

201 §3.1

1

No abs. min  
Abs max @ (4, 5)

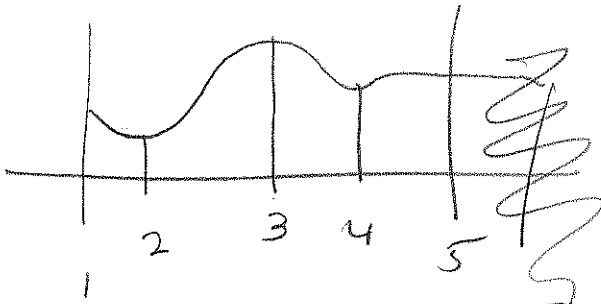
Local min @ (1, 3)  
(2, 2), (5, 3)

Local max @  
(4, 5), (6, 4)

2

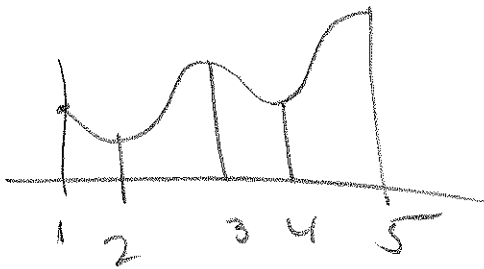
cont'd on [1, 5],

Abs min @ 2, Abs max @ 3, Local min  
at 4



3

Abs max @ 5, abs min @ 2  
loc max @ 3, local min @ 2, 4

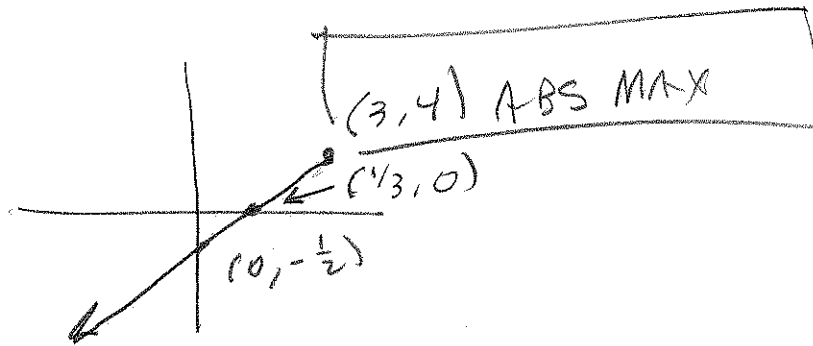


sketch & use it to find  
Abs & local extremes

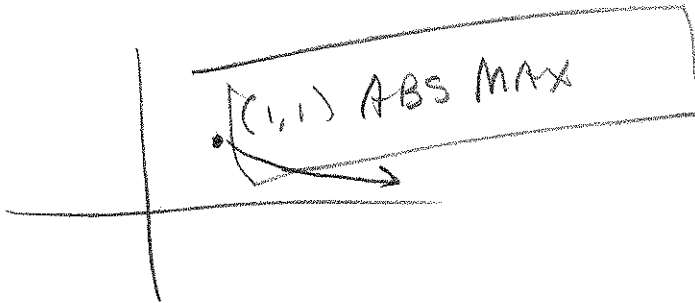
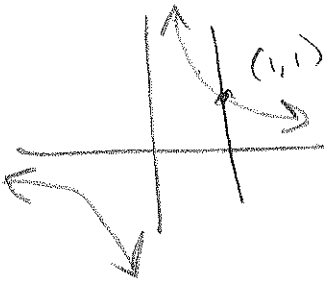
4  $f(x) = \frac{1}{2}(3x - 1) \quad x \leq 3$

