201 §2.9

Find Linearization @ a.

1 $f(x) = x^4 + 3x^2$ @ $a=1$

$f'(x) = 4x^3 + 6x \Longrightarrow f'(1) = 4 + 6 = 10 = m$

$\Longrightarrow L_1(x) = 10(x-1) + y_1$

$y_1 = f(a) = 1^4 + 3(1)^2 = 1 + 3 = 4 = y_1 \Longrightarrow$

$\boxed{L_1(x) = 10(x-1) + 4}$

2 $f(x) = \sqrt{x}$, $a = 4$ $\quad f(a) = f(4) = \sqrt{4} = 2 = y_1$

$f(x) = x^{\frac{1}{2}} \longrightarrow f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} \Longrightarrow$

$f'(a) = f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{2 \cdot 2} = \frac{1}{4} = m$

$y = L_1(x) = m(x - x_1) + y_1$

$\boxed{L_1(x) = \frac{1}{4}(x-4) + 2}$

3 Use $g(x) = \sqrt{x+1}$ @ $a=0$ to approximate $\sqrt{1.1}$ and $\sqrt{.95}$; (In class $a=1$ and $\sqrt{x}$)

$g(x) = (x+1)^{\frac{1}{2}} \Longrightarrow g'(x) = \frac{1}{2}(x+1)^{-\frac{1}{2}} = \frac{1}{2\sqrt{x+1}}$

@ $a=0$, $g(a) = \sqrt{0+1} = \sqrt{1} = 1 = y_1 \qquad a = 0 = x_1$

& $g'(0) = \frac{1}{2\sqrt{1}} = \frac{1}{2} = m$

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$y = m(x - x_1) + y_1$

$L_1(x) = \frac{1}{2}(x - 0) + 1$

$L_1(x) = \frac{1}{2}x + 1$

$L_1(.95) = \frac{1}{2}(.95) + 1 = .475 + 1 = \boxed{1.475 \approx \sqrt{.95}}$

$L_1(1.1) = \frac{1}{2}(1.1) + 1 = .55 + 1 = \boxed{1.55 \approx \sqrt{1.1}}$

ACTUAL: $\sqrt{.95} \approx .9746794345$ & $\sqrt{1.1} \approx 1.048808848$

Calculator

Oops! What's wrong with this?

It's just unnatural to use $\sqrt{x+1}$

$x + 1 = .95 \Rightarrow$

$x = -.05$ is what should go in for

$\sqrt{.95} = \sqrt{-.05 + 1}$

$x + 1 = 1.1 \Rightarrow$

$x = 1.1 - 1 = .1$ is what should go in for $\sqrt{1.1}$

So $L_1(-.05) = \frac{1}{2}(-.05) + 1 = -.025 + 1$

$= \boxed{.975 \approx \sqrt{.95}}$

& $L_1(.1) = \frac{1}{2}(.1) + 1 = .05 + 1 = \boxed{1.05 \approx \sqrt{1.1}}$

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#s 11–14 Find the differential of each func.

4 $y = x^2 \sin(2x)$

$\Rightarrow dy = \boxed{\left[2x\sin(2x) + 2x^2\cos(2x)\right]dx}$

5 $y = \sqrt{1+t^2} = (t^2+1)^{\frac{1}{2}}$

$\Rightarrow dy = \left[\frac{1}{2}(t^2+1)^{-\frac{1}{2}}(2t)\right]dt$

$= \boxed{\frac{t}{\sqrt{t^2+1}}\,dt = dy}$

6 $y = \frac{s}{2s+1} \Rightarrow dy = \frac{2s+1-(s)(2)}{(2s+1)^2}\,ds$

$= \boxed{\frac{ds}{(2s+1)^2} = dy}$

7 $y = u\cos(u) \longrightarrow$

$\boxed{dy = \left[\cos(u) - u\sin(u)\right]du}$

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Compute $\Delta y$ & $dy$ for given values of $x$ & $\Delta x$. Then draw the picture

