201 §2.8

1 Each side of a square is increasing @ 6 cm/s. At what rate is the area of the square increasing, when the area of the square is 16 cm^2?

$x = x(t)$ = length of a side in cm, as a function of

t = time in s.

$$A = A(x) = A(x(t)) = x^2 \implies$$

$$\frac{dA}{dt} = 2x\frac{dx}{dt} \implies$$

$$\left.\frac{dA}{dt}\right|_{A=16} = \left.\frac{dA}{dt}\right|_{x=4} = 2(4)(6) = 48\ cm^2/s$$

2 Cylindrical tank w/ radius $r = 5$ cm is being filled w/ water at a rate of 3 m^3/min. How fast is the height of H_2O rising?

$$V = \pi r^2 h = \pi(5)^2 h = 25\pi h$$

$$\frac{dV}{dt} = 25\pi\frac{dh}{dt} \overset{SET}{=} 3 \implies$$

$$\frac{dh}{dt} = \frac{3}{25\pi}\ cm/min$$

$$\approx .0381971863\ cm/min$$

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3 § $y=\sqrt{2x+1}$ where x & y are funcs of t.

(a) If $\frac{dx}{dt}=3$, find $\frac{dy}{dt}$ when $x=4$

$$y=(2x+1)^{\frac{1}{2}} \Longrightarrow \frac{dy}{dt}=\frac{1}{2}((2x+1))^{-\frac{1}{2}}\cdot 2\frac{dx}{dt}.$$

$$\Longrightarrow \left.\frac{dy}{dt}\right|_{x=4}=\frac{1}{\cancel{2}}(2(4)+1)^{-\frac{1}{2}}\cdot\cancel{2}\cdot 3$$

$$=\frac{3}{\sqrt{9}}=\frac{3}{3}=\boxed{1}$$

4 § $x^2+y^2+z^2=9$, $\frac{dx}{dt}=5$, $\frac{dy}{dt}=4$.

Find $\left.\frac{dz}{dt}\right|_{(2,2,1)=(x,y,z)}$

$$2xx'+2yy'+2zz'=0$$

$$2(2)(5)+2(2)(4)+2(1)z'=0$$

$$2z'=-20-16=-36$$

$$z'=\boxed{\frac{dz}{dt}=-18}$$

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10 #s 11–14

(a) Quantities given?

(b) What's unknown?

(c) Picture?

(d) Equation(s)?

(e) Solve

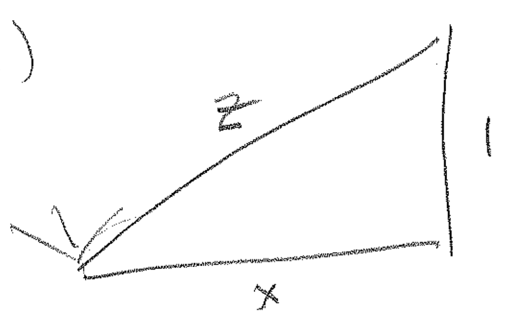
5 Plane flies horizontally @ 1 mi. altitude, and speed 500 mi/h, flying over radio sttn. Find rate at which distance to station is increasing when it's 2 mi from the station

(a) $\frac{dx}{dt} = 500 \ mi/h$ $\quad$ x = horizontal position

$y = 1, \ \frac{dy}{dt} = 0$

(b) Want $\frac{dz}{dt}$, when $z = 2$, where z = distance from plane to station

(c)



