201 §2.6

(a) Find y' by implicit differentiation

(b) Solve for y explicitly & find y'

(c) Check your sol'ns (compare.)

1 (a) $9x^2 - y^2 = 1$

$$18x - 2yy' = 0$$

$$-2yy' = -18x$$

$$y' = \frac{9x}{y}$$

(b) $-y^2 = 1 - 9x^2$

$$y^2 = 9x^2 - 1$$

$$y = \pm\sqrt{9x^2 - 1} = \pm (9x^2 - 1)^{\frac{1}{2}}$$

$$\Rightarrow y' = \pm\tfrac{1}{2}(9x^2-1)^{-\frac{1}{2}}(18x) = \frac{9x}{\pm\sqrt{9x^2-1}} = \boxed{(c)\ \pm\frac{9x}{y}}$$

(c) See above

2 (a) $\frac{1}{x} + \frac{1}{y} = 1$

$$x^{-1} + y^{-1} = 1$$

$$-x^{-2} - y^{-2}y' = 0$$

$$-y^{-2}y' = -x^{-2}$$

$$\boxed{y' = \frac{y^2}{x^2}}$$

(b) $y + x = xy$

$$y - xy = x$$

$$y(1-x) = x$$

$$y = \frac{x}{1-x}$$

$$y' = \frac{1(1-x) - x(-1)}{(1-x)^2}$$

$$= \frac{1}{(1-x)^2} = \frac{x^2}{(1-x)^2} \cdot \frac{1}{x^2}$$

(c) $= \frac{y^2}{}$

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Find y' by implicit differentiation $\xrightarrow{\quad} \frac{dy}{dx}$

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$$x^3 + y^3 = 1$$

$$3x^2 + 3y^2 y' = 0$$

$$3y^2 y' = -3x^2$$

$$\boxed{y' = -\frac{x^2}{y^2}}$$

4

$$x^2 + xy - y^2 = 4$$

$$2x + y + xy' - 2yy' = 0$$

$$y'(x - 2y) = -2x - y$$

$$\boxed{y' = \frac{-2x - y}{x - 2y}}$$

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$$x^4(x + y) = y^2(3x - y)$$

$$4x^3(x + y) + x^4(1 + y') = 2yy'(3x - y) + y^2(3 - y')$$

$$4x^4 + 4x^3 y + x^4 + x^4 y' = 6xyy' - 2y^2 y' + 3y^2 - y^2 y'$$

$$x^4 y' - 6xyy' + y^2 y' = 3y^2 - 4x^4 - 4x^3 y + x^4$$

$$y'(x^4 - 6xy + y^2) = 3y^2 - 3x^4 - 4x^3 y$$

$$\boxed{y' = \frac{3y^2 - 3x^4 - 4x^3 y}{x^4 - 6xy + y^2}}$$

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6 $y\cos x = x^2 + y^2$

$$y'\cos x - y\sin x = 2x + 2yy'$$

$$y'\cos x - 2yy' = 2x + y\sin x$$

$$\boxed{y' = \frac{2x + y\sin x}{\cos x - 2y}}$$

7 $4\cos x \sin y = 1$

$$(-4\sin x)(\sin y) + (4\cos x)(\cos y)y' = 0$$

$$y' = \frac{-4\sin x \sin y}{4\cos x \cos y} = \boxed{-\tan x \tan y}$$ to be cute.

8 $\tan\left(\frac{x}{y}\right) = x + y$

$$\left(\sec^2\left(\frac{x}{y}\right)\right)\left(\frac{y - xy'}{y^2}\right) = 1 + y'$$

$$\sec^2\left(\frac{x}{y}\right)(y - xy') = y^2 + y^2y'$$

$$y\sec^2\left(\frac{x}{y}\right) - xy'\sec^2\left(\frac{x}{y}\right) = y^2 + y^2y'$$

$$-xy'\sec^2\left(\frac{x}{y}\right) = y^2 - y\sec^2\left(\frac{x}{y}\right)$$

$$\boxed{y' = \frac{y\sec^2\left(\frac{x}{y}\right) - y^2}{x\sec^2\left(\frac{x}{y}\right)}}$$