8 $y = 2x - x^2$, $x=2$, $\Delta x = -.4$

$y(2) = 2(2) - 2^2 = 0$

$y(2+\Delta x) = y(2-.4) = y(1.6) = 2(1.6) - 1.6^2$

$= 3.2 - 2.56 = .64$

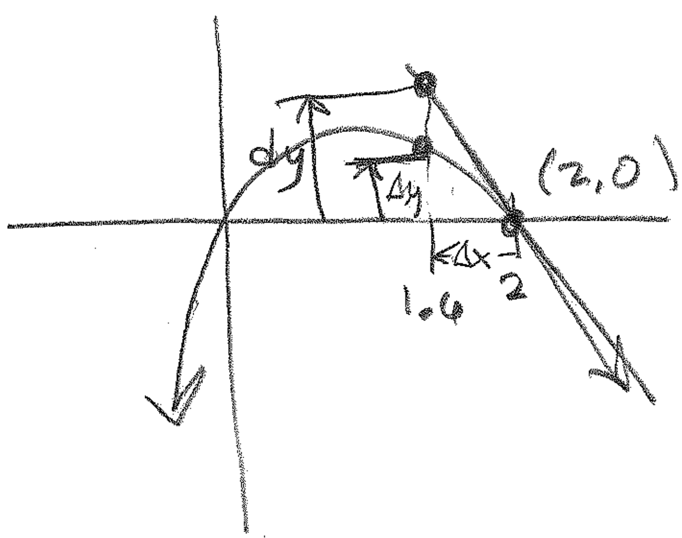
$\Delta y = y(1.6) - y(2) = .64 - 0 = \boxed{.64 = \Delta y}$

$y' = 2 - 2x$

$y'(2) = 2 - 4 = -2$

$\Rightarrow dy = y'dx = -2(-.4) = .8$

$-x^2 + 2x = -x(x-2)$



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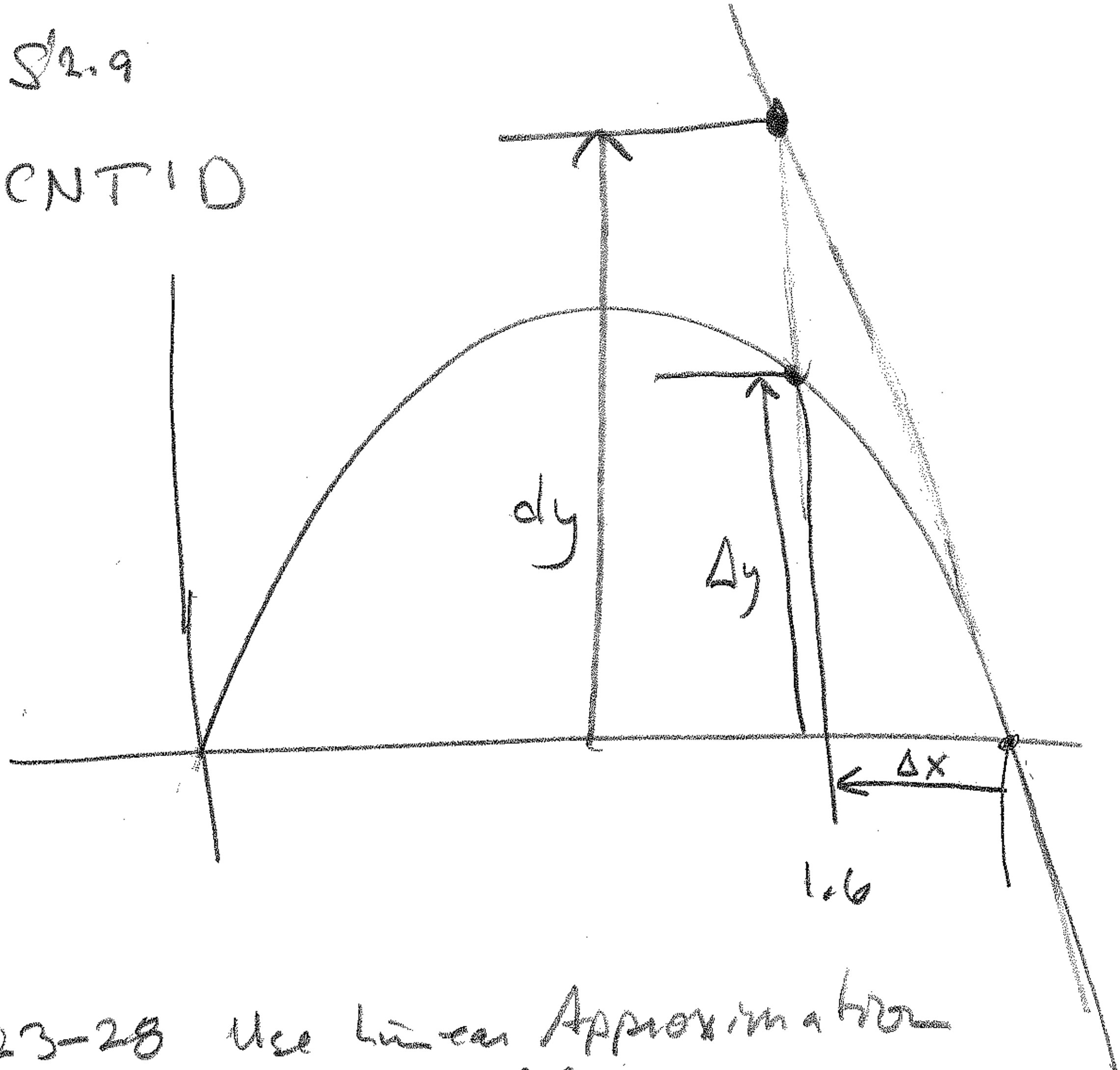
8 CNT'D