$\frac{dx}{dt} = 500 \longrightarrow$

(d) $x^2 + 1^2 = z^2$

(e) $2xx' = 2zz'$

$2(x)(500) = 2(2)z'$

$z = 2 \Longrightarrow x^2 + 1 = 2^2$

$x^2 = 3$

$x = \sqrt{3} \Longrightarrow$

$2\sqrt{3}(500) = 4z' \Longrightarrow \boxed{z' = 250\sqrt{3} \ mi/hr}$

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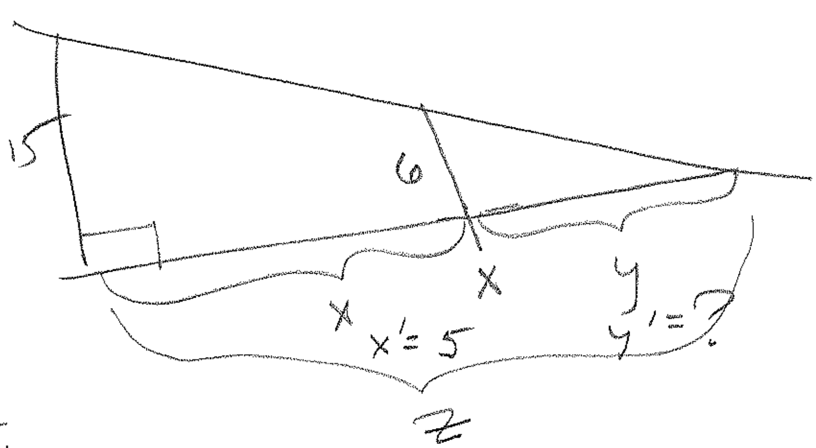
6 A street light's mounted on a 15-ft pole. A man 6 ft tall walks away from the pole w/ speed of 5 ft/s along a straight path. How fast is the tip of ~~th~~ his shadow ~~growing~~, moving, when he's 40 ft from the pole?

(a) $\frac{dx}{dt} = 5$ ft/s. man's 6 ft, pole's 15 ft.

(b) want $\frac{dz}{dt}$ = speed of the tip of shadow



(d) $\frac{15}{x+y} = \frac{6}{y}$

$15y = 6x + 6y$

(e) $15y' = 6x' + 6y'$

$9y' = 6x'$

$y' = \frac{6}{9}x' = \frac{2}{3}x' = \frac{2}{3}(5) = \frac{10}{3}$

$z' = (x+y)' = x' + y' = 5 + \frac{10}{3} = \boxed{\frac{25}{3} \text{ ft/s}}$

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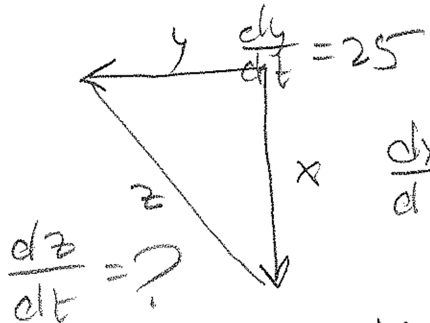
7

2 cars @ same start.

one South @ 60 mi/h,

one West @ 25 mi/h

At what rate is distance between them changing? When $t = 2$ hrs?



$\frac{dx}{dt} = 60$

$x^2 + y^2 = z^2$

$2xx' + 2yy' = 2zz'$

$x(2) = 120$ $\quad 2(120)(60) + 2(50)(25) = 2zz'$

$y(2) = 50$ $\quad 14400 + 2500 = 2zz'$

$50^2 + 120^2 = 2500 + 14400$ $\quad 16900 = 2zz'$

$= 16900$ $\quad 16900 = 2(130)z'$

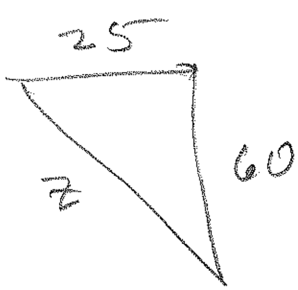
$= 130^2$ $\quad \frac{16900}{260} = z'$

$z = 130$

$\boxed{z' = 65 \text{ mi/hr}}$

Alternate:

rates are constant:



$25^2 + 60^2 = 625 + 3600 = 4225$

$= z^2$ (OR $\left(\frac{dz}{dt}\right)^2$)

$\rightarrow z = 65$ in 1 hr $\rightarrow$

$\frac{dz}{dt} = 65$ only works b/c speeds are held constant.