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$$\sqrt{xy} = 1 + x^2y$$

$$(xy)^{\frac{1}{2}} = 1 + x^2y$$

$$\tfrac{1}{2}(xy)^{-\frac{1}{2}}(y + xy') = 2xy + x^2y'$$

$$\tfrac{1}{2}(xy)^{-\frac{1}{2}}y + \tfrac{1}{2}(xy)^{-\frac{1}{2}}xy' = 2xy + x^2y'$$

$$\tfrac{1}{2}(xy)^{-\frac{1}{2}}xy' - x^2y' = 2xy - \tfrac{1}{2}y(xy)^{-\frac{1}{2}}$$

$$\boxed{y' = \frac{2xy - \frac{1}{2}y(xy)^{-\frac{1}{2}}}{\frac{1}{2}(xy)^{-\frac{1}{2}}x - x^2}}$$

Which could be given a workover, but to what purpose?

10

$$y\cos x = 1 + \sin(xy)$$

$$y'\cos x - y\sin x = (\cos(xy))(y + xy')$$

$$y'\cos x - y\sin x = y\cos(xy) + xy'\cos(xy)$$

$$y'\cos x - xy'\cos(xy) = y\cos(xy) + y\sin x$$

$$\boxed{y' = \frac{y\cos(xy) + y\sin x}{\cos x - x\cos(xy)}}$$

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11) If $f(x) + x^2(f(x))^3 = 10$ and $f(1) = 2$, what's $f'(1)$?

$$f'(x) + 2x\,(f(x))^3 + (x^2)\left(3(f(x))^2\right)(f'(x)) = 0$$

$$\Longrightarrow f'(x) + 3x^2 f'(x)(f(x))^2 = -2x\,(f(x))^3$$

$$\Longrightarrow f'(x) = \frac{-2x\,(f(x))^3}{1 + 3x^2(f(x))^2} \longrightarrow$$

$$\Longrightarrow f'(1) = \frac{-2(1)(2)^3}{1 + 3(1)^2(2)^2} = \frac{-16}{1+12} = \boxed{-\frac{16}{13}}$$

12) Treat y as independent variable and x as dependent. Then find dx/dy with implicit differentiation.

Phil

$$x^4y^2 - x^3y + 2xy^3 = 0$$

$$4x^3x'y^2 + x^4\cdot 2y - 3x^2x'y - x^3 + 2x'y^3 + 2x\cdot 3y^2 = 0$$

$$4x^3y^2x' - 3x^2yx' + 2x'y^3 = -2x^4y + x^3 - 6xy^2$$

$$x'(4x^3y^2 - 3x^2y + 2y^3) = -2x^4y + x^3 - 6xy^2$$

$$\boxed{x' = \frac{-2x^4y + x^3 - 6xy^2}{4x^3y^2 - 3x^2y + 2y^3}}$$

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Use implicit differentiation to find an eq'n of the tangent line to the curve at the given point.

13 $y \sin(2x) = x\cos(2y)$ @ $\left(\frac{\pi}{2}, \frac{\pi}{4}\right)$

$$y'\sin(2x) + y(\cos(2x))(2) = \cos(2y) - x(\sin(2y))(2y')$$

$$y'\sin(2x) + 2xy'\sin(2y) = \cos(2y) - 2y\cos(2x)$$

$$y' = \frac{\cos(2y) - 2y\cos(2x)}{\sin(2x) + 2x\sin(2y)} \longrightarrow$$

$$y'\Big|_{(\pi/2, \pi/4)} = \frac{\cos(2\cdot\frac{\pi}{4}) - 2\cdot\frac{\pi}{4}\cos(2\cdot\frac{\pi}{2})}{\sin(2\cdot\frac{\pi}{2}) + 2\cdot\frac{\pi}{2}\sin(2\cdot\frac{\pi}{4})}$$

