0, \pi, 2\pi \qquad \theta = \pi$

f': 0 $(-)$ π $(+)$ 2π

$-2\sin\frac{\pi}{2}\left(1 + \cos\frac{\pi}{2}\right)$

$= -2(1) = -2 \quad -$

$-2\sin\left(\frac{3\pi}{2}\right)\left(1 + \cos\frac{3\pi}{2}\right)$

$+2(1) = 2 \quad +$

$f''(\theta) = -2\cos\theta - 2\left[-\sin\theta\sin\theta + \cos\theta\cos\theta\right]$

$= -2\cos\theta - 2\left[\cos^2\theta - \sin^2\theta\right]$

$= -2\cos\theta - 2\cos^2\theta + 2(1 - \cos^2\theta)$

$= -2\cos\theta - 2\cos^2\theta + 2 - 2\cos^2\theta$

$= -4\cos^2\theta - 2\cos\theta + 2 \overset{\text{SET}}{=} 0$

$-2(2\cos^2\theta - \cos\theta + 1) = 0$

$b^2 - 4ac = 1 - 4(2)(1)$
$= -7$

$\Longrightarrow 2u^2 - u + 1 = 0 \Longrightarrow u = 1 \pm$

No real zeros $\therefore$ ALWAYS NEGATIVE

Not making sense

201 §3.3 III #39

39 $f'(\theta) = -2\sin\theta + 2\cos\theta(-\sin\theta)$

$= -2\sin\theta - 2\sin\theta\cos\theta = -2\sin\theta(1+\cos\theta) \overset{SET}{=} 0 \longrightarrow$

$-2\sin\theta = 0$ $\qquad$ $\cos\theta + 1 = 0$

$\sin\theta = 0$ $\qquad$ $\cos\theta = -1$

$\theta \in \{0, \pi, 2\pi\}$ $\qquad$ $\theta \in \{\pi\}$

$(2\pi, 3)$ MAX

$(0, 3)$ MAX

$A = (\frac{\pi}{3}, \frac{5}{4})$ IP

$B = (\pi, -1)$ MIN

$C = (\frac{5\pi}{3}, \frac{5}{4})$ IP

$f''(\theta) = -2\cos\theta \cancel{+2\cos^2\theta} + 2\sin^2\theta$

$= -2\cos\theta - 2(\cos^2\theta - \sin^2\theta)$

$= -2\cos\theta - 2(\cos^2\theta - (1-\cos^2\theta))$

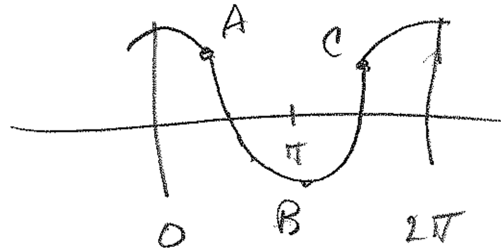
$= -2\cos\theta - 2(\cos^2\theta - 1 + \cos^2\theta)$

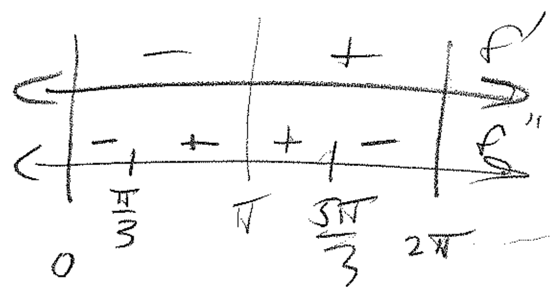
$= -2\cos\theta - 2(2\cos^2\theta - 1)$

$= -4\cos^2\theta - 2\cos\theta + 2$

$= -2(2\cos^2\theta + \cos\theta - 1)$

$= 2(2u^2 + u - 1) \overset{SET}{=} 0$





$\Rightarrow (2u-1)(u+1) = 0$

$(2\cos\theta - 1)(\cos\theta + 1)$

$2u = 1$ $\qquad$ $u = -1$

$u = \frac{1}{2}$ $\qquad$ $\cos\theta = -1$

$\cos\theta = \frac{1}{2}$ $\qquad$ $\boxed{\theta = \pi}$

$\boxed{\theta = \frac{\pi}{3}, \frac{5\pi}{3}}$

@ $\theta = \pi$, $f''(x)$ doesn't change sign.

$2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$