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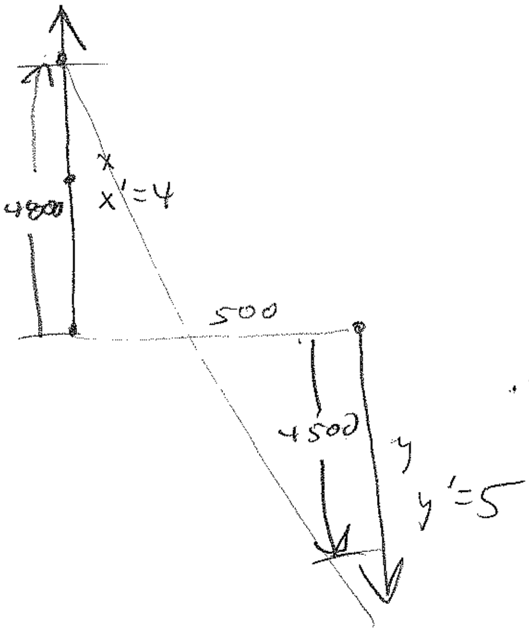
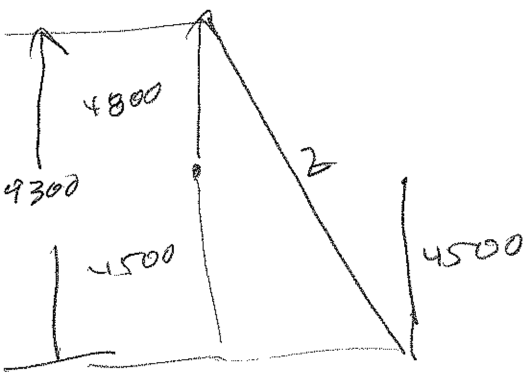
8

$(20)(4)(60)$
$= 4800 \text{ ft}$



$.5(5)(60) = 300(15) = 4500$



$x = 4t$

$y = 5(t-5)$

$(x+y)^2 + 500^2 = z^2$

$2(x+y)(x'+y') = 2zz'$

Check @ $t-5 = 15$ &

$x = 4800 \qquad x' = 4$

$y = 4500 \qquad y' = 5$

$z = 9300$

$\longrightarrow 2(9300)(4+5) = 2(9300)z'$

$18 = 2z'$

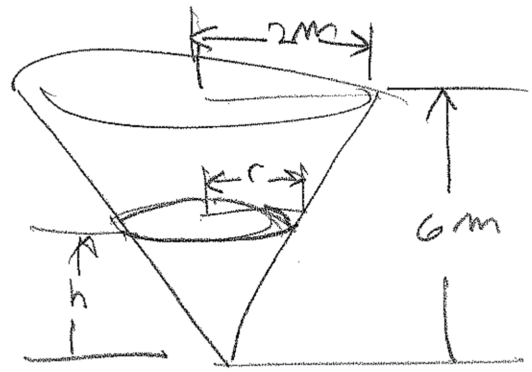
$\boxed{9 = z'}$

I get EXACTLY 9.

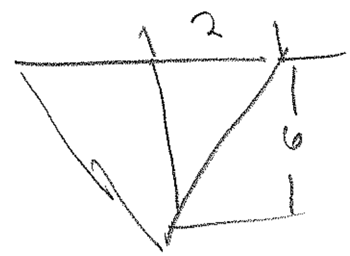
201 §2.8 #23

9) Water leaking out @ 10,000 cm^3/min.

Water coming in @ $\frac{dI}{dt} = I' =$ In take.



Given $\left.\frac{dh}{dt}\right|_{h=200} = 20$ cm/min



$\frac{r}{h} = \frac{2}{6}$

$r = \frac{1}{3}h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{1}{3}h\right)^2 h$

$= \frac{1}{27}\pi h^3 \Longrightarrow$

$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt}$

$= \frac{1}{9}\pi (200)^2 (20) = \frac{800000\pi}{9}$

We have In $-$ Out $= \frac{800000\pi}{9} \Longrightarrow$

In $- 10000 = \frac{800,000\pi}{9} \Longrightarrow$

In $= \frac{800,000\pi}{9} - 10000 \approx 269252.6803$ cm^3/min