201 §2.5

Write as $f(g(x))$

1 $y = \sqrt[3]{1+4x}$ Let $f(u) = \sqrt[3]{u}$

$y' = \frac{1}{3}(4x+1)^{-2/3}(4)$ $g(x) = 4x+1$

2 $y = \tan(\pi x)$ $f(u) = \tan u$

$g(x) = \pi x$

$y' = (\sec^2(\pi x))(\pi)$

3 $y = \sqrt{\sin x} = (\sin x)^{\frac{1}{2}}$ $f(u) = \sqrt{u} = u^{\frac{1}{2}}$

$g(x) = \sin x$

$y' = \left(\frac{1}{2}(\sin x)^{-\frac{1}{2}}\right)(\cos x)$

Find the derivative

4 $F(x) = (x^4+3x^2-2)^5 \longrightarrow$

$F'(x) = \left(5(x^4+3x^2-2)^4\right)(4x^3+6x)$

5 $F(x) = \sqrt{-2x+1} = (-2x+1)^{\frac{1}{2}} \longrightarrow$

$F'(x) = \frac{1}{2}(-2x+1)^{-1/2}(-2)$

6 $f(z) = \frac{1}{z^2+1} = (z^2+1)^{-\frac{1}{2}} \longrightarrow$

My -1/2 power should be -1 power. What was I thinking? Thanks, Greg Rupp.

$f'(z) = -\frac{1}{2}(z^2+1)^{-3/2}(2z)$

7 $y = \cos(x^3+a^3) \longrightarrow$

This one is poorly posed, because it's not clear which is the variable with respect to which we're differentiating. Author assumes it's *x*.

$y' = (-\sin(x^3+a^3))(3x^2)$

K

8 $y = x\sec(kx) \longrightarrow y' = 1\sec(kx) + x(\sec(kx)\tan(kx))(k)$

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9 $f(x) = (2x-3)^4(x^2+x+1)^5 \Longrightarrow$

$$f'(x) = \left(4(2x-3)^3(2)\right)(x^2+x+1)^5 + (2x-3)^4\left(5(x^2+x+1)^4(2x+1)\right)$$

$$(2x-3)^4\left(5(x^2+x+1)^4\right)(2x+1)$$

10 $h(x) = (t+1)^{\frac{2}{3}}(2t^2-1)^3 \Longrightarrow$

$$h'(x) = \tfrac{2}{3}(t+1)^{-1/3}(2t^2-1)^3 + (t+1)^{2/3}\left(3(2t^2-1)^2(4t)\right)$$

11 $y = \left(\dfrac{x^2+1}{x^2-1}\right)^3 \Longrightarrow$

$$y' = 3\left(\frac{x^2+1}{x^2-1}\right)^2\left(\frac{2x(x^2-1)-(x^2+1)(2x)}{(x^2-1)^2}\right)$$

12 $y = \sin(x\cos x) \Longrightarrow$

$$y' = \left(\cos(x\cos x)\right)\left(\cos x - x\sin x\right)$$

13 $F(z) = \sqrt{\dfrac{z-1}{z+1}} = \left(\dfrac{z-1}{z+1}\right)^{\frac{1}{2}} \Longrightarrow$

$$F'(z) = \frac{1}{2}\left(\frac{z-1}{z+1}\right)^{-\frac{1}{2}}\left(\frac{1(z+1)-(z-1)(1)}{(z+1)^2}\right)$$

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14 $y = \frac{r}{\sqrt{r^2+1}} = r(r^2+1)^{-\frac{1}{2}} \longrightarrow$

$$y' = \boxed{(r^2+1)^{-1/2} + r\left(-\frac{1}{2}(r^2+1)^{-3/2}\right)(2r)}$$

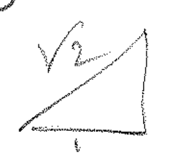
15 $y = \sin\sqrt{x^2+1} \longrightarrow$

$$y' = \left(\cos\sqrt{x^2+1}\right)\left(\frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x)\right)$$

16 $y = \sin(\tan(2x)) \longrightarrow$

$$y' = \cos(\tan(2x))\,(\sec^2(2x))(2)$$

17a (a) Find eq'n of tan line to the curve $y = \tan\left(\frac{\pi x^2}{4}\right)$ @ $(1, 1)$



$$y' = \left(\sec^2\left(\frac{\pi x^2}{4}\right)\right)\left(\frac{2\pi x}{4}\right) = \sec^2\left(\frac{\pi x^2}{4}\right)\left(\frac{\pi}{2}x\right)$$

$$y'(1) = \left(\sec^2\left(\frac{\pi}{4}\right)\right)\left(\frac{\pi}{2}\right) = (\sqrt{2})^2 \cdot \frac{\pi}{2} = \pi = m_{tan}$$

$$\boxed{y = \pi(x-1)+1}$$

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17b (b) Illustrate w/ graph (See Notes)

18 If g is twice-diff'bl and $f(x) = x\,g(x^2)$, find f'' in terms of g, g', g'':

$$f'(x) = g(x^2) + x\,g'(x^2)(2x) = g(x^2) + 2x^2 g'(x^2)$$

$$f''(x) = g'(x^2)\cdot 2x + 4x\,g'(x^2) + 2x^2 g''(x^2)(2x)$$

$$= \boxed{2x\,g'(x^2) + 4x\,g'(x^2) + 4x^3 g''(x^2)}$$

19 Find $D^{103}\cos(2x)$ by seeing the pattern.

$$D^1\cos(2x) = -2\sin(2x)$$

$$D^2\cos(2x) = -4\cos(2x) = -2^2\cos(2x)$$

$$D^3\cos(2x) = +8\sin(2x)$$

$$D^4\cos(2x) = +16\cos(2x)$$

$$D^5(\cos(2x)) = -32\cos(2x)$$

$103 = \underline{100} + 3$

$\downarrow$ (mult. of 4)

My guess:

$$+\boxed{2^{103}\sin(2x)}$$