

#51-16 Differentiate

$$1 \quad f(x) = 3x^2 - 2\cos x \rightarrow f'(x) = 6x + 2\sin x$$

$$2 \quad f(x) = \sin x + \frac{1}{2} \cot x \rightarrow f'(x) = \cos x - \frac{1}{2} \csc^2 x$$

$$3 \quad y = \sec \theta \tan \theta \rightarrow y' = \sec \theta \tan \theta \tan \theta + \sec \theta \sec^2 \theta \\ = \sec \theta \tan^2 \theta + \sec^3 \theta$$

$$4 \quad y = e \cos t + t^2 \sin t \rightarrow$$

$$y' = -e \sin t + 2t \sin t + t^2 \cos t$$

$$5 \quad y = \frac{x}{2 - \tan x} \rightarrow y' =$$

$$y' = \frac{1(2 - \tan x) - x(-\sec^2 x)}{(2 - \tan x)^2} = \frac{2 - \tan x + x \sec^2 x}{(2 - \tan x)^2}$$

$$6 \quad f(\theta) = \frac{\sec \theta}{1 + \sec \theta}$$

$$\rightarrow f'(\theta) = \frac{(\sec \theta \tan \theta)(1 + \sec \theta) - \sec \theta (\sec \theta \tan \theta)}{(1 + \sec \theta)^2}$$

$$= \frac{\sec \theta \tan \theta + \sec^2 \theta \tan \theta - \sec^2 \theta \tan \theta}{(1 + \sec \theta)^2}$$

$$= \frac{\sec \theta \tan \theta}{(1 + \sec \theta)^2}$$

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$$\boxed{7} \quad y = \frac{t \sin t}{1+t} \Rightarrow y' = \frac{(\sin t + t \cos t)(1+t) - (t \sin t)(1)}{(1+t)^2}$$

$$= \frac{\sin t + t \sin t + t \cos t + t^2 \cos t - t \sin t}{(1+t)^2}$$

$$= \frac{t^2 \cos t + t \cos t + \sin t}{(t+1)^2}$$

$$\boxed{8} \quad h(\theta) = \theta \csc \theta - \cot \theta \Rightarrow$$

$$h'(\theta) = \csc \theta - \theta \csc \theta \cot \theta - \csc^2 \theta$$

sign is wrong on this last term, courtesy of Greg Rupp.

$$\boxed{9} \quad \frac{d}{dx} (\csc x) = -\csc x \cot x$$

$$\boxed{PF} \quad \frac{d}{dx} [\csc x] = \frac{d}{dx} \left[\frac{1}{\sin x} \right] = \frac{0 \cdot \sin x - 1 \cdot \cos x}{\sin^2 x}$$

$$= \frac{-\cos x}{\sin^2 x} = -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} = \boxed{-\csc x \cot x}$$

$$\boxed{10} \quad \frac{d}{dx} [\cot x] = -\csc^2 x$$

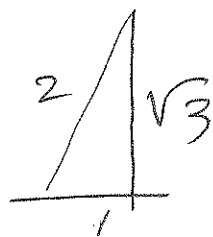
$$\boxed{PF} \quad \frac{d}{dx} \left[\frac{\cos x}{\sin x} \right] = \frac{-\sin x \sin x - \cos x \cos x}{\sin^2 x}$$

$$= -\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\csc^2 x$$

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#s 21-24 Find tan. line @ given (x_1, y_1) .

11 $y = \sec x, (\frac{\pi}{3}, 2)$



$$y' = \sec x \tan x \Rightarrow y'(\frac{\pi}{3}) = \sec \frac{\pi}{3} \tan \frac{\pi}{3} = 2 \cdot \sqrt{3} = m$$

