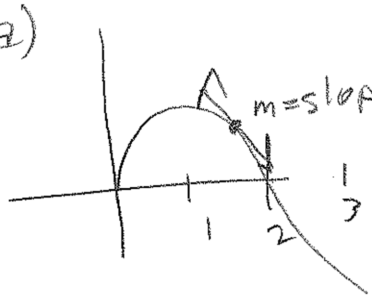
201 §2.7

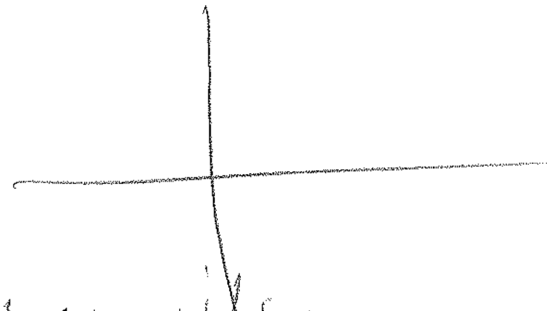
1 Graphs of 2 velocity funcs.
when is each slowing / speeding

(a)



m = slope of velocity func = acceleration

(b)



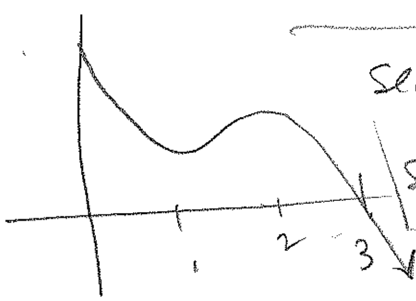
Slowing down: Acceleration is negative

Acceleration $= a = \frac{dv}{dt}$

on $\boxed{(1,3) \text{ is slowing}}$

Speeding up: $(0,1)$ tangent line has positive slope.

(b)



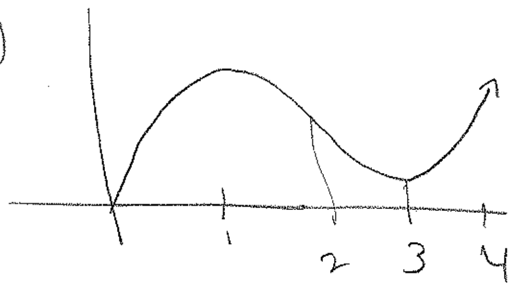
$\boxed{\begin{array}{l} \text{Slowing: } (0,1) \cup (2,3) \\ \text{Speeding-up: } (1,2) \end{array}}$

201 §2.7

2

Graphs o POSITION funcs

(a)

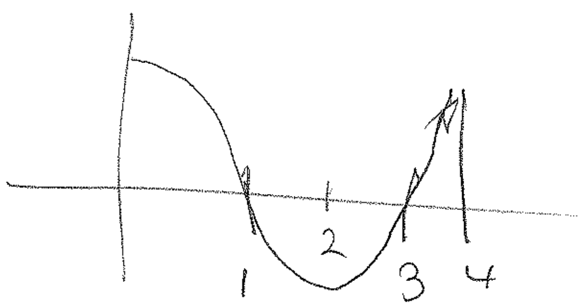


1st Look @ concavity

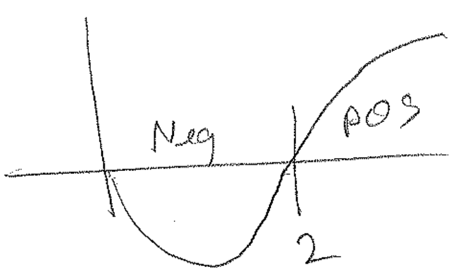
Speeding-up : (2,4) concave up

slowing-down : (0,2) concave down.

Look @ velocity :



Look @ accelerations :



slowing accelerating

201 § 2.7

3

$v_0 = +80$ ft/s

$s_0 = 0$

$s(t) = \frac{1}{2}at^2 + v_0 t + s_0$

$= -\frac{32}{2}t^2 + 80t + 0$

$= -16t^2 + 80t$

$s(t)$ = height

$v(t) = s'(t)$ = velocity

$a(t) = s''(t) = v'(t)$

= acceleration

(a) Max height : $s'(t) = 0$

$-32t + 80 = 0$

$t = \frac{80}{32} = \frac{5}{2}$ s

$s(\frac{5}{2})$:

$$\begin{array}{c|ccc} \frac{5}{2} & -16 & 80 & 0 \\ & & -40 & 100 \\ \hline & -16 & 40 & 100 \end{array}$$

$\boxed{100 = s(\frac{5}{2})}$

100 ft is max ht.

(b) Find velocity when $s(t) = 96$ on its way up/ down.