#s 23–28 Use Linear Approximation to estimate the following.

9 $(1.999)^4$ $\quad f(x) = x^4 \quad x = 2$

$f'(x) = 4x^3 \quad \Delta x = 1.999 - 2 = -.001$

$f(2) = 2^4 = 16 = y_1$

$f'(2) = 4(2)^3 = 32 = m$

$\therefore \; L(x) = m(x - x_1) + y_1$

$= f'(2)(x-2) + 16$

$= 32(x-2) + 16 \longrightarrow L(1.999)$

$\longrightarrow L(1.999) = 32(-.001) + 16 = -.032 + 16$

$= \boxed{15.968 \approx 1.999^4}$

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10 $\frac{1}{4.002}$ Looks like $\frac{1}{x}$ situation

$x_1 = 4$ $\Longrightarrow$ $\Delta x = .002 = x_2 - x_1$
$x_2 = 4.002$

$f(x) = \frac{1}{x} = x^{-1} \longrightarrow f'(x) = -x^{-2} = -\frac{1}{x^2} \Rightarrow$

$f'(4) = -\frac{1}{4^2} = -\frac{1}{16}$ & $f(4) = \frac{1}{4}$

$\therefore$ $L(x) = -\frac{1}{16}(x-4) + \frac{1}{4}$
$= f'(4)(x-4) + f(4)$
$= m(x - x_1) + y_1$

Extra writing a LEARNER should do.

$\therefore$ $L(4.002) = -\frac{1}{16}(4.002 - 4) + \frac{1}{4}$
$= -\frac{1}{16}(.002) + \frac{1}{4}$
$= .249875$, exactly

Calculator sez: $= .2498750625$, exactly.