$$\rightarrow \boxed{y = 2\sqrt{3}(x - \frac{\pi}{3}) + 2} \text{ does it}$$

$$= 2\sqrt{3}x - \frac{2\pi\sqrt{3}}{3} + 2$$

$$= 2\sqrt{3}x + \frac{6 - 2\pi\sqrt{3}}{3}$$

12 $y = \cos x - \sin x, (\pi, 1)$

$$y' = -\sin x - \cos x \rightarrow y'(\pi) = -\sin \pi - \cos \pi$$

$$= -0 - (-1)$$

$$= 1 = m$$

$$\boxed{y = 1(x - \pi) + 1}$$

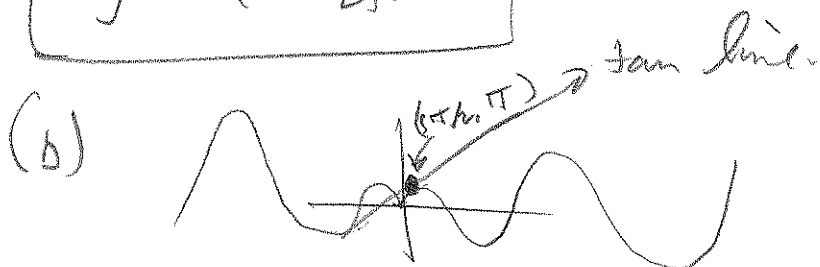
$$= x - \pi + 1$$

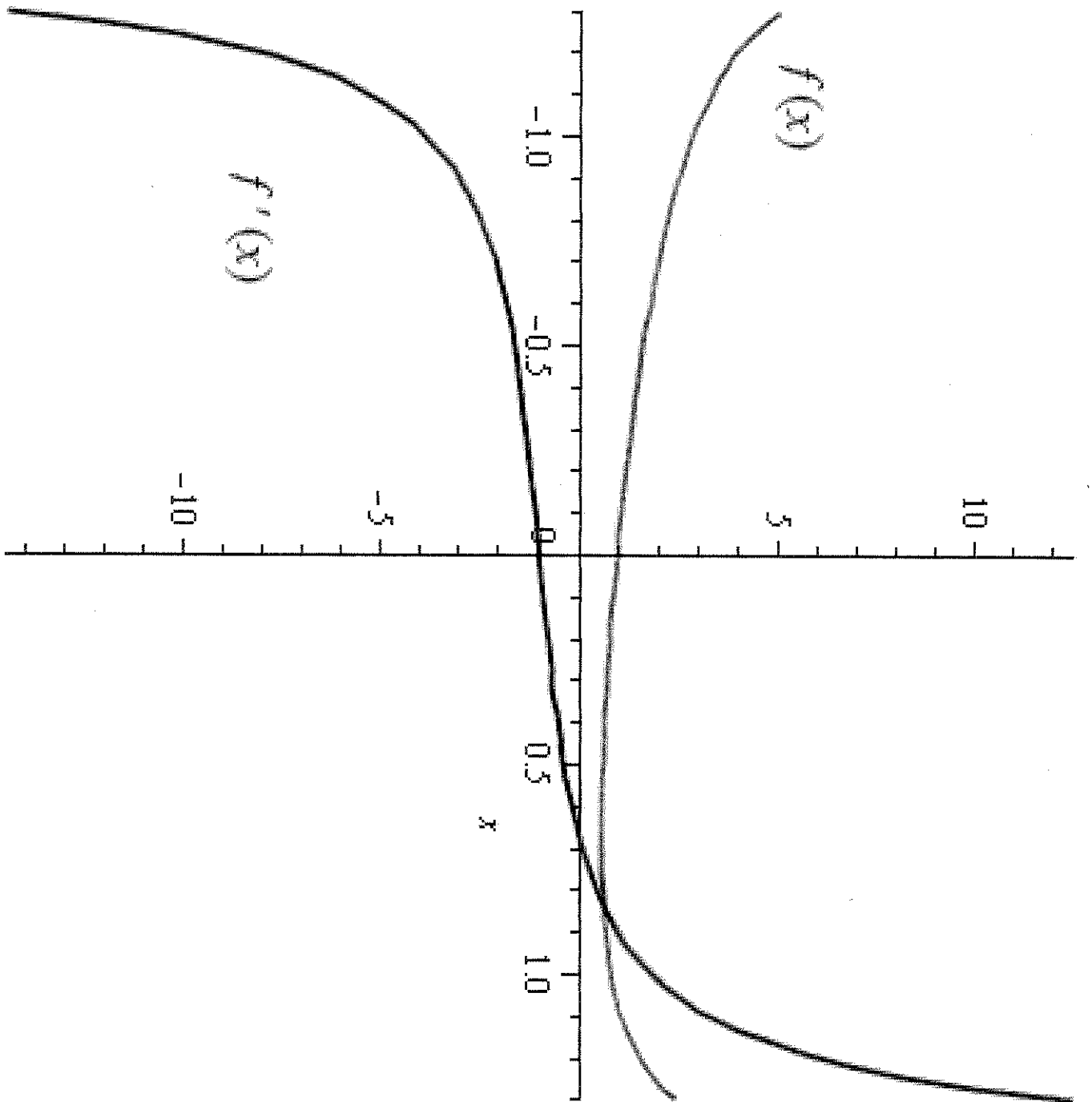
13 (a) Find eq'n of L tan line to $y = 2x \sin x$