$s(t) = 96$

$-16t^2 + 80t = 96$

$-16t^2 + 80t - 96 = 0$

$t^2 - 5t + 6 = 0$

$t = 2, 3$

$s'(2) = v(2) = -32(2) + 80$

$= -64 + 80$

$= \boxed{16 \text{ ft/s}}$ on way up.

$s'(3) = -32(3) + 80$

$= \boxed{-16 \text{ ft/s}}$ on way down

§ 2.7

4) Computer chips in square wafers. Wants to keep side length close to 15 mm and wants to know how the area $A(x)$ changes when side length x changes

(a) Find $A'(15)$ & explain its meaning.

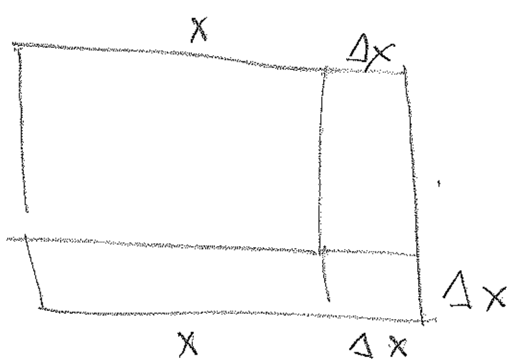
$A(x) = x^2$ $A(x)$ in cm^2, x in cm

$A'(x) = 2x \Longrightarrow A'(15) = 30 \; cm^2/cm$

This means rate of Area increase wrt length of a side is $30 \; cm^2/cm$.

(b) The rate of change of area wrt length of a side is $\frac{1}{2}$ its perimeter, $4x$, since

$$A'(x) = 2x = \tfrac{1}{2}[4x] = \tfrac{1}{2}[\text{perimeter}]$$



Geometrically, you can see the area increasing by an amount

$x\Delta x + x\Delta x + (\Delta x)^2$
$= 2x\Delta x + (\Delta x)^2$

If $\Delta x = 1$, then we get $2x+1$, close to $2x$

If Δx = small, we get $2x\Delta x + (\Delta x)^2$, which, neglecting $(\Delta x)^2$ as small is a <u>lot</u> like what we'll do in §2.9!

201 §2.7

5

(a) Spherical balloon $V = \frac{4}{3}\pi r^3$, where radius ~~at~~ r is in μm. Find AVG RATE OF CHANGE in V when r changes from

(i) 5 μm to 8 μm

(ii) 5 to 6 μm

(iii) 5 to 5.1 μm

(i) $\dfrac{V(8) - V(5)}{8-5} = \dfrac{\frac{4}{3}\pi\left[8^3 - 5^3\right]}{3} = \dfrac{\frac{4}{3}\pi\left[(8-5)(8^2+40+25)\right]}{3}$

$= \frac{4}{9}\pi\left[3(64+65)\right] = \left(\frac{4}{9}\right)(3)\pi\left[129\right] = \frac{4}{3}\pi\left[129\right]$

$= 4\pi[43] = 172\pi \ \mu m^3/\mu m \approx 540.354 \ \mu m^3/\mu m$

(ii) $\dfrac{V(6)-V(5)}{6-5} = \dfrac{\frac{4}{3}\pi\left[6^3-5^3\right]}{6-5} = \frac{4}{3}\pi\left[(6-5)(6^2+30+5^2)\right]$

$= \frac{4}{3}\pi\left[36+30+25\right] = \frac{4}{3}\pi\left[66+25\right] = \frac{4}{3}\pi\left[91\right] = \dfrac{364\pi}{3} \ \mu m^3/\mu m$

$\approx 381.1799 \ \mu m^3/\mu m$

(iii) $\dfrac{V(5.1)-V(5)}{.1} = \dfrac{\frac{4}{3}\pi\left[5.1^3-5^3\right]}{.1} = 10\left(\frac{4}{3}\pi\right)(7.651)$

~~ft~~ $= \dfrac{306.04\pi}{3} \ \mu m^3/\mu m \approx 320.484 \ \mu m^3/\mu m$

201 §2.7 #15

(15) A spherical balloon's ~~volume is $V = \frac{4}{3}\pi r^3$ ft^3,~~
~~where r is radius in feet~~ #16 stuff.

Surface area is $4\pi r^2 = S$ in ft^2

Find rate of increase in S.A. when $r =$

(a) 1 ft : $\left.\frac{dS}{dr}\right|_{r=1} = 8\pi r\Big|_{r=1} = 8\pi$

(b) 2 ft : $\left.\frac{dS}{dr}\right|_{r=2} = 8\pi(2) = 16\pi$

(c) 3 ft : $\left.\frac{dS}{dr}\right|_{r=3} = 8\pi(3) = 24\pi$

Rate of increase is, apparently a linear function! $(1, 8\pi)$, $(2, 16\pi)$, $(3, 24\pi)$

$m = 8\pi$!