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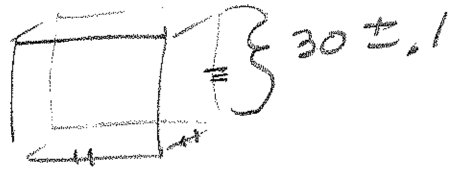
11 Edge of a cube is 30 cm, ± .1 cm.
(Use differentials to)(Estimate the error, relative error and percentage error in
... (a) volume
(b) surface area




11a $V = x^3 \longrightarrow$

$dV = 3x^2 dx$

$\longrightarrow \Delta V \approx 3(30)^2(\pm.1) = (2700)(\pm.1) = \boxed{\pm 270 \text{ cm}^3}$

Relative Error $= \dfrac{\Delta V}{V} \approx \dfrac{\pm 270}{30^3} = \pm \dfrac{270}{27000}$

$= \boxed{\pm .01}$ unitless measure

Percent Error $= (\pm.01)(100\%) = \boxed{\pm 1\%}$

11b $S = 6x^2$

$dS = 12x\,dx = 12(30)(\pm.1) = \boxed{\pm 36 \text{ cm}^2}$

$\boxed{\approx \Delta S}$

Relative Error $= \dfrac{\Delta S}{S} \approx \dfrac{dS}{S} = \dfrac{\pm 36}{6(30)^2}$

$= \dfrac{\pm 36}{5400} = \boxed{\pm .00\overline{6} \approx \dfrac{\Delta S}{S}}$

Percent Error $= \left(\dfrac{\Delta S}{S}\right)(100\%) = \boxed{\pm .\overline{6}\ \%}$

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12 (Use differentials to!) Estimate the amount of paint needed to apply a coat that is 0.05 cm thick to a hemispherical dome of diameter 50 m.

This is a change in volume deal, although it might at first seem like a surface area thing.

HEMI sphere

D = 50 $\Rightarrow r = 25$!

$$V = \frac{1}{2}\cdot\frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$$

$$dV = 2\pi r^2 dr = 2\pi(25)^2 \cancel{(.05)}$$

$$= 62.5\pi \ \ldots \text{ OOPS!}$$

r is meters.

.05 is cm! $(.05\text{ cm})\left(\frac{1\text{ m}}{100\text{ cm}}\right) = .0005\text{ m}$

$$dV = 2\pi(25)^2(.0005)\pi = .625\pi = \frac{5\pi}{8} \approx$$

$$\boxed{1.963495408\text{ m}^3}$$

$$\approx (1.963495408\text{ m}^3)\left(\frac{1\text{ gal}}{.003785411784\text{ m}^3}\right)$$

$\approx$ 519 gallons!

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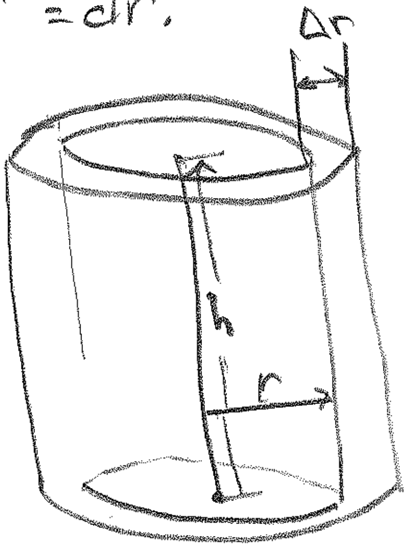
13 Consider a thin cylindrical shell of inner radius $r$, height $h$, and thickness $\Delta r = dr$.




13a Ans:

$V = \pi r^2 h = V(r)$

$\boxed{dV = 2\pi r h\, dr}$

Towards the end of the semester, this "cylindrical shell" method will figure prominently in finding volumes of bodies with circular cross-section. The delta-r will approach zero, and this estimate will approach exactitude.

(a) Estimate volume of the shell with a differential.

(b) What's the error in using a differential?

13b Ans: $V(r+\Delta r) = \pi (r+\Delta r)^2 h$

$\Delta V = V(r+\Delta r) - V(r)$

$= \pi h[r^2 + 2r\Delta r + (\Delta r)^2 - r^2]$

$= \pi h[2r\Delta r + (\Delta r)^2]$

$|\text{Error}| = |\Delta V - dV| = \pi h[2r\Delta r + (\Delta r)^2 - 2r\,dr]$ $(dr \equiv \Delta r)$

$= \pi h[(\Delta r)^2] = \boxed{\pi h (\Delta r)^2}$

Error Associated w/ Differential.