(b) $(\frac{\pi}{2}, \pi)$: $y' = 2\sin x + 2x \cos x$

$$y'(\frac{\pi}{2}) = 2(1) + \frac{\pi}{2}(0) = 2$$

$$\boxed{y = 2(x - \frac{\pi}{2}) + \pi}$$



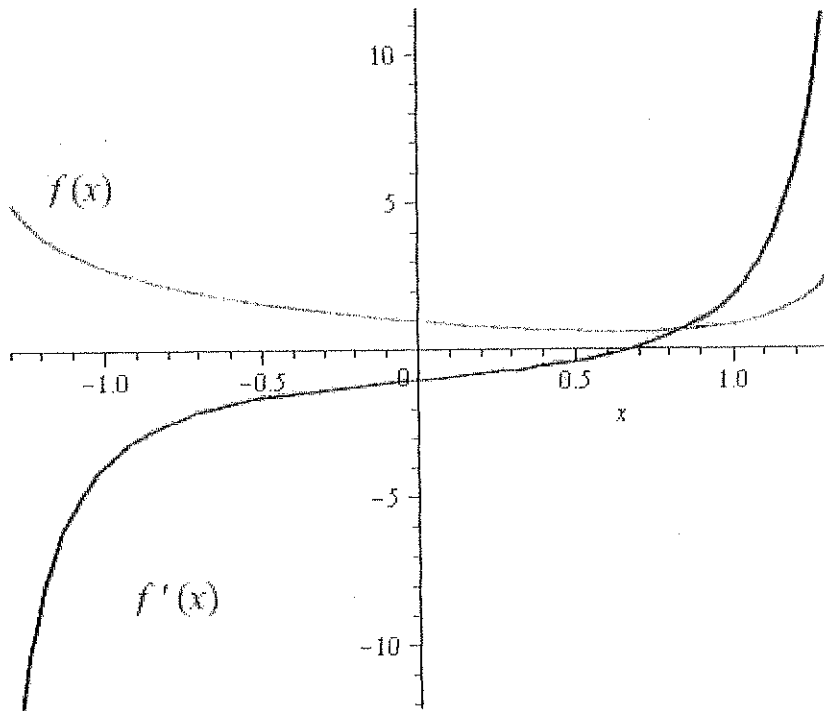


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a) $f(x) = \sec x - x \rightarrow f'(x) = \sec x \tan x - 1$

(c) Check w/ graphs of f, f' on $\{x \mid |x| < \frac{\pi}{2}\}$



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$H(\theta) = \theta \sin \theta, \quad \frac{d}{d\theta} \rightarrow$

$H'(\theta) = \sin \theta + \theta \cos \theta \quad \rightarrow$

$H''(\theta) = \cos \theta + \cos \theta - \theta \sin \theta$
 $= 2\cos \theta - \theta \sin \theta$

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(a) Quotient Rule $\frac{d}{dx} f(x) = \frac{\tan x - 1}{\sec x}$

$\rightarrow f'(x) = \frac{(\sec^2 x)(\sec x) - (\tan x - 1)(\sec x \tan x)}{\sec^2 x}$

$= \frac{\sec^2 x - \tan^2 x + \tan x}{\sec x}$

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$$\textcircled{31} \text{ (b) } f(x) = \frac{\tan x - 1}{\sec x} = \frac{\frac{\sin x}{\cos x} - 1}{\frac{1}{\cos x}}$$
$$= \frac{\cos x}{1} \cdot \frac{\sin x - \cos x}{\cos x} = \sin x - \cos x$$

$$\rightarrow f'(x) = \cos x + \sin x$$

(c) We show (a) & (b) are the same.

$$\frac{\sec^2 x - \tan^2 x + \tan x}{\sec x} = \frac{1 + \tan x}{\sec x} = \frac{1}{\sec x} + \frac{\tan x}{\sec x}$$
$$= \cos x + \tan x \cos x = \cos x + \frac{\sin x}{\cos x} \cos x$$

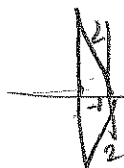
17 For what values of x does $x + 2\sin x$ have a horizontal tangent?

$$f'(x) = 1 + 2\cos x \stackrel{\text{SET}}{=} 0$$

$$\Rightarrow 2\cos x = -1$$

$$\Rightarrow \cos x = -\frac{1}{2}$$

~~SOLN: $\{x \mid x = 2\}$~~



$$\pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

$$\text{So, } \left\{ x \mid x = \frac{2\pi}{3} + 2n\pi \text{ OR } x = \frac{4\pi}{3} + 2n\pi, n \in \mathbb{Z} \right\}$$

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Mass on a spring vibrates horizontally on smooth level surface. Its eq'n of motion is $x(t) = 8 \sin t$, where t is secs & x is cm.