$$f(3) = \frac{1}{2}(9 - 1) = \frac{1}{2}(8) = 4$$

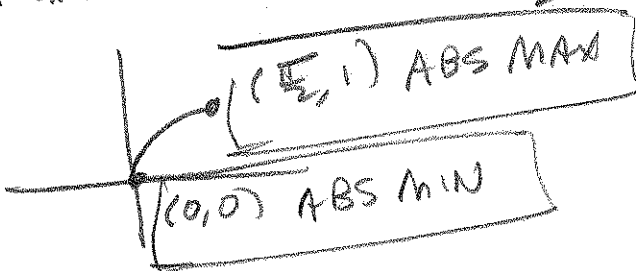
$$f(0) = -\frac{1}{2}$$



5  $f(x) = \frac{1}{x}, \quad x \geq 1$

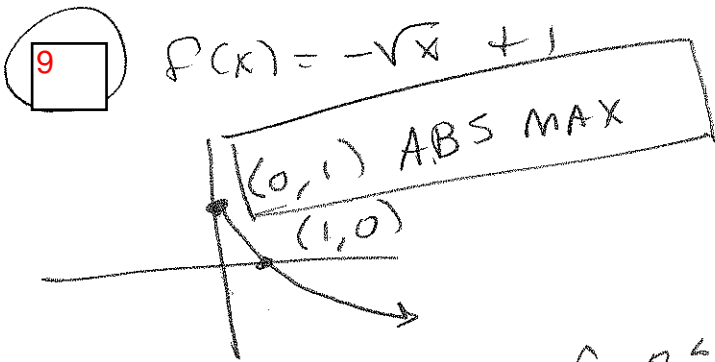
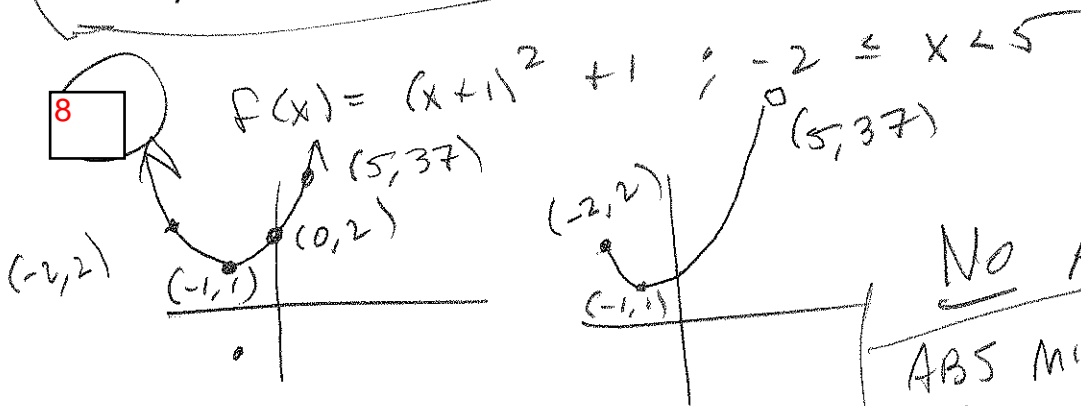
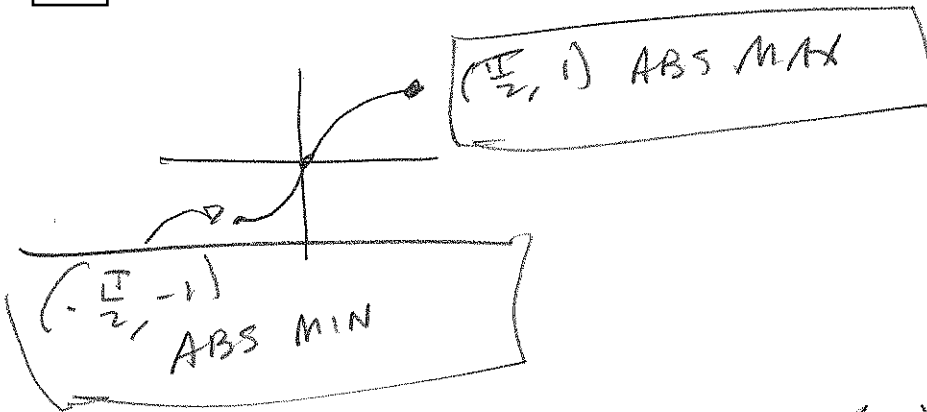


6  $f(x) = \sin x \quad 0 \leq x \leq \frac{\pi}{2}$



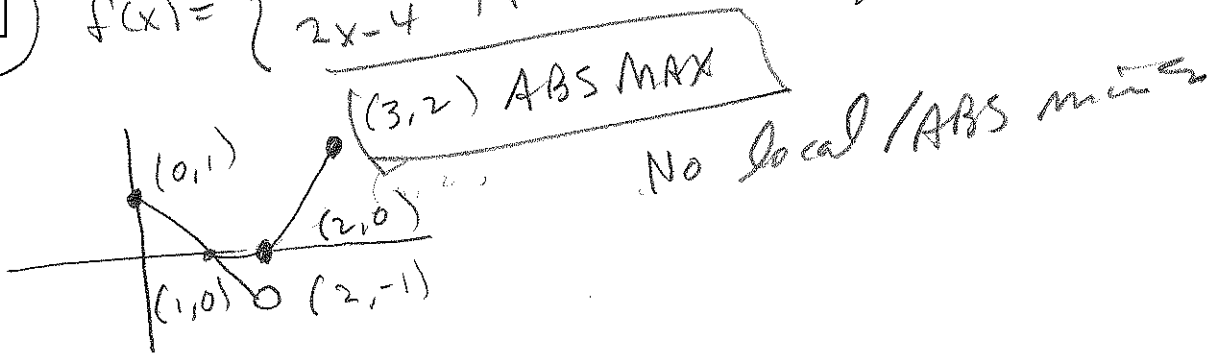
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7  $f(x) = \sin x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$



10  $f(x) = \begin{cases} 1-x & \text{if } 0 \leq x < 2 \\ 2x-4 & \text{if } 2 \leq x \leq 3 \end{cases}$

$1-2 = -1$   
 $2(2)-4 = 0$   
 $2(3)-4 = 2$



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Find critical pts

11

$$f(x) = -\frac{1}{2}x^2 + \frac{1}{3}x + 4$$

$$f'(x) = -x + \frac{1}{3} \stackrel{\text{SET}}{=} 0 \rightarrow x = \frac{1}{3}$$