$$= \frac{\cos(\frac{\pi}{2}) - \frac{\pi}{2}\cos(\pi)}{\sin(\pi) + \pi\sin(\frac{\pi}{2})} = \frac{0 - \frac{\pi}{2}\cdot(-1)}{0 + \pi} = \frac{1}{2}$$

$$\boxed{y = \frac{1}{2}\left(x - \frac{\pi}{2}\right) + \frac{\pi}{4}} = \frac{1}{2}x - \frac{\pi}{4} + \frac{\pi}{4} = \frac{1}{2}x$$

$$y = m(x - x_1) + y_1 \qquad y = \frac{1}{2}x \text{ !?}$$

14 $x^2 + xy + y^2 = 3$ @ $(1,1)$

$$2x + y + xy' + 2yy' = 0$$

$$xy' + 2yy' = -2x - y$$

$$y' = \frac{-2x - y}{x + 2y} \Longrightarrow y'\Big|_{(1,1)} = \frac{-2-1}{1+2} = \frac{-3}{3}$$

$$y = -1(x-1) + 1 \quad \text{STOP!}$$

$$\boxed{y = -x + 2}$$

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15 Find eq'n of tangent to ~~the~~ the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad @ \quad (x_0, y_0)$$

$$\frac{1}{a^2}x^2 - \frac{1}{b^2}y^2 = 1$$

$$\frac{2}{a^2}x - \frac{2}{b^2}yy' = 0$$

$$-\frac{2}{b^2}yy' = -\frac{2}{a^2}x$$

$$y' = \frac{-\frac{2}{a^2}x}{-\frac{2}{b^2}y} = \frac{b^2}{a^2} \cdot \frac{x}{y} \longrightarrow$$

$$y'\Big|_{(x_0, y_0)} = \frac{b^2}{a^2} \cdot \frac{x_0}{y_0} = \frac{b^2 x_0}{a^2 y_0} \longrightarrow$$

$$\boxed{y = \frac{b^2 x_0}{a^2 y_0}(x - x_0) + y_0}$$ IS FINE BY ME

$$y = \frac{b^2 x_0}{a^2 y_0}x - \frac{b^2 x_0^2}{a^2 y_0} + y_0$$

OR

$$y = \frac{b^2}{a^2}\frac{x_0}{y_0}x + \frac{a^2 y_0^2 - b^2 x_0^2}{a^2 y_0}$$

Book Says $\frac{x_0 x}{a^2} - \frac{y_0 y}{b^2} = 1$ Are these the same?

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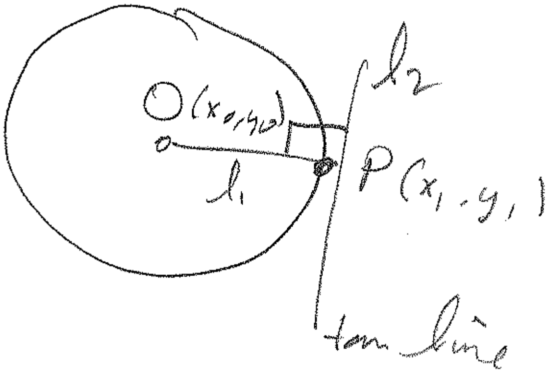
16 Show, using implicit differentiation, that any tangent to a point p is perpendicular to the radius OP



l_1: $m_1 = \dfrac{y_1 - y_0}{x_1 - x_0}$

We see that $m_2 = -\dfrac{1}{m_1}$, if we take the center to be $(x_0, y_0) = (0,0)$.

$x^2 + y^2 = r^2$

$2x + 2yy' = 0$

$y'\Big|_{(x_1,y_1)} = -\dfrac{x}{y}\Big|_{(x_1,y_1)} = -\dfrac{x_1}{y_1} = m_1$

$m_1 = -\dfrac{x_1}{y_1}$

$m_2 = \dfrac{y_1}{x_1}$

So we could make this argument for a circle centered at the origin. Then argue that the geometry's the same, regardless of where the circle is moved to.

Anybody else have a good argument?