(a) Find Velocity = $x'(t) = 8 \cos t$ and Acceleration = $x''(t) = -8 \sin t$

(b) Find x, x', x'' @ $t = \frac{2\pi}{3}$

$$x\left(\frac{2\pi}{3}\right) = 8 \sin\left(\frac{2\pi}{3}\right) = 8 \cdot \frac{\sqrt{3}}{2} = 4\sqrt{3} \text{ cm}$$

$$x'\left(\frac{2\pi}{3}\right) = 8 \cos\left(\frac{2\pi}{3}\right) = 8 \left(-\frac{1}{2}\right) = -4 \text{ cm/s}$$

$$x''\left(\frac{2\pi}{3}\right) = -8 \sin\left(\frac{2\pi}{3}\right) = -8 \left(\frac{\sqrt{3}}{2}\right) = -4\sqrt{3} \text{ cm/s}^2$$

Find the limit.

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$$\lim_{x \rightarrow 0} \frac{\sin(3x)}{x} = \lim_{x \rightarrow 0} 3 \left(\frac{\sin(3x)}{3x} \right) = 3$$

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$$\lim_{x \rightarrow 0} \frac{\sin(3x)}{5x^3 - 4x} = \lim_{x \rightarrow 0} \frac{\sin 3x}{x} \cdot \frac{1}{5x^2 - 4}$$

$$= \lim_{x \rightarrow 0} \frac{\sin(3x)}{3x} \cdot \frac{3}{5x^2 - 4} = 1 \cdot \frac{3}{-4} = -\frac{3}{4}$$

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$$\lim_{x \rightarrow \infty} x \sin\left(\frac{1}{x}\right) = \lim_{x \rightarrow \infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}}$$

$$= \lim_{u \rightarrow 0} \frac{\sin u}{u} = \left[\frac{0}{0} \right]$$

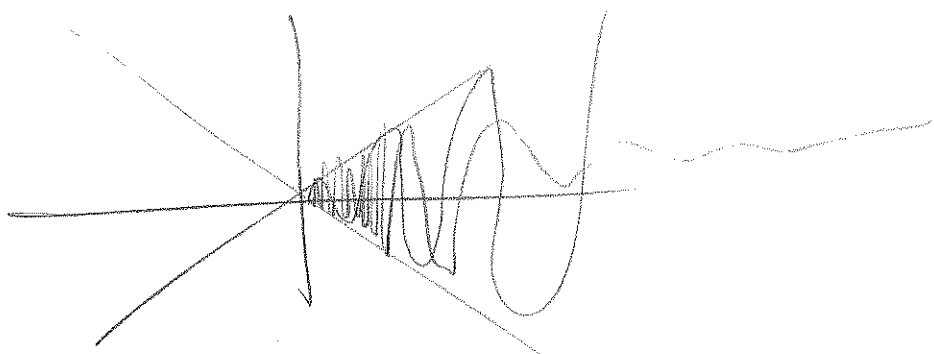
Not sure I'm buying this.
 Hmm $\sin\left(\frac{1}{x}\right) \rightarrow 0$ as $x \rightarrow \infty$ } Maybe so.

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$$\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right) = 0, \text{ by squeezing.}$$

$$-x \leq x \sin \frac{1}{x} \leq x$$

(c) Graph Lem



Hmmm. I'll put the graphs in the notes.

Close to $x=0$, $\sin \frac{1}{x}$ is ~~MMMMM~~

Dampen it by $x \cdot \sin \frac{1}{x}$ ~~MMMMM~~

Then look @ it as $x \rightarrow$ big

$$\sin\left(\frac{1}{x}\right) \rightarrow \sin(0) = 0$$

$$x \rightarrow \infty$$

It's an $\infty \cdot 0$ thing, which balances out (this time) @ $y=1$!