12

$$f(x) = 2x^3 - 3x^2 - 36x$$

$$f'(x) = 6x^2 - 6x - 36 \stackrel{\text{SET}}{=} 0$$

$$\Rightarrow x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$\Rightarrow x = -2, 3$$

13

$$g(y) = \frac{y-1}{y^2y+1}$$

$$y-1 = 0$$
$$y = 1$$

x-axis

$$y^2y+1 = 0$$

$$y^2y + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + 1$$

$$= \left(y - \frac{1}{2}\right)^2 + \frac{3}{4} \stackrel{\text{SET}}{=} 0$$

~~\*~~ Never, for  $y \in \mathbb{R}$

$$g'(y) = \frac{1(y^2y+1) - (y-1)(2y-1)}{(y^2y+1)^2} = \frac{y^2y+1 - (2y^2-3y+1)}{c^2}$$

$$= \frac{y^2y+1 - 2y^2+3y-1}{c^2} = \frac{-y^2+2y}{c^2} \stackrel{\text{SET}}{=} 0$$

$$\Rightarrow y^2-2y = y(y-2) = 0 \Rightarrow y = 0, 2$$

critical pts.

14

$$f(\theta) = 2 \cos \theta + \sin^2 \theta$$

$$\Rightarrow f'(\theta) = -2 \sin \theta - 2 \sin \theta \cos \theta$$

$$= -2 \sin \theta (1 + \cos \theta)$$

$$\sin \theta = 0$$

$$\theta = \pm \pi, \pm 2\pi, \dots$$

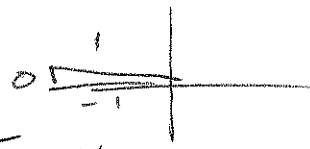
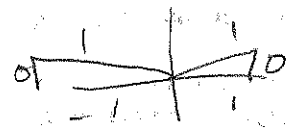
$$n\pi, n \in \mathbb{Z}$$

$$\cos \theta = -1$$

$$\theta = \pm \pi, \pm 3\pi, \dots$$

$$\theta = (2n+1)\pi, n \in \mathbb{Z}$$

$$\text{C.V.s: } \{n\pi, n \in \mathbb{Z}\}$$



45-56 Find abs max, min on [a,b]

15

$$f(x) = 2x^3 - 3x^2 - 12x + 1 \text{ on } [2, 3]$$

$$f'(x) = 6x^2 - 6x - 12 \stackrel{\text{SET}}{=} 0$$

$$\Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow (x-2)(x+1) = 0$$

$$x = -1, 2$$



$$\begin{array}{r} 2 \overline{) 2 \quad -3 \quad -12 \quad 1} \\ \underline{-2 \quad 5 \quad 7} \\ 2 \quad -5 \quad -7 \end{array} \quad \boxed{8 = f(-1)} \quad \text{ABS MAX}$$

$$\begin{array}{r} -2 \overline{) 2 \quad -3 \quad -12 \quad 1} \\ \underline{-4 \quad 14 \quad -4} \\ 2 \quad -7 \quad 2 \end{array} \quad \boxed{-3 = f(2)}$$

$$\begin{array}{r} 2 \overline{) 2 \quad -3 \quad -12 \quad 1} \\ \underline{-4 \quad 2 \quad -20} \\ 2 \quad -10 \end{array} \quad \boxed{-19 = f(2)}$$

$$\text{ABS MIN}$$

$$\begin{array}{r} 3 \overline{) 2 \quad -3 \quad -12 \quad 1} \\ \underline{-6 \quad 9 \quad -9} \\ 2 \quad -3 \quad -3 \end{array} \quad \boxed{-8 = f(3)}$$

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16  $f(t) = t\sqrt{4-t^2} \quad [-1, 2]$

$$= t(4-t^2)^{\frac{1}{2}}$$

$$\Rightarrow f'(t) = \sqrt{4-t^2} + t \left( \frac{1}{2}(4-t^2)^{-\frac{1}{2}} \right) (-2t)$$

$$= \sqrt{4-t^2} - \frac{t^2}{\sqrt{4-t^2}} = \frac{4-t^2-t^2}{\sqrt{4-t^2}} = \frac{4-2t^2}{\sqrt{4-t^2}}$$

$$4-2t^2=0$$

$$2t^2=4$$

$$t^2=2$$

$$t = \pm\sqrt{2}$$

$$t = +\sqrt{2} \in [-1, 2]$$

$$\sqrt{4-t^2}=0$$

$$4-t^2=0$$

$$t = \pm 2$$

$$t = +2 \in [-1, 2]$$

$$f(-1) = -1\sqrt{4-(-1)^2} = -1\sqrt{3} = -\sqrt{3}$$

MIN  
 $(-1, -\sqrt{3})$

$$f(\sqrt{2}) = \sqrt{2}\sqrt{4-\sqrt{2}^2} = \sqrt{2}\sqrt{4-2} = \sqrt{2}\sqrt{2} = 2$$

MAX  
 $(\sqrt{2}, 2)$

$$f(2) = 2\sqrt{4-2^2} = 2\sqrt{0} = 0 \quad (2, 0)$$